

THE DEFLECTIONS AND SLOPES OF SHAFTS OR BEAMS OF CONSTANT OR STEPPED SECTION

1. NOTATION

A, B, C	coefficients defining second moment of area distribution	in^{-4}	m^{-4}
a	distance from left-hand support to point load	in	m
b	distance from left-hand support to applied moment	in	m
c	length of uniformly distributed load	in	m
d	distance of section step from left-hand support (see illustration)	in	m
E	modulus of elasticity of shaft material	lbf/in^2	N/m^2
I	second moment of area	in^4	m^4
L	length of shaft between supports	in	m
l	length of shaft of constant cross section	in	m
M	applied moment, clockwise positive	lbf in	N m
n	total number of sections of shaft		
P	point load, downwards positive	lbf	N
p	uniformly distributed load, downwards positive	lbf/in	N/m
R_1, R_2, R_3	reactions at bearings 1, 2 and 3	lbf	N
x	distance from left-hand support	in	m
y_x	deflection at x , downwards positive	in	m
α, β	functions of d and l		
Δ_M	coefficient in the expression $y_x = \frac{ML^2}{E} \Delta_M$	in^{-4}	m^{-4}
Δ_P	coefficient in the expression $y_x = \frac{PL^3}{E} \Delta_P$	in^{-4}	m^{-4}
θ_x	slope at x , clockwise positive	rad	rad

Φ_M coefficient in the expression $\theta_x = \frac{ML}{E}\Phi_M$ in⁻⁴ m⁻⁴

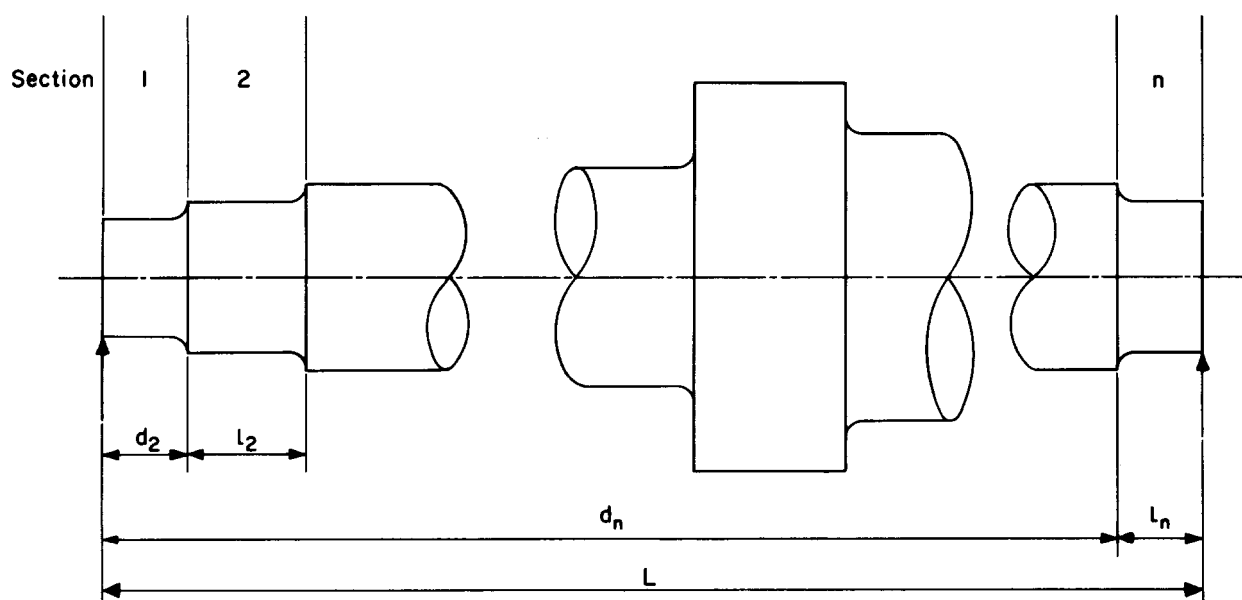
Φ_P coefficient in the expression $\theta_x = \frac{PL^2}{E}\Phi_P$ in⁻⁴ m⁻⁴

