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Lift-curve slope and aerodynamic centre position of wings in inviscid supersonic flow

Associated software: ESDUpac A7012



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THE PREPARATION OF THIS DATA ITEM

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LIFT-CURVE SLOPE AND AERODYNAMIC CENTRE POSITION OF WINGS IN INVISCID SUPERSONIC FLOW

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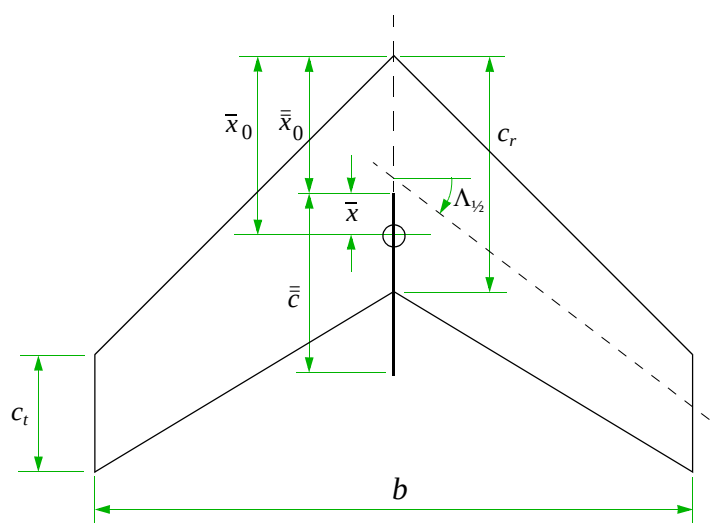
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LIFT-CURVE SLOPE AND AERODYNAMIC CENTRE POSITION OF WINGS IN INVISCID SUPERSONIC FLOW

1. NOTATION AND UNITS

		SI	British
A	aspect ratio, b^2/S		
b	wing span, see Sketch 1.1	m	ft
C_L	lift coefficient		
c_r	root chord, see Sketch 1.1	m	ft
c_t	tip chord, see Sketch 1.1	m	ft
$\bar{c}$	aerodynamic mean chord, see Sketch 1.1	m	ft
M	free-stream Mach number		
n	fraction of chord		
S	wing area	m ²	ft ²
$\bar{x}$	distance of aerodynamic centre aft of leading edge of aerodynamic mean chord, see Sketch 1.1	m	ft
$\bar{x}_0$	distance of aerodynamic centre aft of leading edge of root chord, see Sketch 1.1	m	ft
$\bar{\bar{x}}_0$	distance of leading edge of aerodynamic mean chord aft of wing apex, see Sketch 1.1	m	ft
α	wing incidence	rad, deg	rad, deg
β	compressibility parameter, $(M^2 - 1)^{1/2}$		
Λ_n	sweepback of n^{th} -chord line	deg	deg
$\Lambda_{1/2}$	sweepback of mid-chord line, see Sketch 1.1	deg	deg
$\Lambda_{1/2M}$	$\tan^{-1}((\tan \Lambda_{1/2})/\beta)$, see Appendix A	deg	deg
λ	taper-ratio, c_t/c_r		

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Sketch 1.1 Wing notation

2. GENERAL

Figures 1a to 1d and 2a to 2d present, respectively, data for the lift-curve slope and aerodynamic centre position of wings with straight leading and trailing edges and streamwise tips (see Sketch 1.1) at low incidence in supersonic flow.

The lift-curve slope parameter, $(1/A)(dC_L/d\alpha)$, is plotted in Figures 1a to 1d in carpet form against βA and $A \tan \Lambda_{1/2}$ for values of taper ratio $\lambda = 0, 0.25, 0.5$ and 1.0 . The aerodynamic centre parameter, $\bar{x}/\bar{c}$, is plotted in Figures 2a to 2d in pseudo-carpet form in which the curves for constant βA have been omitted for clarity of presentation. Because of this and to aid cross-plotting on these Figures the abscissa, $\beta A - A \tan \Lambda_{1/2}$, has been introduced.

