

DESIGN FOR MINIMUM WEIGHT. COMPRESSION PANELS WITH UNFLANGED INTEGRAL STIFFENERS HAVING DIMENSIONAL RESTRICTIONS

1. NOTATION

b	stiffener pitch	m	in
B	$(\eta_b b)/(\eta_{b0} b_0)$		
E	Young's modulus of panel material	N/m ²	lbf/in ²
E_t	tangent modulus of panel material at stress f	N/m ²	lbf/in ²
f	average compressive stress in panel at failure	N/m ²	lbf/in ²
F	$(\eta_{a0} f)/(\eta_a f_0)$		
f_E	overall buckling stress of panel	N/m ²	lbf/in ²
f_L	local buckling stress of panel	N/m ²	lbf/in ²
f_n	stress at which $E_t = \frac{1}{2}E$ (see Item No. 76016)	N/m ²	lbf/in ²
h	depth of stiffener (measured from skin centre-line, see Figure 3)	m	in
H	$(\eta_b h)/(\eta_{b0} h_0)$		
l'	equivalent length of panel (see Data Sheet Struct. 01.01.01)	m	in
m	material characteristic (see Item No. 76016)		
P	compressive load per unit width acting on panel	N/m	lbf/in
t	thickness of skin	m	in
T	$(\eta_a t)/(\eta_{a0} t_0)$		
t_s	thickness of stiffener	m	in
T_s	$(\eta_a t_s)/(\eta_{a0} t_{s0})$		
w	mass per unit area of panel	kg/m ²	lb/in ²
W	$(\eta_a w)/(\eta_{a0} w_0)$		
η_a	$(\eta_E \eta_L)^{1/4}$ when η_E and η_L are calculated at stress f for panel of least weight having dimensional restrictions		

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η_b	$(\eta_E^3/\eta_L)^{1/8}$ when η_E and η_L are calculated at stress f for panel of least weight having dimensional restrictions		
η_{a0}, η_{b0}	plasticity correction factors at stress f_0 for panel of least weight having no dimensional restrictions		
η_E	plasticity correction factor for overall buckling		
η_L	plasticity correction factor for local buckling		
ρ	density of panel material	kg/m ³	lb/in ³

Suffixes

0	suffix added to b, f, h, t, t_s and w when they relate to panel of least weight having no dimensional restrictions
e	suffix added to $b, f, f_L, f_E, h, t, t_s$ and w indicating that they are calculated on assumption that panel is elastic

Both SI and British units are quoted but any coherent system of units may be used.

2. INTRODUCTION

This Item gives the cross-sectional dimensions for minimum weight of uniform compression panels having unflanged integral stiffeners. Data are also given for determining the dimensions of the panel of least weight when one or more of the cross-sectional dimensions is fixed or restricted. An allowance is made for the effects of initial irregularities and plasticity correction factors are included.

The lower of f_L or f_E has been taken to be the limiting stress and, if no restrictions are imposed and no initial irregularities are present, the least-weight panel is found to occur when the local and overall buckling modes coincide. In practical panels as f approaches f_L the presence of initial irregularities considerably reduces the overall stiffness of the panel. This effect has been taken into account by replacing E with $0.6E/(f/f_L)$ for overall buckling when $0.6 \leq f/f_L \leq 1.0$. Below $f/f_L = 0.6$ the effects of initial irregularities on panel stiffness are disregarded. These assumptions provide a satisfactory representation of the panel behaviour if irregularities typical of aircraft construction are present and result in a family of panels of equal least weight with $f = f_E$ and $0.6 \leq f/f_L \leq 1.0$.

For panels having dimensional restrictions the condition $f = f_E \leq f_L$ will usually provide the least-weight panel. However, if h is restricted to a value above h_0 the least-weight panel may occur when $f = f_L \leq f_E$.

The panel is assumed to be sufficiently wide for the effects of the side restraints to be negligible. Provided that the panel width is at least one third the panel length no significant error results from this assumption. Allowance for the type of edge restraint acting at the loaded ends is made by adjustment of the length as indicated by the table on Page 2 of Data Sheet Struct. 01.01.01.

