

## A METHOD FOR ESTIMATING THE PRESSURE DISTRIBUTION ON THE SURFACE OF A TWO-DIMENSIONAL AEROFOIL IN A SONIC STREAM

### SUMMARY

A method is presented for calculating the pressure distribution on an aerofoil in a sonic stream. The method, the derivation of which is given in Reference 1, is based on theoretical and empirical relationships and comprises a number of separate procedures which are set out in this Memorandum. To apply these procedures it is necessary to specify only the aerofoil geometry and the angle of incidence for which the calculation is to be made.

Each calculation is made essentially in four main stages. The first stage determines the stagnation point, the second and third stages evaluate the subsonic flow over appropriate portions of the aerofoil and the fourth stage evaluates the region of supersonic flow.

The method may be applied to a wide range of both round- and sharp-nosed, lifting and non-lifting sections including that category of aerofoils exhibiting a velocity peak in the vicinity of the leading edge.

### 1. INTRODUCTION

At sufficiently high subsonic speeds the flow past the surface of an aerofoil is characterised by the presence of one or more regions of local supersonic flow each enclosed between a sonic line, the aerofoil surface and generally, at the downstream end, a terminating shock wave; at supersonic speeds the flow is characterised by the presence of one or more small regions of subsonic flow at the leading edge embedded in an otherwise wholly supersonic flow. Transition between these two regimes is achieved by a growth of the major local supersonic region outwards and rearwards, the rearwards growth being controlled by the movement of the terminating shock (Figure 1a) which approaches the trailing edge as the stream speed nears sonic. The corresponding development of the surface pressures is illustrated by an example in Figure 1b.

The pressure distribution that is established in the case when the free stream is sonic ( $M_0 = 1.0$ ) has special significance because

- (i) it is essentially invariant for a small range of free-stream speeds on either side of sonic speed and is often referred to as the “sonic-range” pressure distribution; the corresponding near-stationary point in the variation of surface pressure with  $M_0$  at any given fixed point upstream of the shock is often referred to as the “sonic freeze”,
- (ii) the form of that part of the pressure distribution between the crest<sup>†</sup> (and indeed forward of the crest for some aerofoils) and the shock wave terminating the major region of local supersonic flow remains similar for an even wider range of speeds below  $M_0 = 1.0$ .

Sinnott<sup>2</sup> shows how the part of the pressure distribution between the crest<sup>†</sup> and the terminating shock may be derived from the sonic-range pressure distribution as a stage in his method for predicting mixed supersonic/subsonic pressure distributions which, in turn, form the basis<sup>3,4</sup> for predicting the onset of the effects of shock-induced separation, such as buffeting. This is one context in which the need to estimate the sonic-range pressure distribution has arisen. However, Sinnott's method is limited to predicting local pressures downstream of the crest on a particular class of pressure distributions (in which the local supersonic flow velocity increases continuously between the sonic point and the terminating shock) and

<sup>†</sup> The crest is defined as the point at which the aerofoil surface is tangential to the undisturbed free-stream direction.

cannot be used to evaluate the pressure distribution in the leading-edge region, nor to predict the onset of buffeting should this condition correspond to a shock location ahead of the crest.

T.D. Memo. 6511 presented a development of Sinnott's method for predicting the sonic-range pressure distribution that removed the restriction on the class of pressure distributions that could be considered; the present Memorandum not only removes this restriction but also allows the complete sonic-range pressure distribution from the stagnation point to the trailing edge to be calculated. However, although it may thus ultimately provide, irrespective of the shock location, a basis from which to predict the onset of buffeting, at present its use in this way is still restricted to the one class of pressure distributions (see preceding paragraph) for which, when the shock is located downstream of the crest, Sinnott's approach is valid. Then criteria for the divergence of trailing-edge pressure presented in Reference 4 may be used. Means of extending the concepts of the present method to free-stream Mach numbers less than unity are being investigated and it is hoped that from this further work a more general basis for predicting the onset of buffeting will ensue.

In the method presented here the pressure distribution around the leading edge is obtained with sufficient accuracy for evaluating pressure drag and therefore has relevance in the wider context of section design for swept wings that are to operate through the condition for which the leading edge is sonic.

The method presented here is founded on theoretical and empirical relationships established in Reference 1 and comprises a number of separate procedures. The main procedures are:

- (i) the determination of the stagnation point,
- (ii) the determination of the subsonic flow in the leading-edge region where the local radius of curvature is small,
- (iii) the determination of the subsonic flow at points on the aerofoil surface where the local radius of curvature is relatively large,
- (iv) the determination of the supersonic region of flow.

The method takes account of the different ways in which the flow accelerates from the stagnation point to establish local supersonic velocities. The possible developments of the flow from the stagnation point are described in Section 3.

To apply the procedures, which are detailed in Section 5, it is necessary to specify only the aerofoil geometry and the geometric angle of incidence for which the calculation is to be made.

## 2. NOTATION

|                     |                                                                                                                                                                                                                   |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $c$                 | aerofoil chord (see Section 2.3)                                                                                                                                                                                  |
| $C_p$               | pressure coefficient                                                                                                                                                                                              |
| $f$                 | thickness similarity parameter applicable to local supersonic flow                                                                                                                                                |
| $g$                 | thickness similarity parameter applicable to local small-disturbance subsonic flow                                                                                                                                |
| $H$                 | stagnation pressure in undisturbed stream                                                                                                                                                                         |
| $k$                 | factor used in determining $f_L$ that extends application of correlations in Figures 14 and 15 to thin aerofoils                                                                                                  |
| $K$                 | factor that extends correlations in Figure 10 to aerofoils on which the <i>subsonic</i> small-disturbance flow that may exist influences the upstream large-disturbance flow near a round leading edge            |
| $M$                 | local Mach number                                                                                                                                                                                                 |
| $M_0$               | free-stream Mach number                                                                                                                                                                                           |
| $n$                 | $-[d\Theta_e/d\alpha]_{\theta = \text{const}}$ ; rate of change of effective surface slope with incidence at a given chordwise station (local values of $n$ lie between $n_{st}$ and 1.0)                         |
| $n_{st}$            | $d(\delta\theta_{st})/d\alpha$ ; angular rate of movement of stagnation point with incidence expressed in terms of local surface slope                                                                            |
| $p$                 | local static pressure                                                                                                                                                                                             |
| $t$                 | aerofoil maximum thickness                                                                                                                                                                                        |
| $U$                 | local velocity                                                                                                                                                                                                    |
| $U_0$               | free-stream velocity                                                                                                                                                                                              |
| $x$                 | chordwise distance measured from leading edge                                                                                                                                                                     |
| $z$                 | aerofoil ordinate measured from and normal to chord                                                                                                                                                               |
| $\alpha$            | angle of incidence in degrees                                                                                                                                                                                     |
| $\Delta$            | factor that extends application of large-disturbance correlations in Figure 10 to leading-edge profiles having a region where $\rho/\rho_{LE} \leq 1.0$ (local values of $\Delta$ lie between $\Delta_A$ and 1.0) |
| $\Delta_A$          | particular value of $\Delta$ on that part of leading-edge profile where $\rho/\rho_{LE} \leq 1.0$                                                                                                                 |
| $\delta\theta_{st}$ | displacement of stagnation point from leading edge in terms of surface slope (on lower surface if positive)                                                                                                       |
| $\theta$            | actual aerofoil surface slope relative to chord line                                                                                                                                                              |

