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Lift-curve slope and aerodynamic centre position of wings in inviscid subsonic flow

Associated software: ESDUpac A7011



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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU W.01.01.01 and W.08.01.01 and in part ESDU W.S.01.03.03-06 and W.S.08.01.02, was monitored and guided by the Aerodynamics Committee, which first met in 1942 and now has the following membership:

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LIFT-CURVE SLOPE AND AERODYNAMIC CENTRE POSITION OF WINGS IN INVISCID SUBSONIC FLOW

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LIFT-CURVE SLOPE AND AERODYNAMIC CENTRE POSITION OF WINGS IN INVISCID SUBSONIC FLOW

1. NOTATION AND UNITS

		SI	British
A	aspect ratio, b^2/S		
b	wing span, see Sketch 1.1	m	ft
C_L	lift coefficient		
c_r	root chord, see Sketch 1.1	m	ft
c_t	tip chord, see Sketch 1.1	m	ft
$\bar{c}$	aerodynamic mean chord, see Sketch 1.1	m	ft
M	free-stream Mach number		
n	fraction of chord		
S	wing area	m ²	ft ²
$\bar{x}$	distance of aerodynamic centre aft of leading edge of aerodynamic mean chord, see Sketch 1.1	m	ft
$\bar{x}_0$	distance of aerodynamic centre aft of leading edge of root chord, see Sketch 1.1	m	ft
$\bar{\bar{x}}_0$	distance of leading edge of aerodynamic mean chord aft of wing apex, see Sketch 1.1	m	ft
α	wing incidence	rad	rad
β	compressibility parameter, $(1 - M^2)^{1/2}$		
Λ_n	sweepback of n^{th} -chord line	deg	deg
$\Lambda_{1/2}$	sweepback of mid-chord line, see Sketch 1.1	deg	deg
$\Lambda_{1/2M}$	$\tan^{-1} [(\tan \Lambda_{1/2})/\beta]$, see Appendix C	deg	deg
λ	taper ratio, c_t/c_r		

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2. GENERAL

The position of the aerodynamic centre, which is that point on the axis of symmetry of the wing about which the rate of change of pitching moment with incidence is zero, is given in this Item as a fraction of the aerodynamic mean chord aft of the leading edge of the aerodynamic mean chord. The distance of the leading edge of the aerodynamic mean chord aft of the wing apex may, if required, be obtained either from ESDU 76003 (Reference 6) or from the expression

Similarly, the value of the aerodynamic mean chord in terms of the root chord may be found either from ESDU 76003 or from the expression

The data in this Item are given in terms of the sweepback of the mid-chord line, $\Lambda_{1/2}$, which is related to

the sweepback of the n^{th} -chord line, Λ_n , via the expression

$$A \tan \Lambda_{1/2} = A \tan \Lambda_n + 2(2n - 1) \left(\frac{1 - \lambda}{1 + \lambda} \right). \quad (2.3)$$

3. BASIS OF DATA

The calculations and data on which the curves of Figures 1a to 1e and 2a to 2g are based over the range $1.5 \leq \beta A \leq 8$ are detailed in Derivation 1 which also contains full tabulations of the computer solutions together with the spanwise centre of pressure position. The method used was basically the subsonic lifting-surface theory of Reference 4 with the increased rounding of the central kink recommended in Reference 5. For lift-curve slope some additional cases were calculated, using the method of Derivation 2, to extend the applicability up to $\beta A = 12$. Corresponding calculations for aerodynamic centre position were not available and the extensions shown in Figures 2a to 2g were obtained by extrapolation in terms of $1/\beta A$ together with the known asymptote as $\beta A \rightarrow \infty$. Slender wing theory (Reference 3) was used to provide solutions for $\beta A = 0$ and the curves in Figures 1a to 1e and 2a to 2g were faired into these data.

4. APPLICABILITY OF DATA

The theory from which the data were derived is linearised, with the implication that its application is limited to wings at low incidence and with small thickness, camber and twist. The theory also relates only to inviscid flow. These restrictions are, however, known to be relatively unimportant for most practical purposes in calculating the lift and position of the aerodynamic centre *provided* that the flow remains fully attached over the wing surface, see Appendix A. In particular this condition is not likely to apply, at other than low incidences, to wings with highly swept leading-edges. In addition the theory applies only to those cases for which the flow is subsonic over the whole wing.

Although the Figures were derived from calculations for straight-tapered planforms with streamwise tips, they may be applied to other types of planform using the concept of an “equivalent” wing planform* defined in Addendum A of ESDU 76003. If the ratio of body diameter to wing span is small, the data of Figures 1a to 1e may also be used to approximate the lift-curve slope of a wing-body combination (see Appendix A to the present Item) but the aerodynamic centre position (Figures 2a to 2g) requires a correction for the effect of a body.