In determining the local buckling stresses full account has been taken of coupling between the skin and the stiffeners. The effect of shear deflection on the overall buckling mode is negligible and is therefore excluded from the calculations. Poisson's ratio below the limit of proportionality is taken as 0.3 throughout the calculations; likely variation will not significantly alter the weight of the optimum panel.

3. THE PANEL OF LEAST WEIGHT HAVING NO RESTRICTIONS ON CROSS-SECTIONAL DIMENSIONS

Table 3.1 gives expressions for f_0 , w_0 , t_0 , t_{s0} , b_0 and h_0 for the family of panels of least weight that occurs when $0.6 \leq f/f_L \leq 1.0$. The expressions show w_0 to be unaffected by f/f_L . At the extremes of this range the data will be accurate but with values of f/f_L between about 0.8 and 0.85 very well made panels will have a small reserve (not greater than 5 per cent) against f_E .

For an elastic panel $\eta_{a0} = \eta_{b0} = 1.0$, and so f_{0e} , w_{0e} , t_{0e} , t_{s0e} , b_{0e} and h_{0e} can be evaluated. For the inelastic panel the values of η_{a0} and η_{b0} may be obtained from Figures 1 and 2 by using f_{0e} , obtained assuming the panel to be elastic, and the appropriate values of f_n and m .

TABLE 3.1 Equations for Stress and Dimensions for the Family of Panels of Least Weight for which
 $0.6 \leq f/f_L \leq 1.0$

f_0	$0.7151\eta_{a0}E(P/EI')^{1/2}$
w_0	$1.3985l'\rho(1/\eta_{a0})(P/EI')^{1/2}$
t_0	$0.5852l'(1/\eta_{a0})(P/EI')^{1/2}$
t_{s0}	$1.3077l'(1/\eta_{a0})(P/EI')^{1/2}$
b_0	$1.6903l'(1/\eta_{b0})(f/f_L)^{1/2}(P/EI')^{1/4}$
h_0	$1.0514l'(1/\eta_{b0})(f/f_L)^{1/2}(P/EI')^{1/4}$

4. THE PANEL OF LEAST WEIGHT HAVING DIMENSIONAL LIMITATIONS

4.1 The General Case - Elastic Panels

It is first necessary to determine the dimensions and failing stress of the panel of least weight having no dimensional restrictions to ascertain whether or not any of the limitations is violated. Restrictions on b and h only can sometimes be met by varying f/f_L between 1.0 and 0.6. If, for the unrestricted panel, $\eta_{a0} = \eta_{b0} = 1$ it follows that $\eta_a = \eta_b = 1$ since $f_e < f_{0e}$. If η_{a0} and η_{b0} are less than 1.0 then η_a and η_b may also be below 1.0.

A panel of least weight having dimensional restrictions will satisfy one of the following conditions.

- (a) $f = f_E \leq f_L$. This case is treated in two parts:
 $0.6 \leq f/f_L \leq 1.0$; in this range initial irregularities may affect the overall buckling characteristics of the panel
 and $f/f_L < 0.6$; in this range initial irregularity effects are negligible.
- (b) $f = f_L \leq f_E$. Initial irregularity effects are always present in this case.

The least-weight panel for most restrictions satisfies one of the two conditions of case (a). If h is restricted to a value above h_0 then the least-weight panel satisfies case (b). If the imposed restrictions affect h and other dimensions both conditions (a) and (b) must be investigated to discover the least-weight panel.

Figures 3, 4 and 5 are provided for case (a), Figure 3 being constructed for $0.6 \leq f/f_L \leq 1.0$ and Figures 4 and 5 for $f/f_L = 0.5$ and 0.4 respectively. Figures 6, 7 and 8 are provided for case (b) and are constructed for $f/f_E = 0.8, 0.6$ and 0.4 respectively. All the figures show contours of F and lines of constant B and H on grids of T_s against T . Any imposed set of restrictions will eliminate parts of Figures 3 to 8 but for consistency there must remain on at least one of the figures a region in which they can be met. Figure 3 applies to a range of values of f/f_L and consideration of the conditions at the ends of the range will show whether or not a region exists within the range.