|                    |                                                                                                                                                                                                                                                                                                                     |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\Theta$           | $\theta - \alpha$ ; actual aerofoil surface slope relative to free-stream direction                                                                                                                                                                                                                                 |
| $\Theta_e$         | $\theta - (\psi + \alpha n)$ ; <i>effective</i> aerofoil surface slope relative to free-stream direction (at stagnation point $\Theta_e = 90^\circ$ whilst away from leading-edge region of high curvature $\Theta_e = \Theta$ )                                                                                    |
| $\tilde{\Theta}_e$ | <i>effective transformed</i> value of $\Theta$ that identifies point on circular cylinder where local value of $(K/\Delta) (U/U_0)$ is the same as at local point on aerofoil                                                                                                                                       |
| $\Theta_{e1}$      | parameter in relationship between $\Theta_e$ and $\Theta$ that accounts for change in local aerofoil surface curvature from leading-edge value. It is hypothetical value of $\Theta_e$ that would occur at point where $(K/\Delta) (U/U_0) = 1.0$ if curvature downstream of point under consideration was constant |
| $\lambda$          | factor in correlations of Figures 14 and 15 that allows for influence of local subsonic flow on downstream supersonic flow                                                                                                                                                                                          |
| $\rho$             | local surface radius of curvature                                                                                                                                                                                                                                                                                   |
| $\bar{\rho}$       | mean surface radius of curvature between $\phi = 0$ and $70^\circ$                                                                                                                                                                                                                                                  |
| $\phi$             | angle between local flow direction at surface and tangent at stagnation point (see Figure 2)                                                                                                                                                                                                                        |
| $\psi$             | angle which allows for effect of nose camber in evaluation of $\Theta_e$ at a given chordwise station (local values of $\psi$ lie between $\psi_{st}$ and 0)                                                                                                                                                        |
| $\psi_{st}$        | angle defining location of stagnation point on cambered aerofoil at zero incidence (see Figure 2)                                                                                                                                                                                                                   |
| $\omega$           | Prandtl-Meyer angle (see Section 2.2)                                                                                                                                                                                                                                                                               |
| <b>Suffixes</b>    |                                                                                                                                                                                                                                                                                                                     |
| $f$                | pertaining to point at which $f_L$ begins to vary with $x/c$ for thin aerofoils. The point corresponds to value of $x/c$ at sonic point or point where $\rho/c = 0.2$ , whichever is larger                                                                                                                         |
| $l$                | denotes lower surface                                                                                                                                                                                                                                                                                               |
| $L$                | denotes local value                                                                                                                                                                                                                                                                                                 |
| $LE$               | denotes leading edge                                                                                                                                                                                                                                                                                                |
| $p$                | pertaining to point, $P$ , at which upstream large-disturbance and downstream small-disturbance flows are matched (see Figures 3a, 3e)                                                                                                                                                                              |
| $s$                | pertaining to sonic point                                                                                                                                                                                                                                                                                           |
| $st$               | denotes stagnation point                                                                                                                                                                                                                                                                                            |
| $u$                | denotes upper surface                                                                                                                                                                                                                                                                                               |
| 1                  | denotes shock upstream condition                                                                                                                                                                                                                                                                                    |
| 2                  | denotes shock downstream condition                                                                                                                                                                                                                                                                                  |

## 2.1 Units

Throughout this document all angles are given in degrees.

## 2.2 Definition of Prandtl-Meyer Angle

The Prandtl-Meyer angle,  $\omega$ , is defined as the angle through which a streamline would have to be deflected in a simple-wave (or Prandtl-Meyer) expansion in order to accelerate the flow from an initial Mach number of unity to the value in question.

## 2.3 Definition of Aerofoil Chord

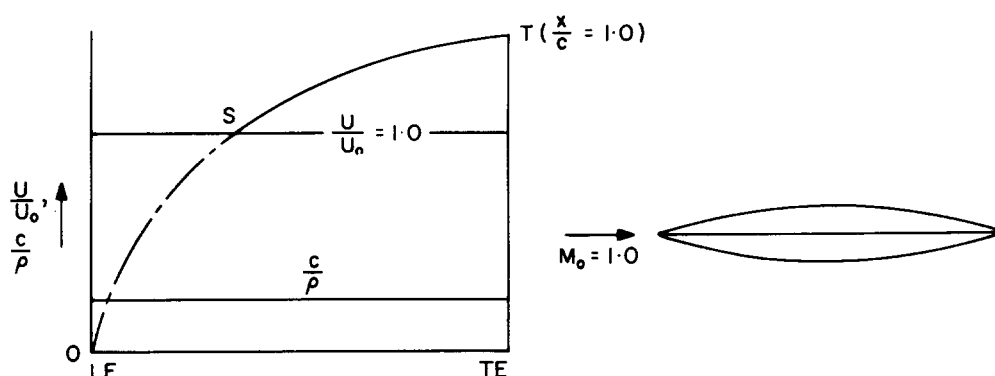
In the context of this Memorandum the aerofoil chord is defined as the radius through the unique point at which a circle centred at the trailing edge (or mid-point of a finite trailing edge) is, at that unique point, tangential to the aerofoil.

# 3. THE VELOCITY DISTRIBUTION AND ITS COMPONENT PARTS

The basis of the method described in this Memorandum derives from a consideration and definition of the various flow regimes to be observed at the surface of an aerofoil in a sonic stream. Examples of particular types of flow pattern for the upper surface of a series of aerofoil shapes are described in Sections 3.1 to 3.3. The accompanying sketches show how each velocity distribution may be subdivided into component parts. The sketches also illustrate the development of a leading-edge velocity peak as the leading-edge shape is progressively blunted and the varying forms of leading-edge flow compression that may occur with certain combinations of surface-shape and incidence. In each sketch chordwise distance is distorted in such a way that the rapid chordwise variation of curvature and velocity near the leading edge is more clearly illustrated.

## 3.1 Sharp-Nosed Aerofoil

Consider firstly the relatively simple case of a sharp-nosed symmetrical aerofoil (for which the local surface curvature is everywhere fairly small) at zero or very small incidence as illustrated in Sketch 3.1.



Sketch 3.1

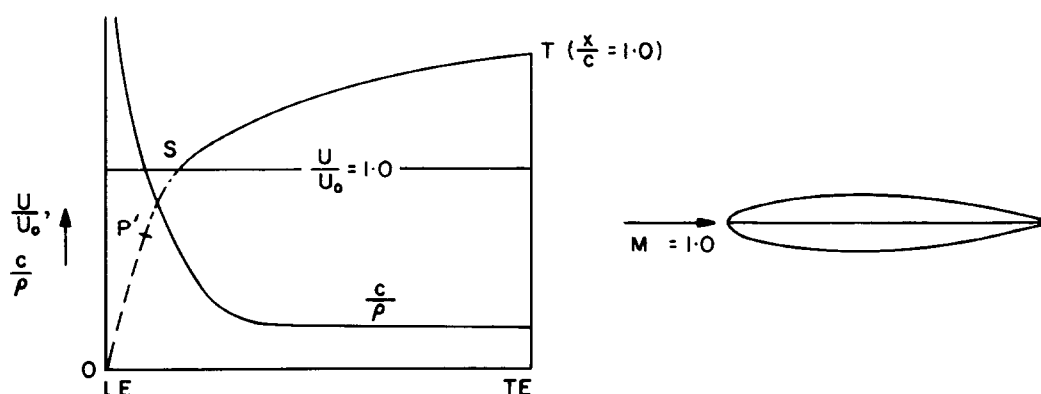
The flow expansion from the stagnation point, O, to the trailing edge can be considered to consist of two component parts, namely:

- (i) the flow region ahead of the sonic point, S (where the local velocity reaches sonic speed),

- (ii) the region of supersonic flow between the sonic point,  $S$ , and the shock wave terminating the local region of supersonic flow which, for  $M_0 = 1.0$ , occurs at the trailing edge,  $T$ .

### 3.2 Round-Nosed Aerofoil Without Leading-Edge Compression

Consider now a development from the sharp-nosed aerofoil described above in which the sharp leading edge is rounded by modifying the leading-edge region so that the surface curvature decreases uniformly with increasing  $x/c$  as illustrated in Sketch 3.2.



Sketch 3.2

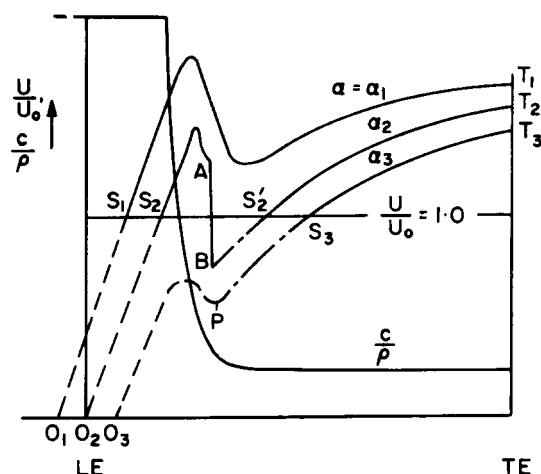
In this case also, with the stagnation point situated close to the leading edge, the flow expansion from the stagnation point to the trailing edge is smooth. The velocity distribution is similar to that depicted for the sharp-nosed aerofoil except that in this case the rounded leading edge produces a more rapidly expanding region of so-called “large-disturbance” flow near to the leading edge where the surface curvature, slope and local perturbations are all large. The component parts of the velocity distribution may, therefore, be considered to consist of:

- (i) the region of large-disturbance subsonic flow,  $OP'$ , between the stagnation point and a downstream point  $P'$ ; this point cannot, at this stage, be clearly defined but must occur where the surface curvature and slope are fairly small.
- (ii) in the general case, a region of small-disturbance subsonic flow,  $P'S$ , assuming that the large-disturbance flow terminates before sonic velocity is attained,
- (iii) a region of supersonic flow,  $ST$ .