$\delta_i, \delta_{ii}, \delta_{iii}, \phi_i, \phi_{ii}, \phi_{iii}$ functions of position of point load

$\delta_{iv}, \delta_v, \delta_{vi}, \phi_{iv}, \phi_v, \phi_{vi}$ functions of position of applied moment

Suffixes

1, 2, 3, ..., n of section 1, 2, 3, ..., n



Sketch 1.1 Dimensions of a shaft with n sections

2. INTRODUCTION

This Data Item gives a method of calculating the slope and deflection at any point along the length of a shaft or beam* of constant or stepped cross section under any number of point loads, distributed loads and applied moments.

A calculation procedure is described in Section 3 for cases in which the shaft is simply-supported and carries point loads and moments in one plane. In Section 4 details are given of means of extending the calculation procedure to include treatment of uniformly distributed loads, overhung loads, shafts supported on three bearings and out-of-plane loads.

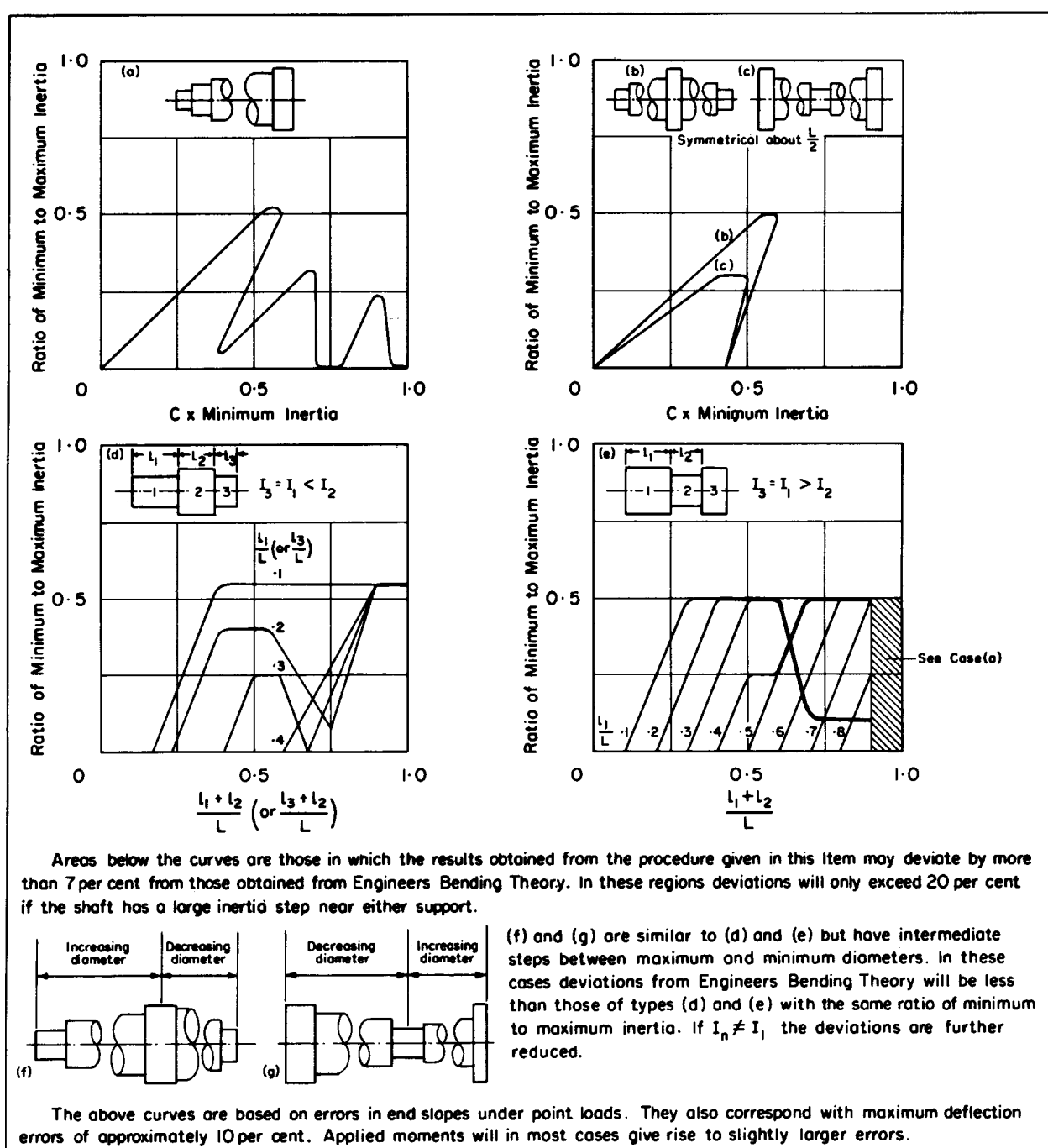
The procedure is based on an approximation to Engineers Bending Theory whereby the inertia of the shaft