The following sub-sections give detailed instructions on the use of Figures 3 to 8. The range of h on each of the figures is somewhat limited and when h is restricted it is often preferable to ignore this restriction until the final cross-plot of the dimensions and stresses is drawn up.

4.1.1 The Method for Case (a) - $f/f_L \leq 1.0$ - Elastic Panels

Figures 3, 4 and 5 are provided for this case. The steps listed below give the method of determining the least-weight panel.

- (i) Take $f/f_L = 1.0$.
- (ii) Use Table 3.1 to calculate f_{0e} , t_{0e} , t_{s0e} , b_{0e} and h_{0e} .
- (iii) Use the results of step (ii) to express the imposed restrictions in terms of T , T_s , B and H .
- (iv) Use the results of step (iii) to locate on Figure 3 the highest value of F within the restrictions. Read off F and the corresponding values of T , T_s , B and H . Calculate f , t , t_s , b and h .
- (v) Take $f/f_L = 0.6$. Repeat steps (ii) to (iv).
- (vi) It will then be possible to see if a range of values of f/f_L within the bounds $0.6 < f/f_L < 1.0$ is likely to meet the imposed restrictions. If so, repeat steps (ii) to (iv) at convenient values of f/f_L between 1.0 and 0.6 and plot the resulting stresses and dimensions against f/f_L to determine the range of geometries available within the restrictions. If it is not possible to meet the restrictions reduce f/f_L to below 0.6. Steps (vii) on must then be followed.
- (vii) Take $f/f_L = 0.5$.

- (viii) Use the results of step (ii) **obtained when f/f_L was taken as 0.6** to express the imposed restrictions in terms of T , T_s , B and H .
- (ix) Use the results of step (viii) to locate on Figure 4 the highest value of F within the restrictions. Read off F and the corresponding values of T , T_s , B and H . Calculate f , t , t_s , b and h .
- (x) Take $f/f_L = 0.4$ and repeat step (ix) using Figure 5 in place of Figure 4 and using the values of T , T_s , B and H calculated in step (viii).
- (xi) Plot the dimensions and stresses calculated in step (iv) and steps (ix) and (x) against f/f_L and locate the point of highest stress, and consequently minimum weight. Read off the dimensions and failing stress of the panel of least weight.

4.1.2 The Method for Case (b) - $f/f_E < 1.0$ - Elastic Panels

Figures 6, 7 and 8 are provided for this case. The steps listed below give the method of determining the least-weight panel.

- (i) If not previously executed do steps (i), (ii), (iii) and (iv) of Section 4.1.1.
- (ii) Take $f/f_E = 0.8$.
- (iii) Use the results of step (iii) of Section 4.1.1 to locate on Figure 6 the highest F within the restrictions. Read off F and the corresponding values of T , T_s , B and H . Calculate f , t , t_s , b and h .
- (iv) Take $f/f_E = 0.6$ and repeat step (iii) using Figure 7. Take $f/f_E = 0.4$ and repeat step (iii) using Figure 8.
- (v) Plot the dimensions and stresses calculated in step (iv) of Section 4.1.1 and steps (iii) and (iv) above against f/f_E and locate the point of highest stress, and consequently minimum weight. Read off the dimensions and failing stress of the panel of least weight.

4.2 The General Case - Inelastic Panels

If the procedures of Section 4.1 are followed and η_{a0} and η_{b0} are less than 1.0 the least-weight panel may also be inelastic and η_a and η_b may be below 1.0. Since η_a and η_b must correspond to the stress f none of the ratios T , T_s , B or H is explicitly known. In each case it is therefore necessary to assume a value for f and work towards the correct f by means of an iterative procedure. Figure 13 shows η_a and η_b plotted against f/f_n for various values of m .