An analysis of experimental data in Reference 1 demonstrates that the nature of the small-disturbance subsonic flow is dependent, to a very close approximation, only on the local surface geometry. It is found to be unaffected by disturbances originating in the upstream large-disturbance flow near the leading edge and thus to be independent of the leading-edge shape. The nature of the large-disturbance flow is also, in most cases, determined primarily by local surface geometry but is, in addition, also dependent on the stagnation point location and the downstream region of small-disturbance subsonic flow, should such a flow region exist. Any interaction between the two regions of subsonic flow therefore manifests itself as a readjustment of the large-disturbance flow so that conditions of flow continuity and compatibility between the two flow regions are satisfied.

### 3.3 Round-Nosed Aerofoil With Leading-Edge Compression

Consider now a further development of the round-nosed aerofoil for which the round leading edge has been blunted even further by modifying the leading-edge region to include a region of high curvature maintained for a significant change in surface slope from the stagnation point. This type of leading-edge geometry often leads to the development of the flow phenomenon, illustrated in Sketch 3.3, where a rapid flow expansion from the stagnation point is followed by a flow compression as the local curvature decreases rapidly from the high sustained value near the leading edge to a fairly low value further downstream.



Sketch 3.3

Aerofoils of the type illustrated in Sketches 3.1 and 3.2 also may exhibit this particular flow phenomenon when, at high incidence, with the stagnation point situated on the lower surface well away from the leading edge, a rapid flow expansion is allowed to occur within the region of high curvature round the leading edge.

The examples in Sketch 3.3 represent three cases for the one aerofoil at incidences of  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  where  $\alpha_1 > \alpha_2 > \alpha_3$ . These cases are chosen to illustrate, over an extreme range of incidence, the different forms of leading-edge compression that may follow the initial rapid flow expansion from the stagnation point.

#### 3.3.1 Compression occurring at supersonic velocities

In the first case,  $\alpha = \alpha_1$ , the sonic point  $S_1$  ( $U/U_0 = 1.0$ ) occurs in the initial rapid flow expansion and the flow compression, in the region of rapidly decreasing curvature, occurs at supersonic velocities. The flow mechanism associated with this form of compression is described in Reference 1. The component parts of the velocity distribution differ from the previous case (Section 3.2) in that a region of small-disturbance subsonic flow does not occur.

#### 3.3.2 Shock compression

The second, intermediate, configuration ( $\alpha = \alpha_2$ ) also produces a rapid flow expansion to supersonic velocities but in this case the subsequent flow mechanism through which compatibility with the downstream flow is established is more complex. The incidence, combined with the effects of the rapid decrease in surface curvature downstream of the region of large curvature, will not sustain the presence of local supersonic flow and a local region of small-disturbance subsonic flow,  $BS'_2$ , must exist. The "over-expanded" leading-edge flow thus breaks down and is compressed to a subsonic level, compatible with the local section geometry, through a strong shock wave,  $AB$ . The component parts of the velocity distribution are, therefore:

- (i) the region of large-disturbance subsonic flow originating at the stagnation point and terminating at the first sonic point,  $S_2$ ,
- (ii) a local region of supersonic flow,  $S_2A$ ,
- (iii) a shock wave,  $AB$ , where the shock upstream velocity at  $A$  is supersonic and the shock downstream velocity at  $B$  is subsonic,
- (iv) a region of small-disturbance subsonic flow,  $BS'_2$ ,
- (v) a further region of local supersonic flow,  $S'_2T_2$ .

### 3.3.3 Compression occurring at subsonic velocities

In the final case ( $\alpha = \alpha_3$ ) the rapid increase in surface curvature combined with even lower incidence has the effect of producing an even more extensive region of small-disturbance subsonic flow downstream of the initial peak velocity. The surface in this region is thus able to exert an influence on the upstream flow expansion from the stagnation point. This may still allow, or may prevent, as illustrated in Sketch 3.3; sonic velocity being attained in the initial expansion. In the former case the compression of the flow to velocities compatible with the local surface geometry takes place without a strong shock occurring. In the latter case the compression takes place at an entirely subsonic level. The component parts of the velocity distribution are then:

- (i) the region of large-disturbance flow,  $O_3P$ ,
- (ii) the region of small-disturbance subsonic flow,  $PS_3$ ,
- (iii) the region of supersonic flow,  $S_3T_3$ .

### 3.4 The Three Component Parts

It has so far been established that the sonic-range velocity distribution on a round-nosed section may be considered to consist of a number of component parts which fall into one of three categories:

- (i) a region of large-disturbance subsonic flow originating at the stagnation point near the leading edge,
- (ii) a region of small-disturbance subsonic flow,
- (iii) a region of local supersonic flow.

The nature of the large-disturbance flow is dependent on the stagnation point location, the local surface geometry near the leading edge and, in those cases where a fairly extensive region of small-disturbance subsonic flow also occurs, the surface shape downstream of, but near to, the leading-edge region. The transition from large- to small-disturbance flow cannot, in general, be defined precisely but for the purposes of the method described in this Memorandum it is assumed to occur at a point. Criteria for determining this point, when it is necessary to do so, are given in Section 4.

The magnitudes of small-disturbance subsonic velocities in the present context are determined primarily only by local surface geometry. They can, in practice, be considered as part of the velocity distribution that would exist on the surface of an equivalent sharp-nosed version of the round-nosed aerofoil under consideration.

The supersonic flow expansion is dependent on the change in local surface slope from the value at the sonic point. In addition, for a mixed subsonic/supersonic flow field, the expansion is modified by compressive disturbances which can be considered to originate at the sonic line as illustrated in Figure 5.

Thompson<sup>1</sup> shows how each of the three flow regions may be derived from methods based on theoretical and empirical relationships. He then shows how the component parts may be fitted together to form the complete velocity distribution by taking into account conditions for flow continuity and/or compatibility between the different flow regions. These methods are described briefly in Section 4 and step by step procedures are detailed in Section 5.

#### 4. NOTES ON THE DERIVATION AND APPLICATION OF THE METHOD

The preceding Section has drawn attention to the types of flow pattern that have been observed in experimental data and has shown how the complete sonic-range velocity distribution may be described in terms of three component parts plus a knowledge of the stagnation point. This Section is a summary of Reference 1 and its purpose is (a) to comment on the derivation of the methods for predicting the component parts of the velocity distribution, (b) to explain the physical meaning of the parameters and factors used and (c) to explain the means by which the flow mechanism governing the flow, particularly near the leading edge, is reproduced.

The method is based on theoretical and empirical relationships for sonic flow. The initial stage determines the stagnation point position which forms the origin of subsequent calculations. The next stage evaluates the large-disturbance subsonic flow region near the leading edge, where the surface slope and curvature are large. This stage may or may not determine where the sonic point occurs in the initial flow expansion. If sonic velocity is not reached a further stage is necessary to calculate the small-disturbance subsonic flow region which is terminated by the sonic point. The final stages involve the calculation of the supersonic region of flow, which can only be obtained once the sonic point is known, and the determination of the compressive flow region near the leading edge, should such a flow phenomenon be found to exist.

##### 4.1 Determination of Stagnation Point

The stagnation point is determined by the procedure in Section 5.2 from a knowledge of the leading-edge shape and incidence relative to the free stream using the two empirically derived curves in Figures 6a, 6b. The parameter  $\psi_{st}$  gives the zero-incidence location of the stagnation point and the parameter  $n_{st}$  is the angular rate of movement of the stagnation point with incidence expressed in terms of the local surface slope.