The data for $\beta A < 2$, indicated by broken lines, should be used with caution as there is an element of uncertainty as to how the limiting values for $\beta A = 0$ should be approached. Moreover, on wings with a combination of low aspect ratio and high trailing-edge sweep, a spurious effect of the central rounding may become significant. This is not a serious limitation, however, as most practical planforms lie outside this range. The data for $\beta A > 8$ are shown by the broken lines in Figures 2a to 2g in view of the fact that they were obtained by an extrapolation technique (see Section 3).

The data of Figures 1a to 1e may be applied to tailplanes provided that due allowance is made for airframe interference, see ESDU 89029 (Reference 7). The data of Figures 2a to 2g may be applied directly to tailplanes in the determination of the tail arm for other than very close-coupled configurations (tail arm $< b/2$, say).

* In essence, the equivalent wing planform is straight tapered and has the same overall span and exposed wing planform area (*i.e.* planform area outside the body planform) as the true wing. If the true wing has a streamwise tip chord with no rounded corners then the tip chord of the equivalent wing planform is the same as that of the true wing. If, however, the true wing has a non-streamwise tip chord or a small curved tip then the linear leading and trailing edges just inboard of the tip are extrapolated outboard to the full span to define an equivalent tip chord.

5. DERIVATION AND REFERENCES

The Derivation lists selected sources that have assisted in the preparation of this Item.

Derivation

1. GARNER, H.C.
INCH, S.M. Subsonic theoretical lift-curve slope, aerodynamic centre and spanwise loading for arbitrary aspect ratio, taper ratio and sweepback.
ARC CP 1137, 1970.
2. LEHRIAN, D.E.
GARNER, H.C. Theoretical calculation of generalised forces and load distribution on wings oscillating at general frequency in a subsonic stream.
ARC R & M 3710, 1971.

The References lists sources of information supplementary to that given in this Item.

References

3. MANGLER, K.W. Calculation of the pressure distribution over a wing at sonic speeds.
ARC R & M 2888, 1951.
4. GARNER, H.C.
FOX, D.A. Algol 60 programme for Multhopp's low-frequency subsonic lifting-surface theory. ARC R & M 3517, 1966.
5. ZANDBERGEN, P.J.
LABRUJERE, T.E.
WOUTERS, J.G. A new approach to the numerical solution of the equation of subsonic lifting surface theory. NLR Report TR G.49, 1967.
6. ESDU Geometrical properties of cranked and straight-tapered wings.
ESDU 76003, London, 1976.
7. ESDU Installed tailplane lift-curve slope at subsonic speeds.
ESDU 89029, London, 1989.

6. EXAMPLE

Find the lift-curve slope and position of the aerodynamic centre relative to the root chord of a wing at low incidence at a Mach number of 0.6. The wing has the following geometrical properties:

$$A = 3, \Lambda_{1/2} = 60 \text{ degrees}, \lambda = 1.0.$$

$$\text{For this wing, } A \tan \Lambda_{1/2} = 3 \times \tan 60^\circ = 5.2$$

$$\text{and } \beta A = (1 - M^2)^{1/2} A = (1 - 0.6^2)^{1/2} \times 3 = 0.8 \times 3 = 2.4.$$

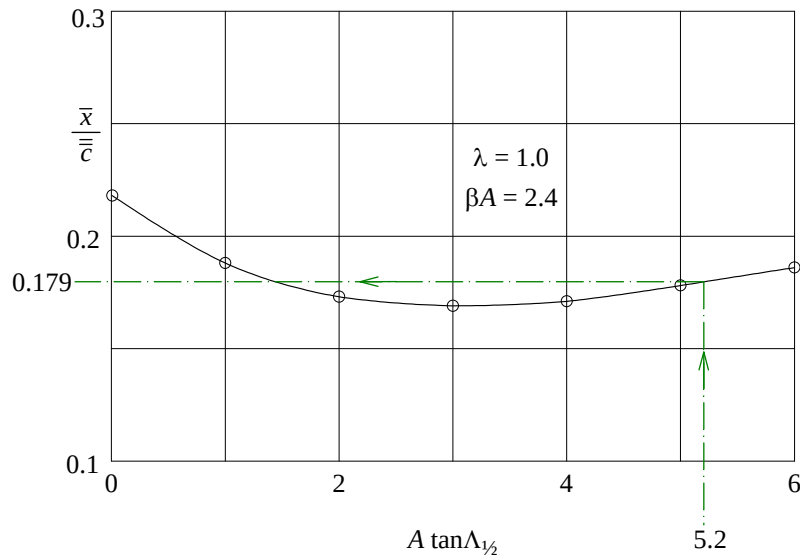
Thus from Figure 1e, for $\lambda = 1.0$, with $A \tan \Lambda_{1/2} = 5.2$ and $\beta A = 2.4$

$$\frac{1}{A} \frac{dC_L}{d\alpha} = 0.773 \text{ per rad.}$$

Therefore the lift-curve slope, $\frac{dC_L}{d\alpha} = 3 \times 0.773 = 2.32 \text{ per rad.}$

