

ELASTIC STRESSES AND DEFLECTIONS FOR LONG RECTANGULAR PLATES WITH SMALL INITIAL CURVATURE UNDER UNIFORMLY DISTRIBUTED NORMAL PRESSURE ON THE CONCAVE FACE

1. NOTATION

a	length of plate	m	in
b	width of plate	m	in
E	Young's modulus	N/m ²	lbf/in ²
f_E	maximum total tensile stress at edge of plate	N/m ²	lbf/in ²
f_m	tensile stress at middle surface of plate	N/m ²	lbf/in ²
f_T	maximum total tensile stress at centre of plate	N/m ²	lbf/in ²
p	normal pressure on plate	N/m ²	lbf/in ²
r	ratio $(1 - \nu^2)/0.91$ (when $\nu = 0.3$, $r = 1$)		
t	plate thickness	m	in
δ	maximum deflection of plate from unloaded form	m	in
δ_0	maximum initial bow	m	in
ν	Poisson's ratio		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

Curves are given that enable the maximum deflections and stresses to be determined for rectangular plates of uniform thickness, initially curved across the width b , under uniformly distributed normal pressure on the concave face.

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In the figures, δ/t and $\frac{f}{p}\left(\frac{t}{b}\right)^2$ are plotted against $\frac{b}{t}\left(r\frac{p}{E}\right)^{1/4}$ for various values of δ_0/t . Table 2.1 shows the figures to be used for a particular stress or deflection and a given form of edge restraint.

TABLE 2.1

Type of edge restraint		Number of figure giving			
Rotation	In-plane translation	δ	f_E	f_T	f_m
Fixed	Fixed	1	2	3	6
Free	Fixed	4		5	7

The bending stresses may be obtained by subtracting f_m from f_E or f_T ; when the edge is free in rotation the edge stresses due to bending are zero and therefore $f_E = f_m$.

When there is no in-plane translational restraint the deflections and stresses can be obtained using the equivalent flat plate values from Figures 1 and 2 of Data Item No. 69018.

For values of δ_0/t up to 5 two initial shapes have been considered. On each figure the full line curves relate to plates having a circular arc form and the broken line curves relate to plates having a sine wave form. For greater values of δ_0/t curves for circular arc form only are given as this form is thought to be more typical.

The curves are derived for infinitely long plates but may be used with reasonable accuracy for values of a/b not less than 3. They are also derived on the basis of thin plate theory which is known to be valid for b/t not less than 20.

It is assumed that the plate has small initial curvature across width b with the maximum initial bow at the centre. The curves may be used with reasonable accuracy for values of δ_0/b not greater than 0.1. It is further assumed that the edge restraint is uniform along the long edges and that these edges remain in their original plane.

The curves are derived for a pressure normal to the plane of the two edges but may be applied with sufficient accuracy to the case where the pressure is normal to the surface.

The curves apply only to stresses within the elastic range of the material but are considered reasonably accurate beyond this provided the stresses do not exceed the 0.2 per cent proof stress.

The stresses and deflections for long flat rectangular plates under uniformly and linearly varying pressure and square plates under uniform pressure are given in Engineering Sciences Data Items Nos. 69018 and 70001 respectively.

3. DERIVATION

1. TIMOSHENKO, S. *Theory of plates and shells*. Second edition. Chapter 1.
WOINOWSKY-KRIEGER, S. McGraw-Hill, New York, 1959.

4. EXAMPLE

A rectangular plate with its long edges fixed against movement in translation and rotation is subjected to a uniform normal pressure of 7.5 lbf/in^2 .

It is required to find the maximum deflection and the total and middle surface stresses in the plate, (a) when it is initially flat and (b) when it has a circular arc form of initial bow of 0.15 in. The length $a = 48 \text{ in}$, the width $b = 10 \text{ in}$, and the thickness $t = 0.048 \text{ in}$, E for the plate material is $10.4 \times 10^6 \text{ lbf/in}^2$ and Poisson's ratio is 0.33.

From the above information $a/b = \frac{48}{10} = 4.8$ and $\delta_0/b = \frac{0.15}{10} = 0.015$. Therefore this Item may be used for this problem.

The parameters required for use with this Item are tabulated.

	$\frac{b}{t} \left(r \frac{p}{E} \right)^{1/4}$	6.04
Case a	$\frac{\delta_0}{t}$	0
Case b	$\frac{\delta_0}{t}$	3.12
	$p \left(\frac{b}{t} \right)^2$	$32.6 \times 10^4 \text{ lbf/in}^2$

The values obtained from the curves are tabulated.

Figure used	Case a $\delta_0/t = 0$		Case b $\delta_0/t = 3.12$	
	Ratio obtained from curves	Hence stress or deflection	Ratio obtained from curves	Hence stress or deflection
1	$\frac{\delta}{t}$ 3.63	δ 0.174 in	$\frac{\delta}{t}$ 1.55	δ 0.0744 in
2	$\frac{f_E}{p} \left(\frac{t}{b} \right)^2$ 0.162	f_E 52 800 lbf/in ²	$\frac{f_E}{p} \left(\frac{t}{b} \right)^2$ 0.080	f_E 26 100 lbf/in ²
3	$\frac{f_T}{p} \left(\frac{t}{b} \right)^2$ 0.042	f_T 13 700 lbf/in ²	$\frac{f_T}{p} \left(\frac{t}{b} \right)^2$ 0.030	f_T 9800 lbf/in ²
6	$\frac{f_m}{p} \left(\frac{t}{b} \right)^2$ 0.027	f_m 8800 lbf/in ²	$\frac{f_m}{p} \left(\frac{t}{b} \right)^2$ 0.024	f_m 7820 lbf/in ²