



ESDU 69011

Issued June 1969
Supersedes ESDU
Aero S.00.03.19

Heat transfer by forced convection between a two-dimensional turbulent boundary layer and a smooth flat plate, with application to wedges, cylinders and cones

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU Aero S.00.03.19, was monitored and guided by the Aerodynamics Committee. This Committee first met in 1942 and now has the following membership:

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HEAT TRANSFER BY FORCED CONVECTION BETWEEN A TWO-DIMENSIONAL TURBULENT BOUNDARY LAYER AND A SMOOTH FLAT PLATE, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

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HEAT TRANSFER BY FORCED CONVECTION BETWEEN A TWO-DIMENSIONAL TURBULENT BOUNDARY LAYER AND A SMOOTH FLAT PLATE, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

1. NOTATION AND UNITS

Any set of consistent units may be used. Two such sets are indicated below.

		SI	British
a	Reynolds analogy factor based on local coefficients, St/C_f		
a_m	Reynolds analogy factor based on mean coefficients, St_m/C_{fm}		
C_f	local skin friction coefficient		
C_{fm}	mean skin friction coefficient		
c_p	specific heat capacity at constant pressure	J/kg K	Chu/slug K
M_1	Mach number at edge of boundary layer		
M_∞	Mach number in uniform flow upstream		
q	local heat flux intensity	W/m ²	Chu/ft ² s
q_m	mean heat flux intensity	W/m ²	Chu/ft ² s
R_x	Reynolds number based on distance measured from the virtual origin of the boundary layer, $u_1 x / \nu_1$		
r	temperature recovery factor (taken as 0.89)		
St	local Stanton number, $q / \rho_1 u_1 c_p (T_{wr} - T_w)$		
St_m	mean Stanton number, $q_m / \rho_1 u_1 c_p (T_{wr} - T_w)$, $(\equiv \frac{1}{x} \int_0^x St(x') dx')^*$		
St_0	local Stanton number at $M_1 = 0$		
St_{m0}	mean Stanton number at $M_1 = 0$		
T_w	wall temperature	K	K

For footnote refer to end of Notation.

T_{wr}	recovery temperature (<i>i.e.</i> temperature of the wall for zero heat transfer), $T_1 \left[1 + r \frac{(\gamma-1)}{2} M_1^2 \right]$	K	K
T_1	temperature at edge of boundary layer	K	K
T_∞	temperature of uniform flow upstream	K	K
u_1	velocity at edge of boundary layer	m/s	ft/s
x	distance measured along the surface from the virtual origin of the turbulent boundary layer	m	ft
x'	dummy variable of integration	m	ft
γ	ratio of specific heat capacities (taken as 1.4)		
ν_1	kinematic viscosity at edge of boundary layer	m ² /s	ft ² /s
ρ_1	density at edge of boundary layer	kg/m ³	slug/ft ³

Suffixes

c	relating to cone flow
i	incompressible flow ($M_1 = 0$, $T_w = T_{wr} = T_1$).

* For cone flow $(St_m)_c = \frac{2}{x_c^2} \int_0^{x_c} [St(x')]_c x'_c dx'_c$.

2. FLAT PLATE FLOW

Figures 1 and 2 present local and mean Stanton numbers (St_0 and St_{m0}) respectively for the two-dimensional turbulent boundary layer on a smooth flat plate at uniform temperature with zero pressure gradient for low-speed flow ($M_1 \approx 0$). The data are given for a range of Reynolds number, R_x , from 10^5 to 10^9 and ratios of wall temperature to recovery temperature, T_w/T_{wr} , within the range 0.25 to 2.0.

The effect of Mach number on Stanton number is given in Figures 3 and 4 for local and mean values respectively at temperature ratios of 0.25 and 2.0. For temperature ratios intermediate to these the use of linear interpolation is satisfactory. The heat transfer data are normalised by dividing by the relevant Stanton number for low-speed flow at the same Reynolds number and temperature ratio.

The data were obtained (see Derivation 4) from a large number of wind-tunnel experimental data for local heat transfer which were analysed by a Reynolds analogy factor, a , using the Spalding and Chi method (Derivation 2) for skin friction coefficient (see Item No. 68020). The analysis produced the following relations:-

$$St = aC_f = 0.5C_f \left/ \left[1 - 3.2 \sqrt{\frac{C_{fi}}{2}} \right] \right. \quad (2.1)$$

$$\text{and} \quad St_m = a_m C_{fm} = 0.5C_{fm} \left/ \left[1 - 3.2 \sqrt{\frac{C_{fmi}}{2}} \right] \right. \quad (2.2)$$

The Reynolds analogy factors a and a_m are shown as functions of R_x in Figure 5.

If skin friction coefficients are required in addition to Stanton numbers, the curves for Reynolds analogy factor given in Figure 5 may be employed directly to obtain skin friction coefficient from Equations (2.1) and (2.2) having found Stanton number from Figures 1 to 4. This procedure thus obviates the need to use Item No. 68020 in such a case.

3. SUPERSONIC WEDGE FLOW

The supersonic flow over a sharp wedge or a flat plate at incidence is two-dimensional and the properties between the shock, or expansion fan, and the boundary layer may generally be assumed constant. All the data for Stanton number relating to the turbulent flow over a flat plate at zero incidence may therefore be applied directly to these cases, provided that the properties of the flow downstream of the shock, or expansion fan, are used in the calculations.

4. AXISYMMETRIC FLOW

4.1 Parallel Circular Cylinder

Provided that the displacement thickness (see Item No. 68020) is of the order of one per cent or less of the radius then flat plate data should apply with negligible loss of accuracy.

4.2 Right Circular Cone in Supersonic Flow

Wilson (Derivation 3), assuming a 1/7th power law for the velocity profile, gives expressions from which the following relations between flat plate and cone data for identical Reynolds numbers ($R_x \equiv (R_x)_c$), Mach numbers at the edge of the boundary layer ($M_1 = (M_1)_c$) and ratios of wall to recovery temperature ($T_w/T_{wr} \equiv (T_w/T_{wr})_c$) can be deduced:-

$$(St)_c = 1.18 St \quad (4.1)$$

$$(St_m)_c = 1.05 St_m \quad (4.2)$$

These expressions are valid for thin boundary layers, *i.e.*, for displacement thickness (see Item No. 68020) of the order of one per cent or less of the local radius. The factors given in Equations (4.1) and (4.2) vary with the velocity profile exponent, but this variation is small (within about ± 2 per cent) for exponents giving a good fit to velocity profiles over the range of parameters covered by this Item.