##### 4.2 The Large-Disturbance Subsonic Flow

Two empirical methods are available for predicting the large-disturbance flow region over the leading edge of round-nosed aerofoils in a sonic stream. The first of these methods is due to Wilby<sup>5</sup> and the second is due to Thompson<sup>1</sup>. Both methods are of similar accuracy.

Wilby's method requires a prior knowledge of the exact incompressible velocity distribution and stagnation point position. In addition, for those cases in which sonic velocity is not reached in the initial rapid expansion, the velocity distribution that is obtained from Reference 5 requires a correction factor to ensure continuity between the calculated upstream large-disturbance and downstream small-disturbance subsonic flow regions.

Thompson's method only requires a knowledge of the local surface geometry and the stagnation point position and has been evolved in conjunction with other parts of the present overall method. For these reasons, and to simplify the presentation of the overall method, Thompson's approach has been incorporated

into Section 5 and the concept underlying this approach is outlined in Section 4.2.1. The application of the alternative method of Wilby when used in the present context is described in Appendix A.

#### 4.2.1 The concept

In the context of Thompson's method the large-disturbance subsonic flow defines the flow region between the stagnation point and the point corresponding to  $\rho/\rho_{LE} = 100$  or the sonic point, whichever occurs first. Local velocities, expressed in the form  $(K/\Delta)(U/U_0)$ , may be obtained by adopting the concept that each element of surface can be transformed into an element of a circular cylinder around which the solution to the velocity distribution at  $M_0 = 1.0$  is known. The solution used here is that due to Chushkin<sup>6</sup> which in the present concept is to be interpreted as  $(K/\Delta)(U/U_0)$  versus  $\tilde{\Theta}_e$  and is given by the curve in Figure 10 for  $\rho/\rho_{LE} \leq 30$ .

For this concept to represent correctly the flow about a circular cylinder the factors  $K$  and  $\Delta$  are both unity and the parameter  $\tilde{\Theta}_e$  is the surface-slope location of a point on the circular cylinder referred to a line normal to the tangent at the stagnation point, i.e. the value of  $90 - \phi$ .

In the general case, when the concept is applied to the leading edge of an aerofoil,  $K$  and  $\Delta$  are not necessarily unity and are evaluated as part of the procedure in Section 5. Their functions are described in Section 4.2.2. The parameter  $\tilde{\Theta}_e$  is, in the general case, the value of surface slope on a cylinder which gives the same local value of  $(K/\Delta)(U/U_0)$  as experienced by a particular point of surface slope  $\Theta$  on the aerofoil and is given by the empirically-derived transformation relation

$$\tilde{\Theta}_e = 34.3\Theta_e/\Theta_{e1}, \quad (4.1)$$

$$\text{where} \quad \Theta_e = \theta - (\psi + \alpha n) = \Theta - (\psi + \alpha(n - 1)) \quad (4.2)$$

$$\text{and} \quad \Theta_{e1} = F(\rho/\rho_{LE}) \text{ is given by Figure 9.}$$

The parameter  $\tilde{\Theta}_e$  is thus, effectively, the transformed value of  $\Theta$ . The parameter  $\Theta_{e1}$  accounts for the changing curvature of the aerofoil surface from the leading-edge value in the relationship between the effective transformed and actual local surface slope. It is the value of  $\Theta_e$  at the point where  $(K/\Delta)(U/U_0)$  would be unity if the curvature downstream of the point under consideration were constant.

Two special cases embodied in the above transformation are worth noting. They are:

- (a) when  $\rho/\rho_{LE} \leq 1.0$ ,  $\Theta_{e1}$  assumes a value of 34.3 and  $\tilde{\Theta}_e = \Theta_e$  is then simply the value of  $(90 - \phi)$ ,
- (b) when  $\Theta_e = \Theta_{e1}$  then  $\tilde{\Theta}_e = 34.3$  and  $(K/\Delta)(U/U_0) = 1.0$ .

Local values of  $(K/\Delta)(U/U_0)$ , obtained from Chushkin's theoretical relationship for a circular cylinder in Figure 10, are applicable to the transformed angles,  $\tilde{\Theta}_e$ , on the aerofoil surface providing  $\rho/\rho_{LE} \leq 30$ . Reference 1 shows that the transformation relationships may also be applied to points on the aerofoil surface for values of  $\rho/\rho_{LE}$  up to 100 provided the empirically-modified velocity distribution curves of Figure 10 are used at the corresponding value of  $\rho/\rho_{LE}$ .

#### 4.2.2 The factors $K$ and $\Delta$

The empirical factors  $K$  and  $\Delta$  may be regarded as influence functions which represent the way in which disturbances originating downstream of the leading-edge region noticeably affect the velocity distribution upstream. Thus values of  $(K/\Delta)(U/U_0)$  may be considered to be the local velocities that would occur in

the absence of any downstream influence.

The factor  $\Delta$  is introduced<sup>†</sup> to take account of the particular way in which the rapid flow expansion from the stagnation point on sections with partly circular leading-edge regions (or, more explicitly, leading-edge regions for which  $\rho/\rho_{LE} \leq 1.0$ ) generally differs from that on a circular cylinder. The factor  $\Delta = \Delta_A$  (Figure 7a) accounts for this effect in the region where  $\rho/\rho_{LE} \leq 1.0$  and represents an influence effect arising from the changing profile shape immediately downstream of the region where  $\rho/\rho_{LE} \leq 1.0$ . As  $\rho/\rho_{LE}$  increases from unity the process of transforming local surface elements into elements of an infinite series of circular cylinders begins to take account of the effective local influence arising from the changing profile shape. However, experimental data indicate that this will not be fully accounted for by the transformation relationships until  $\rho/\rho_{LE}$  has increased to a value of about 10. In the range  $1 \leq \rho/\rho_{LE} \leq 10$ , therefore, values of  $\Delta$  (given by Figure 7b) vary between  $\Delta = \Delta_A$  for  $\rho/\rho_{LE} \leq 1$  and  $\Delta = 1.0$  at  $\rho/\rho_{LE} = 10$ .

The factor  $K$ , a function of  $K_p$ ,  $\Theta_e$  and  $\Theta_{ep}$  (Figure 12), can be considered to represent an influence of the downstream small-disturbance subsonic region of flow on the upstream velocity distribution arising from a condition for flow continuity between the two regions. The value of the factor  $K = K_p$  is determined by Section 5.6 and ensures that the predicted upstream large-disturbance flow and the downstream small-disturbance subsonic flow are continuous at a specified matching or transition point – the location of this point depending on the type of leading-edge flow obtained, as noted in the following Section and illustrated in Figures 3a and 3e. The magnitude of the factor  $K_p$  (and hence  $K$ ) will, in most cases, only be significantly different from unity for sections exhibiting an “over-expansion” velocity peak near the leading edge, such as the case illustrated in Figure 3e. In those cases where a region of small-disturbance subsonic flow does not exist then, of course,  $K_p$  and  $K$  are unity.

#### 4.3 The Small-Disturbance Subsonic Flow

The small-disturbance subsonic flow, which in any particular case may, or may not, exist, is defined in the present context as the flow region between the point at which transition from large-disturbance to small-disturbance subsonic flow is assumed to occur and a downstream sonic point. For the purposes of the calculation the transition point is normally at the chordwise point corresponding to  $\rho/\rho_{LE} = 100$  (Figure 3a) but, for sections exhibiting a leading-edge velocity peak, downstream of which the flow is subsonic (Figure 3e), transition is at the point where the local velocity in the small-disturbance subsonic flow is a minimum<sup>‡</sup> (i.e. where  $dU/dx = 0$  and  $d^2U/dx^2 > 0$ ).

The local flow characteristics are determined, in terms of the velocity ratio  $U/U_0$ , from a knowledge of the local flow inclination relative to the free-stream direction,  $\Theta$ , the thickness parameter,  $g$ , and the local radius of curvature expressed as  $\rho/c$ . The curves presented in Figure 11 are derived largely from the theoretical results of Spreiter and Alksne<sup>7</sup> for a family of sharp-nosed parabolic-arc sections. The basis of this derivation is given in Reference 1 and it is there demonstrated, from an analysis of extensive experimental data, that the curves are applicable to a wide range of both sharp- and round-nosed, lifting and non-lifting aerofoils in a sonic stream. The analysis also demonstrates that when a region of small-disturbance subsonic flow exists it is independent of the upstream large-disturbance flow (i.e. independent of the leading-edge shape) although, paradoxically, the requirement for flow continuity or compatibility may demand that the small-disturbance subsonic flow exerts a dominating influence on the large-disturbance flow.