In steps (iii) and (viii) of Section 4.1.1 the values of η_a and η_b are unknown. A value of f is therefore assumed ($f = f_0$ is usually a reasonable initial assumption) and f/f_n is calculated and Figure 13 used with the appropriate value of m to obtain values of η_a and η_b . The calculation then proceeds and step (iv) or (ix) in Section 4.1.1 or step (iii) in Section 4.1.2 is executed and the f obtained compared with the assumed f . If the stresses do not agree sufficiently closely a new value for f is assumed and the procedure is repeated until agreement is acceptable.

4.3 Cases in which One of t , t_s or b is Restricted

For these cases Figures 9, 10 and 11 are provided. All three figures are derived from Figure 3 and are therefore constructed for case (a) in which $0.6 \leq f/f_L \leq 1.0$. To use these figures steps (i) to (v) of Section 4.1.1 should be followed except that in (iv) the figure associated with the restricted dimension, that is, Figure 9 for t , Figure 10 for t_s or Figure 11 for b , should be used in place of Figure 3.

4.4 Cases in which h only is Restricted

For $h > h_0$ Figure 12 is provided. It is derived from Figures 3, 6, 7 and 8 and therefore applies to case (b), in which $f/f_E \leq 1.0$. Use steps (i), (ii) and (iii) of Section 4.1.1 to obtain the minimum allowable value of H . The corresponding values of F , W , T , T_s and B can be taken directly from Figure 12 and the failing stress f , the mass per unit area w and the dimensions t , t_s and b can then be evaluated.

If h is less than h_0 a family of least-weight geometries exists which can be determined from Figures 3, 4 and 5 by following the steps of Section 4.1.1.

5. DERIVATION

1. GERARD, G. BECKER, H. Handbook of structural stability. Part I - Buckling of flat plates. NACA Tech. Note 3781, October 1954.
2. CATCHPOLE, E.J. The optimum design of compression surfaces having unflanged integral stiffeners. *Jl R. aeronaut. Soc.*, Vol. 58, pp. 765-768, November 1954.
3. COX, H.L. *The buckling of plates and shells*, Chapter 1. Pergamon Press, 1963.
4. GRAYLEY, M.E. Limitations of the wide strut analysis for determination of the optimum proportions of panels having unflanged integral stiffeners. *Jl R. aeronaut. Soc.*, Vol. 73, pp. 888-891, October 1969.

6. EXAMPLES

6.1 Example 1

A panel is to be manufactured from a material for which $E = 69\,000 \text{ MN/m}^2$, $f_n = 392 \text{ MN/m}^2$, $m = 10$ and $\rho = 2790 \text{ kg/m}^3$. The panel is to be 950 mm square. The following restrictions are to be met; $h \leq 48 \text{ mm}$, $b \geq 100 \text{ mm}$ and $t \geq 2.7 \text{ mm}$. The applied load is 1.0 MN and the panel is simply-supported on all four sides.

It is first necessary to calculate the dimensions of the optimum panel for values of f/f_L between 1.0 and 0.6 to ascertain whether or not they fall within the restrictions.

Since the panel sides are simply-supported $l' = 0.950 \text{ m}$. The panel unit loading $P = 1/0.950 = 1.05 \text{ MN/m}$ giving,

$$P/EI' = 1.05/(69\,000 \times 0.950) = 16.02 \times 10^{-6}.$$

$$\text{Hence, } (P/EI')^{1/2} = 0.004 \text{ and } (P/EI')^{1/4} = 0.0633.$$

Table 6.1 is compiled using the expressions of Table 3.1 and selecting a range of values for f/f_L .

TABLE 6.1

f/f_L	f_{0e} MN/m ²	f_{0e}/f_n^*	w_{0e} kg/m ²	t_{0e} mm	t_{s0e} mm	b_{0e} mm	h_{0e} mm
1.0	197	0.503	14.8	2.22	4.97	102	63.2
0.9	"	"	"	"	"	96.4	60.0
0.8	"	"	"	"	"	90.9	56.6
0.7	"	"	"	"	"	85.0	52.9
0.6	"	"	"	"	"	78.7	49.0
Dimensional restrictions are				$t \geq 2.7$ mm	None	$b \geq 100$ mm	$h \leq 48$ mm

* Value is such that for $m = 10$ Figures 1 and 2 give $\eta_{a0} = \eta_{b0} = 1.0$; therefore, the optimum panel is elastic.