* Although throughout this Item specific reference is made to the analysis of circular shafts, the procedures may be used for shafts or beams of any cross section.

is fitted by a Fourier series. The degree of this approximation for each type of shaft which can be covered by this Item is shown in Table 2.1. In addition the normal limitations of Engineers Bending Theory apply and no allowance is made for deformations due to shear.

The Data Item is equally applicable to shafts stepped by machining and to shafts with shrunk-on collars of the same modulus of elasticity since these behave identically until slip occurs at the mating surfaces. Where the collar is a slide fit over the shaft the effective second moment of area of the assembly will have its value between the inertia of the basic shaft and the inertia of the assembly assuming a shrink fit.

TABLE 2.1



3. THE CALCULATION PROCEDURE

3.1 Summary

The shaft geometry is first analysed and defined by A , B and C (which are the first three coefficients in a Fourier series). The slope or deflection at any point x is then given by the addition of the values obtained when each load and moment is considered to act alone on the shaft.

3.2 Calculation Details

The coefficients A , B and C^* are given by the following equations.

$$A = \frac{\alpha_1}{I_1} + \frac{\alpha_2}{I_2} + \frac{\alpha_3}{I_3} + \dots \frac{\alpha_n}{I_n},$$

where values of α for each length of constant cross section between the supports (denoted by the suffixes 1, 2, 3 ... n) are obtained from Figure 1 with the relevant values of d/L and l/L .

$$B = \frac{\beta_1}{I_1} + \frac{\beta_2}{I_2} + \frac{\beta_3}{I_3} + \dots \frac{\beta_n}{I_n}$$

where values of β for each length of constant cross section are obtained from Figure 2 with the relevant values of d/L and l/L .

$$C = \frac{1}{L} \left\{ \frac{l_1}{I_1} + \frac{l_2}{I_2} + \frac{l_3}{I_3} + \dots \frac{l_n}{I_n} \right\},$$

where, again, the suffixes refer to each length of constant cross section.

The slope and deflection at any point x on a shaft subjected to a single point load P at a distance a from the left-hand support are given by:

$$\theta_x = \frac{PL^2}{E} \Phi_P$$

and
$$y_x = \frac{PL^3}{E} \Delta_P$$

where
$$\Phi_P = A\phi_i + B\phi_{ii} + C\phi_{iii}$$

and
$$\Delta_P = A\delta_i + B\delta_{ii} + C\delta_{iii}.$$

For the location of the requisite values of ϕ and δ , see Table 3.1.

The slope and deflection at any point x on a shaft subjected to a single applied moment M at a distance b from the left-hand support are given by:

* For a shaft having a total length of constant cross section, $A = B = 0$. For a shaft symmetrical about $L/2$, $A = 0$.

$$\theta_x = \frac{ML}{E} \Phi_M$$

and $y_x = \frac{ML}{E} \Delta_M,$

where $\Phi_M = A\phi_{iv} + B\phi_v + C\phi_{vi}$

and $\Delta_M = A\delta_{iv} + B\delta_v + C\delta_{vi}.$

For the location of the requisite values of ϕ and δ , see Table 3.1.