From Figures 2a to 2g, with $\lambda = 1.0$ and $\beta A = 2.4$, the following cross-plot gives, for $A \tan \Lambda_{1/2} = 5.2$,

$$\frac{\bar{x}}{\bar{c}} = 0.179.$$



From Equations (2.1) and (2.2) with $\lambda = 1.0$ and $A \tan \Lambda_{1/2} = 5.2$

$$\frac{\bar{x}_0}{\bar{c}} = 1.30 \text{ and } \frac{\bar{c}}{c_r} = 1.00.$$

Hence, the distance of the aerodynamic centre aft of the wing apex, in terms of the root chord, is (see Sketch 1.1)

$$\begin{aligned} \frac{\bar{x}_0}{c_r} &= \frac{\bar{x} + \bar{x}_0}{c_r} = \frac{\bar{c}}{c_r} \left(\frac{\bar{x}}{\bar{c}} + \frac{\bar{x}_0}{\bar{c}} \right) \\ &= 1.00(0.179 + 1.30) \\ &= 1.48. \end{aligned}$$

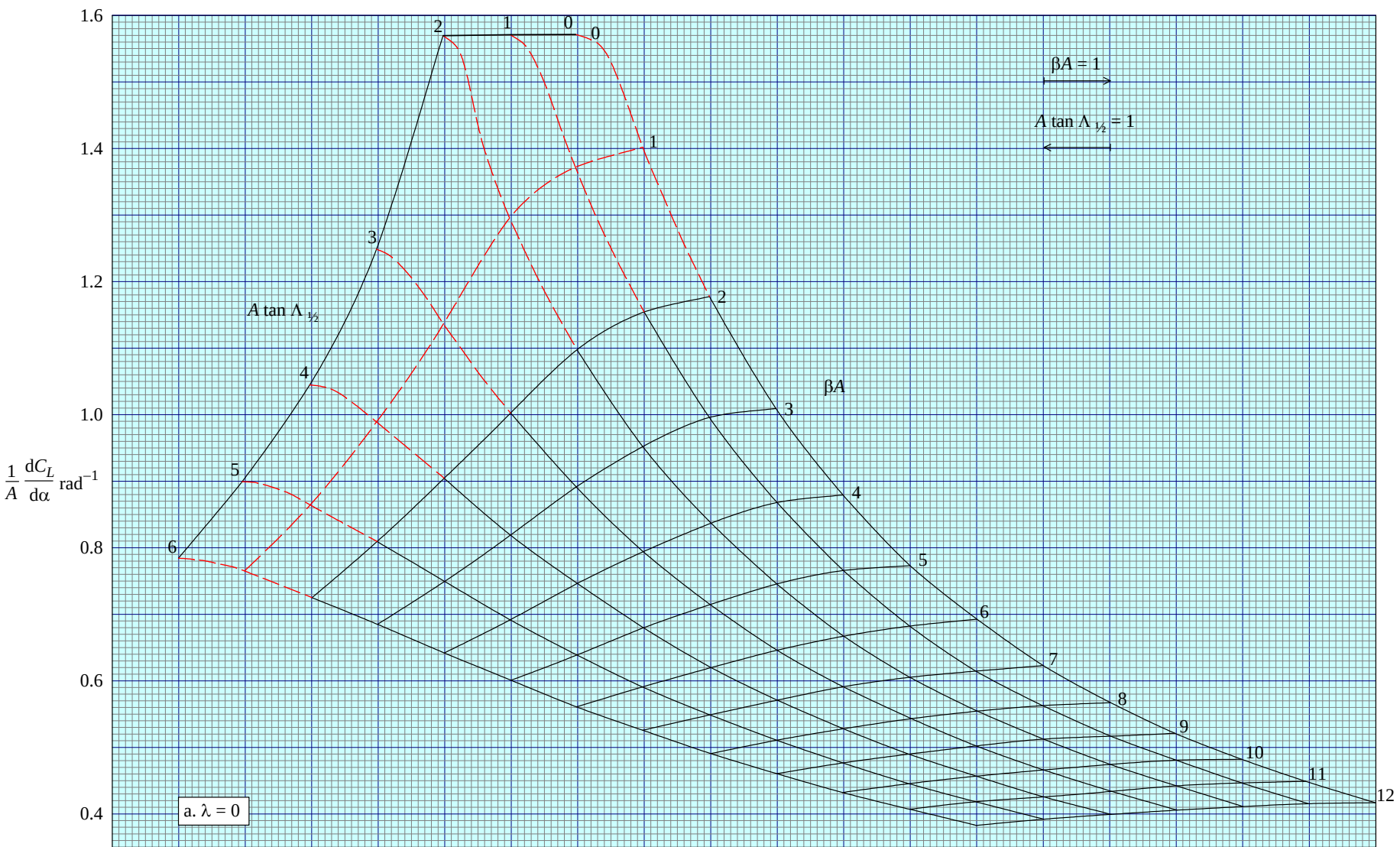


FIGURE 1a LIFT-CURVE SLOPE $\lambda = 0$