In those cases where the small-disturbance subsonic flow demands some form of leading-edge

<sup>†</sup> The factor  $\Delta$  (Figure 7b) is dependent on  $\Delta_A$  and  $\Delta_A$  is determined by the value of  $\bar{\rho}/\rho_{LE}$  (Figure 7a). For  $\bar{\rho}/\rho_{LE} < 1.0$  the curve in Figure 7a is based only on a limited amount of experimental data (see Reference 1).

<sup>‡</sup> If more than one minimum point should occur in this local flow region transition to small-disturbance subsonic flow is assumed to occur at the minimum point nearest the leading edge.

flow-compression (Section 4.5 and 5.5) – and only in such cases – the procedure in Section 5.5 may be applied to any point corresponding to  $\rho/\rho_{LE} < 100$  provided  $U/U_0 \leq 1.0$ .

#### 4.4 The Region of Supersonic Flow

The region of supersonic flow is evaluated from an empirical relationship, derived from the concept of the characteristics network between the aerofoil surface and the sonic line, and certain transonic similarity laws.

This characteristics network can be considered to consist of two hypothetical families of characteristic lines along which are propagated disturbances generated by two wave systems. The first family of characteristic lines originating at the surface carries an outgoing expansive disturbance generated by a change in slope at the surface and along which  $\theta - \omega = \text{constant}$ . The second family of characteristic lines, along which  $\theta + \omega = \text{constant}$ , can be considered to originate at the sonic line and carry a compressive disturbance to account for the non-uniformity of the flow approaching the sonic line. The compressive disturbance may be considered to be a reflection of the expansive disturbance at the sonic line (a constant pressure boundary) which, on returning to the surface, is again reflected as a compressive disturbance as illustrated in Figure 5. The compressive disturbance is determined as the difference between the disturbance that would be generated by a simple-wave expansion (if the flow approaching the sonic line were uniform) and that actually generated by the aerofoil when the approaching flow is not uniform, *i.e.*  $(\theta_s - \theta) - \omega$ . The chordwise variation of  $\theta + \omega$  therefore gives a measure of this total compressive disturbance that reaches the surface from the sonic line and, in particular (as shown by Moulden<sup>8</sup>), the value of  $\partial(\theta + \omega)/\partial(x/c)$  is the strength of the compressive disturbance at any particular point. Reference 1 establishes how the chordwise variation of  $(\theta + \omega)$ , and  $(\Theta + \omega)$  for non-zero incidence cases, for arbitrarily-defined, round- and sharp-nosed symmetrical and cambered aerofoils may be evaluated from the empirically-established relationship of Figure 14 which shows that

$$(\omega + \Theta)f_L\lambda = F(x/c). \quad (4.3)$$

The relationship is valid even during a very rapid flow expansion from the stagnation point (produced by a sustained leading-edge region of high curvature) and during the flow compression (brought about by a rapid decrease in the local surface curvature) that often follows such a rapid expansion. However, should the compression re-establish subsonic velocities then the initial (but only the initial) region of supersonic flow (*i.e.* the region  $S_1A$  in Figure 3d) is influenced by disturbances originating in the downstream region of subsonic flow (the region  $BS_2$  in Figure 3d). In this case values of  $(\omega + \Theta)$  must be evaluated from the empirically-established relationship of Figure 15 which differs from that of Figure 14 only in the region downstream of  $(x/c)^{1/3} = 0.21$ .

The factor  $f_L$  is derived from a consideration of transonic similarity laws and with the exception of the upper surface of thin aerofoils at positive incidence, is defined as

$$f_L = f = 0.1/[2(z/c)_{max}].$$

For thin aerofoils (when  $2(z/c)_{max} < 0.0769$ ) it is necessary to introduce a second term, dependent on  $x/c$  and  $\alpha$ , into the definition of  $f_L$  to represent an increased expansive effect downstream of the sonic point over that given by the correlation curves in Figures 14 and 15 when used in conjunction with the definition of  $f_L$  in the equation above. This increased expansive effect is explained through a consideration of the characteristics network between the aerofoil surface and the sonic line and is attributable to the far forward location of the sonic point on thin round-nosed aerofoils. The general definition of the factor  $f_L$  is

$$f_L = \frac{1}{2(z/c)_{max}} [0.1 - k\alpha/57.3] \quad (4.4)$$

where the chordwise variation of the factor  $k$  can be obtained from Figure 16 and is only different from zero when

$$f = 0.1/[2(z/c)_{max}] > 1.3 \text{ and } \alpha > 0.$$

The factor  $\lambda$ , a constant for any one aerofoil at a particular incidence, is dependent on conditions at the sonic point determined from a priori considerations, and guarantees that the flow should be continuous through this point. Thus  $\lambda$  can be considered to represent the influence of conditions at or near the sonic point (and, in turn, the influence of the subsonic region of flow) on the downstream supersonic region of flow. The value of  $\lambda$ , for any one case, is determined so that at the sonic point the value of  $(\omega + \Theta)f_L\lambda$  lies on the curve of Figure 14 or 15 – whichever is appropriate. To establish the value of  $\lambda$  a necessary first step is, therefore, to obtain a prediction of the subsonic region of flow preceding the sonic point. Sections 4.1 to 4.3 have commented on how this may be achieved.

A point worth emphasising here is that provided the sonic point location is known from some a priori considerations then the simple relationships given in this Section, and the step by step procedure in Section 5.7, provide a rapid but accurate method for estimating the region of local supersonic flow on any arbitrarily-defined aerofoil in a sonic stream.

## 4.5 The Leading-Edge Flow Compression

The rapid flow expansion from the stagnation point exhibited by many sections is invariably followed by a flow compression (as illustrated, for example, in Figures 3b to 3e) as the local surface radius of curvature increases rapidly from a sustained small radius of curvature region near the leading edge. This compression may be achieved through one, or a combination, of several forms.

Reference 1 demonstrates the existence of an apparent paradox when the method is applied to aerofoils which feature a rapid flow expansion followed by a compression, in that the method can predict the occurrence of both subsonic and supersonic flow over a local region near the leading edge. In the context of the method and in the light of experimental data, the apparent paradox can be interpreted as indicating that the leading-edge compression takes one of the particular forms illustrated in Figures 3c to 3e. The criterion used to establish which form of compression occurs is based on whether or no a normal shock wave can be used to establish compatibility between the predicted upstream region of supersonic flow and the downstream region of (small-disturbance) subsonic flow, when such a flow region is predicted to occur.

The various forms of flow compression are discussed in the following Sections.

### 4.5.1 Compression with a strong shock

The existence and location of a strong shock in the leading-edge flow is determined, as illustrated in Figure 3d, by the intersection of the locus of values of possible shock-upstream velocity<sup>†</sup> with the predicted upstream variation of local supersonic velocity, as described in step (v) of Section 5.5. The normal-shock velocity relationship is used in the present context because a study of experimental data<sup>1</sup> shows that when a strong shock wave occurs in the flow near the leading edge the velocity fall through the shock corresponds closely with that due to a theoretical normal shock.

<sup>†</sup> Obtained by interpreting a predicted region of downstream subsonic velocity as the locus of possible normal-shock downstream values.

#### 4.5.2 Compression occurring in a shock-free flow

Consider now the case when the locus of possible values of shock-upstream velocity<sup>†</sup> is always *greater* than the predicted upstream values of supersonic velocity (or, to be precise, values of  $KU/U_0$ ) as illustrated in Figure 3e. This case may be interpreted as indicating that the shock compression that would be required to compress the flow from a supersonic level (achieved in the absence of any influence arising from a requirement for flow compatibility or continuity with a downstream region of local subsonic flow) to the subsonic level is greater than that which can be achieved through the occurrence of a normal shock. In this case it is suggested that the downstream surface shape that supports subsonic velocities exerts an influence on the upstream flow, as noted in Section 3.3.3, reducing the rate of flow expansion from the stagnation point sufficiently either for the remaining compression to subsonic velocity to be achieved in a shock-free flow or, as illustrated in Figure 3e, so that sonic velocity is not achieved in the initial expansion.