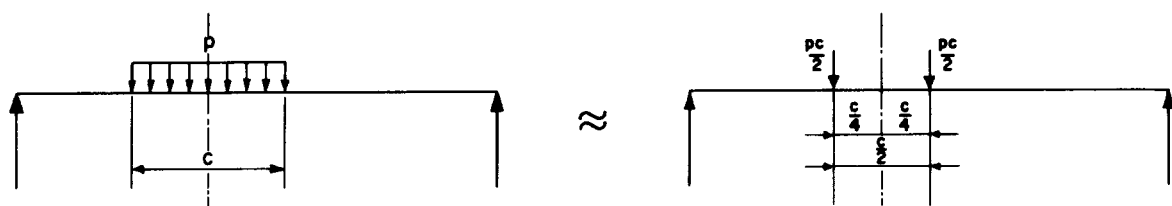
TABLE 3.1

Result	Slope								Deflection					
	$x = 0$		$x \geq 0$											
Loading	P	M	P			M			P			M		
Function	ϕ_i ϕ_{ii} ϕ_{iii}	ϕ_{iv} ϕ_v ϕ_{vi}	ϕ_i	ϕ_{ii}	ϕ_{iii}	ϕ_{iv}	ϕ_v	ϕ_{vi}	δ_i	δ_{ii}	δ_{iii}	δ_{iv}	δ_v	δ_{vi}
Figure	3	4	4	4	7	8	9	10	11	12	13	14	15	16

4. EXTENSIONS TO THE CALCULATION PROCEDURE

4.1 Uniformly Distributed Loads

Uniformly distributed loads may be treated with the following approximation.

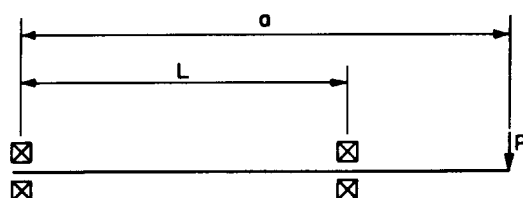


Sketch 4.1

The maximum errors given by this approximation are obtained when $c = L$ and $x = 0$ for slope (12 per cent error) and when $c = L$ and $x = L/2$ for deflection (10 per cent error).

4.2 Overhanging Loads and Moments

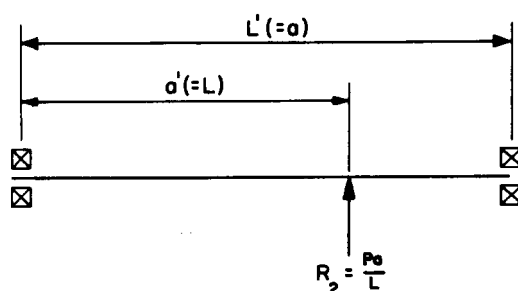
4.2.1 Overhanging point load



Sketch 4.2

To calculate slopes and deflections between the supports, P may be resolved into a moment $M = P(a - L)$ acting at the overhung bearing.

To calculate slopes and deflections between $x = L$ and $x = a$, an equivalent simply-supported shaft may first be analysed. The dimensions of this shaft are distinguished by superscripts.

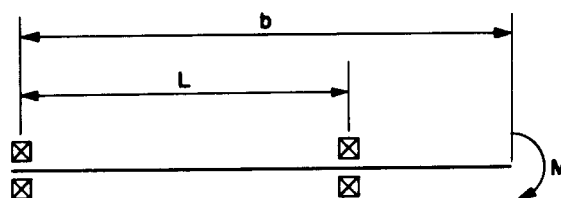


Sketch 4.3

Slopes and deflections (θ'_x and y'_x) obtained from this shaft are then corrected for zero deflection at R_2 .

$$\text{Actual slope } \theta_x = \theta'_x - \frac{y'_x = L}{L} \text{ and actual deflection } y_x = y'_x - \frac{xy'_x = L}{L}$$

4.2.2 Overhanging Applied Moment



Sketch 4.4

To calculate slopes and deflections between the supports, the shaft may be analysed as a simply-supported shaft with moment M at the overhung bearing.