The mass of the optimum panel is therefore $14.8 \times 0.950^2 = 13.4$ kg.

Table 6.1 shows that the restrictions cannot be met by varying f/f_L between 1.0 and 0.6. It is therefore necessary to go to an off-optimum condition. Following the recommendations in the last paragraph of Section 4.1, t and b only will be considered restricted and the restriction on h will be ignored and introduced later. Table 6.2 can now be compiled.



TABLE 6.2

		f/f_L	f/f_E	f_{0e}	t_{0e}	t_{s0e}	b_{0e}	h_{0e}	Restrictions		Figure used	Notes (see below)	Highest F	Ratios at highest F				f	t_e	t_{se}	b_e	h_e
				MN/m ²	mm	mm	mm	mm	$T \geq$	$B \geq$				T	T_s	B	H	MN/m ²	mm	mm	mm	mm
$f = f_E \leq f_L$ (See Section 4.1.1)	Step No.			(ii)					(iii)		(iv)											
	(i)	1.0	1.0	197	2.22	4.97	102	63.2	1.22	0.98	3	1	0.980	1.22	1.02	1.14	1.01	193	2.70	5.07	116	63.8
	(v)	0.6	1.0	197	2.22	4.97	78.7	49.0	1.22	1.27	3	2	0.967	1.29	1.08	1.27	1.01	190	2.86	5.37	100	49.5
	(vi)*	0.9	1.0	197	2.22	4.97	96.4	60.0	1.22	1.04	3	1 and 3	0.980	1.22	1.02	1.14	1.01	193	2.70	5.07	110	60.6
		0.8	1.0	197	2.22	4.97	90.9	56.6	1.22	1.10	3	1 and 3	0.980	1.22	1.02	1.14	1.01	193	2.70	5.07	104	57.2
		0.7	1.0	197	2.22	4.97	85.0	52.9	1.22	1.18	3	4 and 3	0.978	1.22	1.04	1.18	1.01	192	2.70	5.17	100	53.4
	Step No.								(vii)		(ix)											
	(vii)	0.50	1.0	197	2.22	4.97	78.7	49.0	1.22	1.27	4	2	0.910	1.39	1.14	1.27	0.99	179	3.09	5.67	100	48.5
	(x)	0.40	1.0	197	2.22	4.97	78.7	49.0	1.22	1.27	5	2	0.852	1.50	1.22	1.27	0.96	168	3.33	6.06	100	47.0
$f = f_L \leq f_E$ (See Section 4.1.2)	Step No.			(ii)					(iii)		(iii) and (iv)											
	(ii)*	1.0	0.8	197	2.22	4.97	102	63.2	1.22	0.98	6	1 and 3	0.932	1.22	1.07	1.19	1.10	184	2.70	5.32	121	69.5
	(iv)*	1.0	0.6	197	2.22	4.97	102	63.2	1.22	0.98	7	1 and 3	0.874	1.22	1.14	1.29	1.215	172	2.70	5.67	132	76.8
		1.0	0.4	197	2.22	4.97	102	63.2	1.22	0.98	8	5 and 3	0.795	1.26	1.26	1.41	1.41	157	2.80	6.26	144	89.1
Restrictions are																$t \geq 2.70$	None	$b \geq 100$	$h \leq 48.0$			

NOTES

1. The point on the $T = 1.22$ line closest to the eye of the figure does not contravene the restriction on B and is therefore the point of the highest achievable F .
2. The point on the $B = 1.27$ line closest to the eye of the figure does not contravene the restriction on T and is therefore the point of the highest achievable F .

*3. To illustrate the method complete ranges of values of f/f_L and f/f_E for cases (a) and (b) respectively are set out in the table. It would not normally be necessary to investigate the cases marked with an asterisk since it can be seen that the restriction on h will only be met by reducing f/f_L below 0.6.

