

SOLAR HEATING Total Direct Irradiance Within The Earth's Atmosphere

1. NOTATION

d	concentration of dust in atmosphere	particles/m ³
p_0	atmospheric pressure at sea level	N/m ²
q_e	solar spectral irradiance per unit wavelength interval for unit area normal to solar rays, outside earth's atmosphere at mean earth/sun distance	W/m ³
q_θ	solar spectral irradiance per unit wavelength interval for unit area normal to solar rays, for a point within earth's atmosphere and solar zenith angle, θ	W/m ³
q_N	solar spectral irradiance within earth's atmosphere at a point where solar rays are normal to earth's surface ($\theta = 0$, so that $q_\theta = q_N$)	W/m ³
Q_e	solar constant, $\int_0^\infty q_e d\lambda$. Total solar irradiance outside earth's atmosphere at mean earth/sun distance (taken as 1360 W/m ²)	W/m ²
Q_θ	total solar irradiance for a point within earth's atmosphere and solar zenith angle θ , $\int_0^\infty q_\theta d\lambda$	W/m ²
Q_N	total solar irradiance within earth's atmosphere at a point where solar rays are normal to earth's surface, $\int_0^\infty q_\theta d\lambda$	W/m ²
w	water vapour contained by atmosphere above point considered expressed as equivalent depth when precipitated	m
w_0	value of w for a point at sea level	m
w_s	value of w at a point on earth's surface	m
z	quantity of ozone contained by atmosphere expressed as equivalent depth at N.T.P. (0°C; 101.3×10^3 N/m ² or 760 mm Hg)	m
α	hour angle, $15 \times$ number of hours from solar noon, positive in afternoon	degree
δ	seasonal declination of the sun	degree
ε	geographical latitude, positive in northern hemisphere	degree
η	angle between vertical plane through solar rays and true south (azimuth angle)	degree
θ	angle between solar rays and normal to earth's surface (zenith angle)	degree
λ	wavelength of solar irradiance	m
τ	resultant transmission factor for normal radiation defined by $q_N = q_e \tau$, in which $\tau = \tau_a \tau_w \tau_d \tau_z$	
τ_a	transmission factor associated with scattering by a dry atmosphere	

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τ_w	transmission factor associated with scattering and absorption by atmospheric water vapour	
τ_d	transmission factor associated with scattering by dust in atmosphere	
τ_z	transmission factor associated with scattering and absorption by ozone in atmosphere	
ψ	angle between solar rays and normal to a plane surface	degree

SI units are quoted. The following conversion factors apply:

$$1 \text{ W/m}^2 = 0.3170 \text{ Btu/ft}^2\text{h}$$

$$1 \text{ m} = 3.28 \text{ ft.}$$

2. GENERAL NOTES

The curve q_e on Figure 1 shows the spectrum of solar irradiance outside the earth's atmosphere. The integral of this curve, known as the solar constant (Q_e), can be taken as 1360 W/m^2 for the earth/sun distance at its mean. Seasonal variations occur but these are small (± 3.5 per cent) and have been ignored.

The total direct radiation that falls on a point within the earth's atmosphere is less than Q_e , due to atmospheric attenuation of certain wavebands of the spectral irradiance, q_e . This attenuation arises from scattering and absorption by the constituents of the atmosphere as described below, and is represented quantitatively by the transmission factors τ_a , τ_w *et cetera*.

- (i) Scattering by a dry atmosphere (τ_a) is significant at altitudes less than 30 000 m (100 000 ft) and increases with pressure to a maximum at sea level.
- (ii) Scattering and absorption by water vapour (τ_w) is a maximum at sea level where the water vapour content of the atmosphere is greatest. A reduction (assumed to be parabolic) with altitude of the atmospheric water vapour content leads to a corresponding reduction in attenuation, tending to zero somewhere between 9000 m (30 000 ft) and 16 000 m (52 000 ft). The total water vapour content of the atmosphere itself varies with season and position on the globe.
- (iii) Scattering by dust (τ_d) is most significant below 1000 m (3000 ft), although particles may rise to altitudes of 4000 m (13 000 ft).
- (iv) Scattering and absorption by the ozone layer (τ_z) situated between 11 000 m (36 000 ft) and 33 000 m (108 000 ft) is such that the total attenuation is a maximum at and below 11 000 m (36 000 ft).

