

ELASTIC STRESSES AND DEFLECTIONS FOR FLAT SQUARE PLATES UNDER UNIFORMLY DISTRIBUTED NORMAL PRESSURE

1. NOTATION

b	length of edge of plate	m	in
E	Young's modulus	N/m ²	lbf/in ²
f_C	maximum total stress at centre, same in all directions	N/m ²	lbf/in ²
f_c	stress at middle surface of plate at centre, same in all directions	N/m ²	lbf/in ²
f_D	maximum total stress on diagonals	N/m ²	lbf/in ²
f_d	stress at middle surface corresponding to f_D	N/m ²	lbf/in ²
f_E	maximum total stress at mid-point of edge of plate	N/m ²	lbf/in ²
f_e	stress at middle surface of plate corresponding to f_E	N/m ²	lbf/in ²
f_N	maximum total stress on centre-lines	N/m ²	lbf/in ²
f_n	stress at middle surface of plate corresponding to f_N	N/m ²	lbf/in ²
p	normal pressure on plate	N/m ²	lbf/in ²
r	ratio $(1 - \nu^2)/0.91$ (when $\nu = 0.3, r = 1$)		
t	plate thickness	m	in
δ	maximum deflection of plate	m	in
ν	Poisson's ratio		

Suffixes

- 1 denotes stress in direction of edge, diagonal or centre-line
- 2 denotes stress normal to edge, diagonal or centre-line

Convention: Tensile stresses are positive. Compressive stresses are negative.

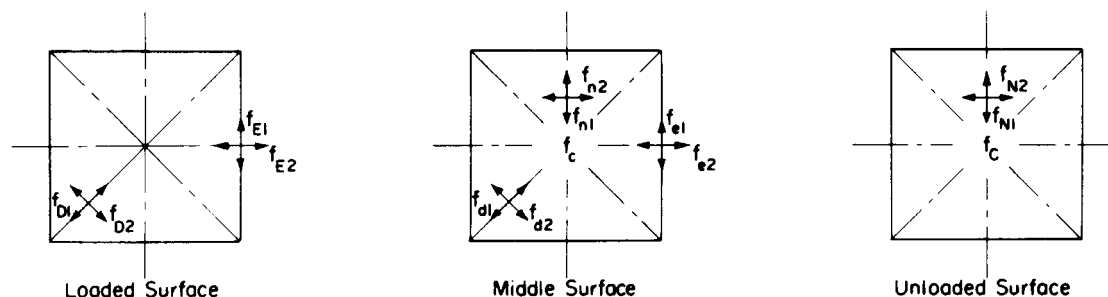
Both SI and British units are quoted but any coherent system of units may be used.

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2. NOTES

Curves are given that enable the stresses and deflections to be determined for initially flat square plates of uniform thickness under uniformly distributed normal pressure. The data given can also be obtained using the computer programs ESDUpacs A9311 and A9433 associated with Data Items Nos. 93011* and 94033† respectively.

In Figure 1 δ/t is plotted against $(b/t)(rp/E)^{1/4}$ for various forms of edge restraint. In Figures 2 to 5 values of $(f/p)(t/b)^2$ are plotted against $(b/t)(rp/E)^{1/4}$ and are grouped according to the form of edge restraint. Curves are given for the maximum total stress at the centre, and at the edge, and for the maximum values on the diagonals or centre-lines of the plate where these are greater. The corresponding middle surface stresses are also given. The centre diagram below shows the locations of the middle surface stresses in the plate, and the side diagrams show the locations of the maximum total stresses.



The total stresses f_C , f_D , f_E and f_N result from the combined effects of bending and middle surface tension or compression. The bending stresses can be found by subtracting the middle surface stresses f_c , f_d , f_e and f_n from the corresponding total stresses. Because of symmetry all the stresses are principal stresses.

In Figure 2 for values of $(b/t)(rp/E)^{1/4} < 1.3$ the maximum surface stress f_{D2} on the loaded face is located at the centre and is to be associated with f_c when calculating the appropriate bending stress.

* Data Item No. 93011 "Flat rectangular orthotropic plates under uniformly distributed normal pressure. Elastic stresses and deflections for various forms of edge restraint." (Structures Sub-series).

† Data Item No. 94033 "Flat rectangular isotropic plates under uniformly distributed normal pressure. Elastic deflections, stresses and strains, moments and forces, for various forms of edge restraint." (Stress and Strength Sub-series).

Table 2.1 shows the figure to be used for a particular stress and a given form of edge restraint.

TABLE 2.1

Type of edge restraint		Number of figure	Stress f (denoted by its suffixes)														Derivations used
			Total							Middle surface							
Rotation	In-plane translation		C	D1	D2	E1	E2	N1	N2	c	d1	d2	e1	e2	n1	n2	
Free	Free	2	x	<	x	x	0	<	<	x	<	x	x	0	<	<	1, 2, 8, 10, 11 and 12
Fixed	Free	3	x	<	<	<	x	<	<	x	<	<	<	0	<	<	2, 7, 8, 9 and 14
Free	Fixed	4	x	<	<	<	x	x	<	x	<	<	<	x	x	<	2, 3, 8, 9 and 13
Fixed	Fixed	5	x	<	<	<	x	<	<	x	<	<	<	x	<	<	2, 4, 5, 6, 7, 8 and 10

x denotes that a curve is given

< denotes that no curve is given because the values are lower than the maximum

0 denotes that the stress is zero.

It is assumed that the plate is initially flat, with no irregularities and that the edges are constrained to remain in the original plane of the undeflected middle surface. It is also assumed that when the edges are free in translation they are free to curve (in plane) along their length.

The curves are derived on the basis of thin plate theory which is certainly valid for b/t greater than 20. They apply only to stresses within the elastic range of the material but are considered to be reasonably accurate beyond this provided the stresses do not exceed the 0.2 per cent proof stress.

The stresses and deflections for long flat rectangular plates under uniform and linearly varying pressure and rectangular plates with initial curvature under uniform pressure are given in Engineering Sciences Data Items Nos 69018 and 69019 respectively.

3. DERIVATION

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4. EXAMPLE

A square plate of width $b = 10$ in, and thickness $t = 0.048$ in, with its edges free in rotation and translation is subjected to a uniform normal pressure of $p = 3.5$ lbf/in². Find the maximum deflection and total, bending, and middle surface stresses in the plate if E for the plate material is 10.4×10^6 lbf/in² and Poisson's ratio is 0.33.

For this material, $\nu = 0.979$.