An analysis<sup>1</sup> of experimental data, for the case referred to above, has suggested that when a region of small-disturbance *subsonic* flow exists, its influence on the upstream flow, demanded by the requirements for flow compatibility or continuity between the two regions, may be represented by the influence factor  $K$ . (Note that this particular influence is similar to that built into the empirical correlation of Figure 15 described in Section 4.4 but is additional to it in those cases where a condition for flow continuity has to be satisfied as in Figure 3e.) Values of  $KU/U_0$  determined by Section 5.3 may be considered to be the local velocities that would occur in the absence of a requirement to satisfy a downstream condition for flow continuity. Local velocities ( $U/U_0$ ) between the stagnation point and the point corresponding to  $Q$  (Figure 3e) are evaluated using Section 5.6.2 by determining local values of  $K$  from Figure 12 with the factor  $K = K_p$  determined so that at the transition point  $Q$  (Figure 3e) the upstream large-disturbance flow ( $OQ$ ) is continuous with the downstream small-disturbance flow ( $QS_2$ ). The correlation in Figure 12 is derived from a comparison of predicted and experimental data exhibiting this particular form of leading-edge flow compression.

For the reasons given above the large-disturbance velocity distribution as predicted by the alternative method of Wilby<sup>5</sup> must also be interpreted as  $KU/U_0$ . However, it should be noted that whilst local values of  $U/U_0$ , evaluated from the variation of  $KU/U_0$  determined by the methods of Wilby<sup>5</sup> and Thompson<sup>1</sup> respectively, will be the same or very nearly so, local values of  $K$ , and hence also  $KU/U_0$ , will not necessarily be the same. This arises because Wilby's method is based on the incompressible flow solution and values of  $KU/U_0$  must, therefore, always include some influence of the downstream surface shape on the upstream flow. Thompson's method, on the other hand, gives values of  $KU/U_0$  which include only a very local downstream influence. In Thompson's method it is the variation of local values of  $K$  from unity that accounts for any more extensive influence arising from the possible existence of a downstream region of subsonic flow.

#### 4.5.3 Compression occurring at supersonic velocities

The case when the locus of possible values of shock-upstream velocity<sup>†</sup> is always *less* than the predicted upstream variation of  $KU/U_0$  is illustrated in Figure 3c. It is suggested that this case should be interpreted as establishing that the region of supersonic velocity is the dominating flow pattern and that the predicted region of small-disturbance subsonic flow does not occur in practice. In this case, as in the case illustrated in Figure 3b when a region of small-disturbance subsonic velocity is not predicted to occur, the factor  $K$  is unity because there is no additional condition of flow continuity to satisfy. The flow compression is then either wholly isentropic (and is evaluated using Figure 14) or is achieved partly through the mechanism of a weak (oblique) shock wave.

<sup>†</sup> Obtained by interpreting a predicted region of downstream subsonic velocity as the locus of possible normal-shock downstream values.

The flow mechanism generating isentropic compression in the supersonic region of flow and, in some cases, causing the formation of weak (oblique) shock waves is discussed in References 1, 4 and 9 from a consideration of the characteristics network in the leading-edge region.

The presence of isentropic compression in the supersonic region of flow and/or of a weak (oblique) shock wave (downstream of which the flow is supersonic) will be given, automatically, by the procedure in Section 5.7. Because a chordwise variation of  $\theta - \omega = \text{constant}$  represents a simple-wave compression (see Reference 10), a chordwise variation of  $\theta - \omega$  which increases with increasing  $x/c$  indicates a compression greater than a simple wave, *i.e.* a shock wave. Applying this criterion, therefore, will indicate whether an oblique shock occurs in the flow field as illustrated in Figure 5. The method given by Section 5.7 is still valid when a weak oblique shock occurs in the flow because the shock, in this particular case, is merely a weak convergence of compression waves. The usual losses associated with the flow through a shock wave are therefore a minimum and the flow compression is nearly isentropic.

## 5. THE METHOD

The procedures in Section 5.1 to 5.7 detail the application of the method to the upper surface of an aerofoil for a given incidence. However, the procedure may readily be applied to the lower surface by inverting the aerofoil and changing the sign of the incidence.

The subsonic region of flow is determined at points corresponding to known values of  $\theta$ . The supersonic region of flow on the other hand, is determined, for the most part, at points corresponding to known values of  $x/c$ . Obviously it is possible to present the calculated pressure distribution either as a function of  $\theta$  or  $x/c$ . Usually it is more convenient to present pressures near the leading edge as a function of  $\theta$  (or indeed  $\phi$  when it is convenient to use the stagnation point as the origin as in the illustrations of Figure 3) and pressures over the remaining part of the surface as a function of  $x/c$ .

The time taken to calculate the velocity distribution on one surface of an aerofoil, without the aid of a computer, will obviously depend on the familiarity of the user with the method and the complexity of the type of flow encountered. Assuming that the chordwise distribution of surface slope and curvature has already been obtained then in the most complex case (*i.e.* a flow with a leading-edge compression at, or to, subsonic velocities (Figure 3e)) a complete calculation for one surface should take approximately 1 – 2 hours when the user is reasonably familiar with the method. However, when such a flow phenomenon does not occur the method is less complicated to use and the calculation time should be halved. Succeeding calculations at different incidences will obviously take less time since many of the parameters will already have been calculated.

### 5.1 Evaluation of Local Radius of Curvature

- (i) Determine local values of  $\theta$  over the whole chord from a knowledge of the aerofoil geometry.
- (ii) Evaluate  $\sin \theta$  at given values of  $x/c$ .
- (iii) Determine  $\rho/c = \left| \frac{d(x/c)}{d(\sin \theta)} \right|$  at sufficient values of  $x/c$  in order to establish a well-defined variation of  $\rho/c$  with  $\theta$ . This will be required in Sections 5.3 to 5.5 but not necessarily for the whole chord.
- (iv) Proceed to Section 5.2.

## 5.2 Determination of Stagnation Point

Since, for each incidence setting, the stagnation point has a unique location it is only necessary to use the following procedure once (*i.e.* when applying the method to the upper surface).

- (i) Evaluate  $z_u/z_l$  at  $x/c = 0.01$ .
- (ii) Determine  $\psi_{st}$  from Figure 6a.
- (iii) Use  $(\rho/c)_{LE}$  to determine  $n_{st}$  from Figure 6b.
- (iv) Determine  $\delta\theta_{st} = (\psi_{st} + \alpha n_{st})$  degrees. (5.1)
- (v) Proceed to Section 5.3.

## 5.3 The First Approximation to the Large-Disturbance Subsonic Flow

Two methods are available for evaluating the large-disturbance flow near the leading edge of round-nosed lifting or non-lifting aerofoils. These are the methods of Wilby<sup>5</sup>, described in Appendix A, and Thompson<sup>1</sup>, detailed in this Section.

Initially, in this Section, values of  $KU/U_0$  are determined which represent a first approximation to the large-disturbance flow. Subsequently values of  $U/U_0$  are found in Section 5.6 after values of  $K$  have been evaluated by the procedure therein.

*Note that steps (i) to (iii) are only necessary for cases where the local radius of curvature, in the region immediately downstream of the stagnation point, either remains constant or decreases with increasing values of  $\phi$ . However, provided  $\rho/\rho_{LE} > 1.0$  when  $\phi$  has increased to a value of about  $15^\circ$ , then it is permissible to assume that  $\Delta_A$ , and hence  $\Delta$ , is unity (unless a detailed velocity distribution is required in the region  $0 < \phi < 15^\circ$ ).*

- (i) Evaluate

$$\bar{\rho}/\rho_{LE} = (1/70) \int_0^{70} (\rho/\rho_{LE}) d\phi. \quad (5.2)$$

(Note that  $\phi = 90^\circ - \theta + \delta\theta_{st}$ .)

- (ii) Use  $\bar{\rho}/\rho_{LE}$  to determine  $\Delta_A$  from Figure 7a.

(Note that if  $\bar{\rho}/\rho_{LE} < 1.0$  then  $\Delta_A > 1.0$ .)

The procedure given in steps (iii) to (x) for evaluating local values of  $KU/U_0$  must be applied to *specific points* on the aerofoil surface, downstream of the stagnation point (*i.e.* to points corresponding to  $\phi > 0$ ), at intervals of  $\theta$  or  $\phi$  sufficiently small to obtain a well-defined velocity distribution (*e.g.*  $\phi = 20^\circ, 40^\circ, 50^\circ, 60^\circ, 65^\circ$  etc. would be suitable). This part of the velocity distribution is most sensibly obtained as a function of  $\theta$  (or  $\phi$ ) since the variation of local velocity with increasing  $x/c$  is very rapid in this region. The procedure is terminated by one of the conditions given in step (x).