4. Satisfaction of both of the restrictions yields the highest achievable F for this case.
5. The eye of the figure lies within the restrictions and is therefore the point of highest achievable F .

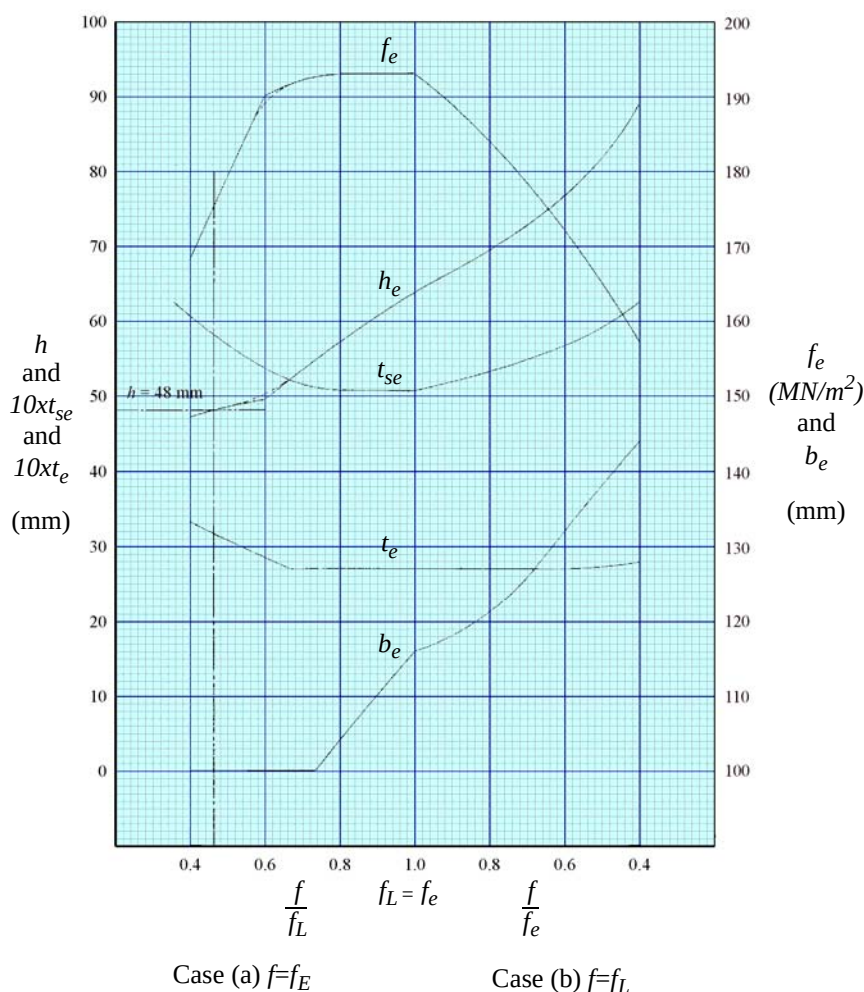


FIGURE 6.1

Figure 6.1 has been constructed from Table 6.2. Discontinuities at $f/f_L = 0.6$ have been smoothed out by the dotted lines (see Section 4.1.1) since they relate to the point where initial irregularity effects have been assumed to terminate. All other discontinuities in the figure are real and relate to changes in the governing restrictions.

The restriction that $h \leq 48$ mm is now introduced and from Figure 6.1 it can be seen that within this restriction the maximum f_e is 175 MN/m^2 at $f/f_L = 0.465$. The panel dimensions are $t_e = 3.17$ mm, $t_{se} = 5.80$ mm, $b_e = 100$ mm and $h_e = 48$ mm.

The panel cross-sectional area per unit width is therefore

$$(0.00317 \times 0.100 + 0.00580 \times 0.048)/0.100 = 0.00595 \text{ m}^2/\text{m}.$$

Checking the panel stress, $f = 1.05/0.00595 = 176 \text{ MN/m}^2$.

The panel mass is $0.00595 \times 0.950^2 \times 2790 = 15.0$ kg.