The cumulative effect of these constituents is illustrated by the curve q_N in Figure 1, typical for normal radiation at sea level. The total irradiance Q_N represented by this curve is 1010 W/m^2 , representing a typical maximum for the tropics.

For the more general case of solar radiation that is inclined to the earth's surface, the path length through the atmosphere of the incident energy is greater than that for normal radiation by the factor $\sec \theta$, and the

transmission factor $\tau^{\sec \theta}$ applies. The total irradiance on a plane normal to the solar rays (see Figure 6) is then less than when the rays are normal to the earth's surface, and is given by the expression in the Derivation. Note that when the *receiving plane* is inclined to the solar rays the irradiance is reduced still further (see Section 3 below).

It should be noted that in estimating the effect of atmospheric attenuation, simplifying assumptions have been made with the objective of obtaining an overestimate rather than an underestimate of solar irradiance. Slightly lower values of irradiance might therefore be expected from a more rigorous approach (see, for example, Reference 7). It should also be noted that no allowance has been made in this Item for the presence of cloud cover or reflected irradiance. These effects, which may in particular circumstances be significant, are treated in Reference 7.

Since this Item was originally issued, worldwide ground-level estimates of direct solar irradiance have been made using a method employing Angstrom turbidity coefficients, and these estimates have been compared with those obtained using this Item (see Reference 8). Within the conditions stated in Sections 2 and 3, the method and data presented in this Item were found to lead to a more representative distribution of insolation values throughout the world than that given by the turbidity coefficients method.

3. USE OF THE CURVES

Figures 2, 3 and 4 enable the total direct solar irradiance to be determined for a point within the earth's atmosphere receiving radiation normal to the earth's surface. The data apply to a cloudless atmosphere. Figure 5 gives mean monthly values of the solar zenith angle θ for use with Figure 6 in the determination of total irradiance when, as is more usual, $\theta \neq 0$. The curves on Figure 5 are symmetric about the axis through solar noon.

The solar heating curves are based on the transmission factors given in Derivation 6. The full lines on Figures 3, 4 and 6 are based on the assumption that water vapour does not rise above 9000 m (30 000 ft). It might equally be assumed that water vapour rises to a height of 16 000 m (52 000 ft), in which case the broken lines on Figures 3 and 4 apply. Figure 6 can be used as before, although this results in a maximum overestimation of 4 per cent of the solar irradiance (for $w_0 = 0.06$ m, $\sec \theta = 5$ at 9000 m). It can be seen from Figure 3 that these two assumptions provide upper and lower limits for an estimate of solar irradiance for altitudes between about 6000 m (20 000 ft) and 16 000 m (52 000 ft). Values of $d = 300 \times 10^6$ particles/m³ and $z = 0.0028$ m have been assumed throughout. For a more precise calculation of solar irradiance involving other values of d or z reference should be made to Derivation 6 or Reference 7.

The first step is to determine a value for w_0 at the locality being considered. For sea level locations, this can be obtained directly from Figure 1 since w_0 is then equal to w_s , and where w_s relates to terrain that is significantly above sea level Figure 4 is used to determine an equivalent value of w_0 . Figure 3 can then be used to find Q_N / Q_e for any given altitude.

Reference is then made to Figure 5 and if this shows that the problem involves $\theta \neq 0$, then Figure 6 is used to determine Q_θ / Q_N for the appropriate values of $\sec \theta$, w_0 and altitude. The range $1 \leq \sec \theta \leq 5$ covers solar zenith angles θ up to 78.5°.

The irradiance falling on a plane that is not normal to the solar rays is determined from the relationship $Q_\psi = Q_\theta \cos \psi$, and depends on the orientation of the receiving plane, the zenith angle θ and the azimuth angle η . The azimuth angle is tabulated in Derivation 4 and is given by the expression

$$\cot \eta = \frac{\sin \epsilon \cos \alpha - \cos \epsilon \tan \delta}{\sin \alpha}.$$

The absorption of radiant energy by some materials is restricted to certain specific wavebands. A table of values is therefore given on Figure 1, showing (for the two spectral distributions) the proportion of the total solar radiation transmitted within the wavebands indicated on the abscissa.