The method assumes that  $(\rho/c)_{st} = (\rho/c)_{LE}$ . If, after step (iv) in Section 5.2 has been evaluated, it is found that this is not so then, for the purposes of determining  $\Delta$ ,  $\psi$  and  $n$  in steps (iii), (iv) and (v) below, it must be assumed that at and between the stagnation point and the leading edge  $\rho/c = (\rho/c)_{LE}$ .

(iii) Use  $\Delta_A$  and  $\rho/\rho_{LE}$  to determine  $\Delta$  from Figure 7b at a specific point on the aerofoil surface (say  $\phi = 20^\circ$ ).

(iv) Use  $\rho/\rho_{LE}$  and  $\psi_{st}$  to determine  $\psi$  from Figure 8a.

(Note that when the procedure is being applied to the lower surface by inverting the aerofoil and changing the sign of the incidence then, by the sign convention adopted in defining the stagnation point location, the sign of  $\psi_{st}$  (calculated for the upper surface) must also be reversed.)

(v) Use  $\rho/\rho_{LE}$  and  $n_{st}$  to determine  $n$  from Figure 8b.

(Note that if  $\rho/\rho_{LE} \leq 1.0$  then  $\Delta = \Delta_A$ ,  $\psi = \psi_{st}$  and  $n = n_{st}$ .)

(vi) Use  $\rho/\rho_{LE}$  to determine  $\Theta_{e1}$  from Figure 9.

(Note that if  $\rho/\rho_{LE} \leq 1.0$  then  $\Theta_{e1} = 34.3^\circ$ .)

(vii) Evaluate

$$\tilde{\Theta}_e = 34.3[\theta - (\psi + \alpha n)]/\Theta_{e1}. \quad (5.3)$$

(viii) Use  $\tilde{\Theta}_e$  and  $\rho/\rho_{LE}$  to determine  $(K/\Delta)(U/U_0)$  from Figure 10.

(ix) Knowing  $\Delta$  (step (iii)) and  $(K/\Delta)(U/U_0)$  evaluate  $KU/U_0$ .

(x) Repeat the procedure given in steps 5.3 (iii) to 5.3 (ix) for pairs of values of  $\theta$  and  $\rho/\rho_{LE}$  at points along the aerofoil surface until one of the following conditions is reached:

(a)  $\rho/\rho_{LE} = 100$  with  $KU/U_0 < 1.0$  (an example of this case is the point  $P$  in Figure 3a),

(b)  $KU/U_0 = 1.0$  with  $\rho/\rho_{LE} < 100$  (an example of this case is the point  $S_1$  in Figure 3b)

**If condition (a) occurs** proceed to Section 5.4 wherein the remaining part of the subsonic-flow region, downstream of the point corresponding to  $\rho/\rho_{LE} = 100$ , is determined prior to the evaluation of local values of  $K$ .

**If condition (b) occurs** proceed directly to Section 5.5 (omitting Section 5.4) because it is now necessary to evaluate any leading-edge flow compression that may be present prior to calculating local values of  $K$ .

In some cases  $K$  is unity (e.g. Figure 3b where a region of small-disturbance subsonic flow does not exist) and in others  $K \neq 1.0$  (e.g. Figures 3a, 3e).

## 5.4 The Small-Disturbance Subsonic Flow

The procedure in steps 5.4 (i) to 5.4 (iv) determines the small disturbance part of the subsonic flow region and is first applied at the point corresponding to<sup>†</sup>  $\rho/\rho_{LE} = 100$  (the point  $P'$  in Figure 3a). The procedure must be repeated for sufficient chordwise stations downstream of this point to define adequately the velocity distribution up to, and including, the sonic point,  $S(U/U_0 = 1.0)$ , in Figure 3a.

### (i) Evaluate

$$g = \left[ \frac{0.1}{2(z/c)_{max} - 0.0070\alpha} \right]^{2/3}. \quad (5.4)$$

### (ii) Evaluate $g^\Theta$ for $\rho/\rho_{LE} = 100$ .

### (iii) Use $\rho/c$ and $g^\Theta$ at $\rho/\rho_{LE} = 100$ to determine $U/U_0$ from Figure 11a.

### (iv) Repeat the procedure given by steps 5.4 (i) to 5.4 (iii) for values of $\theta$ downstream of the point corresponding to $\rho/\rho_{LE} = 100$ until $U/U_0 = 1.0$ .

If  $\rho/c \leq 30$ , before sonic velocity ( $U/U_0 = 1.0$ ) is achieved, then local velocities must be determined from Figure 11b.

**OUTPUT – VALUES OF  $U/U_0$  REPRESENTED BY LOCUS  $P'S$  IN FIGURE 3a**

### (v) Proceed now to step (i) of Section 5.6.1 (omitting Section 5.5) which initiates the evaluation of local values of $K$ and the final calculation of the large-disturbance flow.

## 5.5 The Leading-Edge Flow Compression

The various forms in which the flow compression can occur are discussed in Sections 3 and 4 and are determined primarily by the existence or non-existence of a region of subsonic flow downstream of the point at which  $KU/U_0$  initially becomes unity. Criteria, given in step 5.5 (v), for recognising each type may be applied once the necessary preliminary calculations in steps 5.5 (i) to 5.5 (v) have been made.

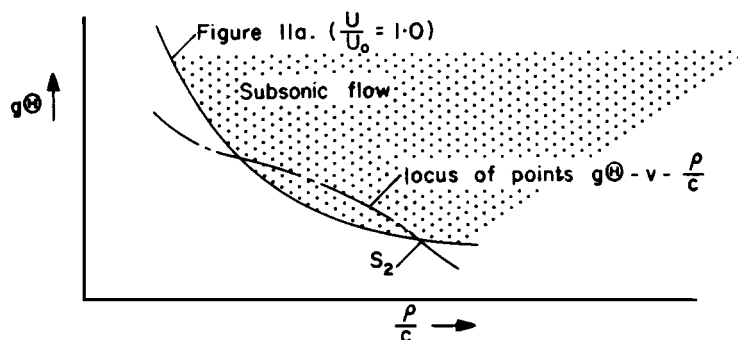
Use the following procedure to establish whether or no subsonic velocities occur at values of  $\theta$  (or  $x/c$ ) downstream of the point corresponding to  $KU/U_0 = 1.0$  (obtained by step 5.3 (x)).

### (i) Evaluate

$$g = \left[ \frac{0.1}{2(z/c)_{max} - 0.0070\alpha} \right]^{2/3} \quad (5.5)$$

### (ii) Evaluate $g^\Theta$ at points on the aerofoil surface downstream of the point corresponding to $KU/U_0 = 1.0$ and construct the locus of points $g^\Theta$ with $\rho/c$ as illustrated in Sketch 5.1. Onto this figure superimpose the variation of $g^\Theta$ with $\rho/c$ for $U/U_0 = 1.0$ , obtained from Figure 11a, as shown in Sketch 5.1 and then proceed according to whether the two curves (a) do, or (b) do not, intersect.

<sup>†</sup> For  $\rho/\rho_{LE} = 100$  read  $\theta = 10^\circ$  if Appendix A has been used.



Sketch 5.1

- (a) **If the two curves intersect** in the manner shown in Sketch 5.1 this establishes that a region of subsonic flow *may* exist downstream of the point at which  $KU/U_0$  initially becomes unity. The second intersection of the two curves determines the downstream limit of this subsonic region and gives the sonic point  $S_2$  where  $U/U_0$  becomes equal to unity (see Figures 3c to 3e). When this circumstance applies the form of the flow compression is determined by the remaining steps in this Section.
- (b) **If the two curves do not intersect** then subsonic velocities, downstream of the point where  $KU/U_0 = 1.0$ , do not occur (as, for example, in Figure 3b). It is then implicit that  $K_p = K = 1.0$ . Values of  $KU/U_0$  determined in Section 5.3 are thus equal to  $U/U_0$ . Any compression that occurs does so in wholly supersonic flow.