4. DERIVATION

The formulae
$$Q_{\theta} = \int_0^{\infty} q_e \tau^{\sec \theta} d\lambda$$

$$\cos \theta = \cos \epsilon \cos \delta \cos \alpha + \sin \epsilon \sin \delta.$$

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| 1. | MOON, P. | Proposed standard solar-radiation curves for engineering use. <i>J. Franklin Inst.</i> , Vol. 230, No. 5, pp. 583-617, November 1940. |
| 2. | GAST, P.R. | <i>Handbook of Geophysics</i> , Chapter 16, pp. 14-32, MacMillan Co., New York, 1960. |
| 3. | BANNON, J.K.
STEELE, L.P. | Average water vapour content of the air. <i>Geophys. Mem.</i> , Vol. 13, No. 102, London, 1960. |
| 4. | Institution of Heating and
Ventilating Engineers | Solar altitude and azimuth angles. I.H.V.E. Guide 1965, Table 6.3, pp. 149-151. |
| 5. | STAIR, R.
ELLIS, H.T. | The solar constant based on new spectral irradiance data from 310 to 530 nanometers. <i>J. appl. Met.</i> , Vol. 7, pp. 635-644, August 1968. |
| 6. | BENOY, M.B. | The determination of solar heating. <i>Aeronaut. J.</i> , Vol. 74, January 1970. |

5. REFERENCES

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|----|---------------|---|
| 7. | ROBINSON, N. | <i>Solar radiation</i> . Elsevier Publishing Co., Amsterdam, 1966. |
| 8. | CLIFTON, C.J. | Worldwide calculations of total direct ground-level radiation. Comparison of 69015 method with one using Angstrom turbidity coefficients. ESDU Memor. No. 15, 1975. |

6. EXAMPLE

6.1 Example 1

It is required to determine the maximum solar irradiance likely during the month of January in Madrid, Spain (latitude 40°20' N). The city is situated in terrain approximately 3000 ft above sea level.

From the data on Figure 2a it will be assumed that $w_s = 0.01$ m and from Figure 4 at an altitude of 3000 ft, $w_s/w_0 = 0.59$.

Hence, the equivalent value

$$w_0 = \frac{0.01}{0.59} = 0.017 \text{ m},$$

and from Figure 3, by interpolation

$$\frac{Q_N}{Q_e} = 0.80,$$

so that $Q_N = 1360 \times 0.80 = 1090 \text{ W/m}^2$.

From Figure 5, at 1200 h on latitude 40°N during the month of January, $\theta = 59^\circ$.

Hence, $\sec\theta = 1.94$, and from Figure 6 by interpolation for an altitude of 3000 ft and $w_0 = 0.017 \text{ m}$

$$Q_\theta/Q_N = 0.89.$$

Thus $Q_\theta = 0.89 \times 1090 = 970 \text{ W/m}^2$ or $307 \text{ Btu/ft}^2\text{h}$.

6.2 Example 2

It is required to determine the total solar radiant heat absorbed during a day in December by the flat top of an upright cylindrical storage vessel situated in Durban, S. Africa (latitude $29^\circ 50'\text{S}$, assuming cloudless conditions from sunrise to sunset. The vessel is 25 ft in diameter and has a surface absorptivity of 0.5.

Now, in time Δt , total heat absorbed by the top of the vessel,

$$\begin{aligned} &= \frac{\pi}{4} \times 25^2 \times 0.5 \Sigma Q_\psi \Delta t, \\ &= \frac{25^2 \pi}{8} \Sigma Q_\theta \cos \psi \Delta t, \\ &= \frac{25^2 \pi}{8} Q_N \Sigma \left(\frac{Q_\theta}{Q_N} \right) \cos \psi \Delta t. \end{aligned}$$

From Figure 2a, $w_0 = w_s = 0.04 \text{ m}$

and from Figure 3 $\frac{Q_N}{Q_e} = 0.706$,

so that $Q_N = 0.706 \times 1360 = 960 \text{ W/m}^2$ or $304 \text{ Btu/ft}^2\text{h}$.

The summation can be performed as follows by utilising the symmetry of the curves on Figure 5, and the fact that for a horizontal surface $\psi = \theta$.