**OUTPUT – VALUES OF  $U/U_0$  REPRESENTED BY LOCUS  $OS_1$  IN FIGURE 3b**

Proceed now to Section 5.7, omitting all the remaining steps in this Section and omitting Section 5.6.

- (iii) Using Figure 11a and, if necessary, Figure 11b determine values of  $U/U_0$  for values of  $\theta$  (or  $x/c$ ) between the point  $S_2$  (where  $U/U_0 = 1.0$ ) and the point  $Q$  corresponding to  $dU/d(x/c) = 0^\dagger$  (or  $\rho/c = 0.2$  should this prove to be the limiting condition for the application of Figure 11a in the present context). (These values, in step 5.5 (v), are identified with  $U_2/U_0$ .)

In this context *only*, the limitation implicit in Section 5.4, that Figure 11a is restricted to points corresponding to  $\rho/\rho_{LE} \geq 100$  does not apply. Figure 11a may here be used for all values of  $\rho/c$  greater than about 0.2 in order to locate the point  $Q$ .

- (iv) Determine the first approximation to the supersonic region of flow for a range of values of  $x/c$  on that part of the aerofoil chords between points  $S_1$  and  $E$  (see footnote<sup>‡</sup>) by assuming, at this stage, that  $K = 1.0$  and then using the procedure given in Section 5.7 in conjunction with Figure 15.

The existence or non-existence of a leading-edge (normal) shock can now be established and, in the former case, its location determined as follows.

<sup>†</sup> If more than one minimum point occurs in this flow region  $Q$  is the minimum point nearest the leading edge.

<sup>‡</sup> Point corresponding to same chordwise point as  $S_2$  (Figure 3c) or point at which  $U/U_0$  falls to unity (Figures 3d, 3e).

- (v) Determine the locus of possible values of shock-upstream velocity  $U_1/U_0$  between  $Q'$  (which has the same chordwise location as  $Q$ ) and  $S_2$  as illustrated in Figures 3c to 3e by using the normal shock relationship, which for  $M_0 = 1.0$  reduces to

$$U_1/U_0 = 1/(U_2/U_0)$$

and the locus of possible values of shock-downstream velocity,  $U_2/U_0$ , represented by the subsonic part of the velocity distribution ( $QS_2$ ) determined by step 5.5 (iii).

The locus of possible values of  $U_1/U_0$  may or may not intersect with the predicted variation of  $U/U_0$  between points  $S_1$  and  $E$ . To determine the existence of a leading-edge shock and the value of the factor  $K_p$  (and hence  $K$ ) discriminate between which of the three following situations occurs.

- (a) If the locus ( $Q'S_2$ ) representing  $U_1/U_0$  intersects with the supersonic part of the velocity distribution between  $S_1$  and  $E$  (as, for example, in Figure 3d) this establishes that  $K_p$  (and  $K$ ) is unity and the intersection determines the shock location<sup>†</sup>. Values of  $KU/U_0$  determined in Section 5.3 and by step 5.5 (iv) are thus equal to  $U/U_0$ .

**OUTPUT – VALUES OF  $U/U_0$  REPRESENTED BY LOCUS  $OS_1ABS_2$  IN FIGURE 3d**

Return to Section 5.7 (omitting Section 5.6) to evaluate the remaining supersonic region of flow ( $S_2T$ ) to the trailing edge.

- (b) If the locus ( $Q'S_2$ ) representing  $U_1/U_0$  fails to intersect with the velocity distribution between  $S_1$  and  $E$ , and takes the form illustrated in Figure 3c, this establishes that (i)  $K_p$  (and  $K$ ) is unity and (ii) that the region of small-disturbance subsonic flow ( $QS_2$ ) does not occur. Values of  $KU/U_0$  determined in Section 5.3 are thus equal to  $U/U_0$ .

**OUTPUT – VALUES OF  $U/U_0$  REPRESENTED BY LOCUS  $OS_1$  IN FIGURE 3c**

Discard the previously calculated velocity distribution  $S_1E$  and return to Section 5.7 (omitting Section 5.6) to evaluate the supersonic region of flow between  $S_1$  and the trailing edge using Figure 14.

- (c) If the locus ( $Q'S_2$ ) representing  $U_1/U_0$  fails to intersect with the velocity distribution between  $S_1$  and  $E$  and takes the form illustrated in Figure 3e then this indicates that a shock wave does not occur, that the velocity distribution  $QS_2$  exists and that the first approximation to the region of large-disturbance flow ( $OS_1P$ ) must be modified to satisfy conditions for flow continuity with the region of small-disturbance subsonic flow ( $QS_2$ ).

**OUTPUT – VALUES OF  $U/U_0$  REPRESENTED BY LOCUS  $QS_2$  IN FIGURE 3e**

The results obtained up to step 5.5 (iv) must now be re-interpreted, with  $K \neq 1.0$ , as described in Section 5.6.2 omitting Section 5.6.1.

<sup>†</sup> If the shock-upstream velocity at  $A$  is greater than about  $U/U_0 = 1.15$  for a shock near the leading edge a shock-induced separation may well occur. The calculated velocity distribution in the vicinity of the shock in this particular circumstance may therefore be unrealistic although the separation, initially, is likely to take the form of a local bubble so that the velocity distribution over most of the chord is unaffected.

## 5.6 The Final Calculation of the Large-Disturbance Flow

The procedure in this Section determines the extent to which the first approximation to the large-disturbance flow, calculated in Section 5.3, is affected by the presence of a downstream region of small-disturbance subsonic flow. Local values of  $K$  are determined for those cases where  $K \neq 1.0$  and are then used to evaluate the final stage of the large-disturbance flow from the calculated values of  $KU/U_0$  determined in Section 5.3 and by step 5.5 (iv).

### 5.6.1 Transition to small-disturbance subsonic flow at point where $\rho/\rho_{LE} = 100$

#### (i) Evaluate

$$K_p = [KU/U_0]_p / (U/U_0) \quad (5.6)$$

where the numerator is the value obtained<sup>†</sup> in step 5.3 (x) and the denominator is the corresponding value obtained<sup>‡</sup> in step 5.4 (iii).

- (ii) Use  $\Theta_e$ ,  $\Theta_{ep}$  at  $\rho/\rho_{LE} = 100$  and  $K_p$  to determine local values of  $K$  from Figure 12 at the values of  $\theta$  used in step (x) of Section 5.3 and use these local values of  $K$  to determine  $U/U_0$  from values of  $KU/U_0$  determined by step (x) in Section 5.3.

**OUTPUT – VALUES OF  $U/U_0$  REPRESENTED BY LOCUS  $OP'$  IN FIGURE 3a**

- (iii) Proceed directly to Section 5.7 (omitting Section 5.6.2) to evaluate the supersonic region of flow (ST) to the trailing edge.

### 5.6.2 Transition to small-disturbance subsonic flow at Q

If the circumstance should arise that the point E lies to the left of the point Q in Figure 3e then this part of the method for the final calculation of the large-disturbance flow OQ breaks down. However, the procedure in this Section may be used to obtain an approximation to this flow region by extrapolating the small-disturbance subsonic velocity locus in Figure 3e towards the leading edge (ignoring the region upstream of, and just downstream of, the minimum point). The transition point Q is then taken as the point on the extrapolated locus corresponding to the same chordwise location as the point E where, in this case,  $KU/U_0 = 1.0$ .

- (i) Interpret the calculated velocity distribution (as determined in Section 5.3 and by step 5.5 (iv)) between the stagnation point and the point corresponding to E (Figure 3e) as  $KU/U_0$  (i.e.  $K \neq 1.0$ ) and note the value of  $KU/U_0 = [KU/U_0]_p$  at the point P (the same chordwise location as Q).
- (ii) Obtain the value of  $U/U_0$  at the point Q (Figure 3e) as determined by step 5.5 (iii) or, if necessary, from the extrapolation procedure described above.
- (iii) Evaluate  $K_p = [KU/U_0]_p / (U/U_0)$  where the numerator is given by step 5.6.2 (i) and the denominator by step 5.6.2 (ii).

<sup>†</sup> At the point P (Figure 3a) where  $\rho/\rho_{LE} = 100$ ; or  $\theta = 10^\circ$  if Appendix A has been used.

<sup>‡</sup> At the point P' (Figure 3a) where  $\rho/\rho_{LE} = 100$ ; or  $\theta = 10^\circ$  if Appendix A has been used.