The position of the aerodynamic centre, which is that point on the axis of symmetry of the wing about which the rate of change of pitching moment with incidence is zero, is given in this Item as a fraction of the aerodynamic mean chord aft of the leading edge of the aerodynamic mean chord. The distance of the leading edge of the aerodynamic mean chord aft of the wing apex may, if required, be obtained either from ESDU 76003 (Reference 18) or from the expression

$$\frac{\bar{x}_0}{\bar{c}} = \frac{(1 + 2\lambda)(1 + \lambda)}{8(1 + \lambda + \lambda^2)} \left[A \tan \Lambda_{1/2} + 2 \left(\frac{1 - \lambda}{1 + \lambda} \right) \right]. \quad (2.1)$$

Similarly, the value of the aerodynamic mean chord in terms of the root chord may be found either from ESDU 76003 or from the expression

$$\frac{\bar{c}}{c_r} = \frac{2}{3} \left(\frac{1 + \lambda + \lambda^2}{1 + \lambda} \right). \quad (2.2)$$

The data in this Item are given in terms of the sweepback of the mid-chord line, $\Lambda_{1/2}$, which is related to the sweepback of the n^{th} -chord line, Λ_n , via the expression

$$A \tan \Lambda_{1/2} = A \tan \Lambda_n + 2(2n - 1) \left(\frac{1 - \lambda}{1 + \lambda} \right). \quad (2.3)$$

Appendix A presents a computer program that reproduces the data of this Item, with extensions to cover higher values of aspect ratio, $\beta A > 12$, and a limited amount of forward mid-chord sweep.

3. BASIS OF DATA

The data given in this Item are based on Derivations 1 to 6 for lift-curve slope and Derivations 7 to 10 for aerodynamic centre position. References 11 and 17 and Derivation A1 give useful collections of many of the formulae derived for the lift-curve slope and aerodynamic centre position of wings in supersonic flow.

It should be noted that the lift-curve slope data for wings such that $A \tan \Lambda_{1/2} < 2$ and $\beta A < 1$ have been omitted from Figures 1a to 1d due to the complications arising in the theory for this region largely as a result of wing tip interaction effects. The computer program in Appendix A interpolates between the data of the carpets and the sonic speed value of $(1/A)dC_L/d\alpha [= \pi/2 \text{ at } \beta A = 0 \text{ for } |A \tan \Lambda_{1/2}| < 2]$.

For the construction of the carpets in Figures 2a to 2d few theoretical data were available for the aerodynamic centre position of wings with subsonic trailing edges. Consequently many of the data for this region are simply presented as straight broken lines joining up with the slender-wing values, and as such should be treated with caution.

4. APPLICABILITY OF DATA

The data presented in this Item apply only to wings with straight leading and trailing edges and streamwise tips. The theory from which the data were derived is linearised with the implication that its application is limited to wings at low incidence and with sharp leading-edges and with small thickness, camber and twist. The theory also relates only to inviscid flow. These restrictions are, however, known to be relatively unimportant for most practical purposes, except at high Mach numbers, provided the flow remains fully attached over the wing surface.

Linearised theory shows that as Mach number becomes very large the lift-curve slope becomes asymptotic to zero and the aerodynamic centre becomes asymptotic to a position at the mid-point of the aerodynamic mean chord. At high Mach numbers ($M > 5$, say) the effects of boundary layers, shock wave detachment, heat transfer and "real gas" phenomena become more important and can have a considerable effect on the wing surface pressure distribution and therefore on the lift-curve slope and aerodynamic centre position. The linearised theory values presented in this Item should therefore be considered as rough approximations at such Mach numbers. A brief discussion of possible effects at hypersonic Mach numbers follows.

In general, because of the high altitudes associated with hypersonic flight the Reynolds number tends to be low and the boundary layer thick. The displacement of the inviscid flow region by the boundary layer has a direct effect on the shock wave pattern over the wing and hence on the pressure distribution.

A lifting surface designed for hypersonic speeds would normally have blunt leading edges to reduce the structural problems of the high aerodynamic heating associated with sharp leading edges. The detached shock-waves resulting from the use of blunt leading edges lead to surface pressures near the leading edges

which are large compared to the corresponding attached-shock values for sharp leading edges. Away from the leading-edge region the differences are less significant. References 13 and 14 provide an empirical method which gives a rough guide to the effects of Reynolds number and detached shocks.

Heat transfer between the air and the lifting surface has an influence on the boundary-layer thickness and hence the pressure distribution. The high surface temperatures associated with hypersonic flight increase the boundary-layer thickness and in addition the assumption of a constant ratio of the specific heat capacities becomes invalid. Some indication of the effect of surface temperature on pressure distribution is given in Reference 14.

The high temperatures behind detached shock waves at hypersonic speeds can also lead to dissociation of the air molecules into their atomic constituents and, at very high temperatures, the atoms tend to ionise to create a conductive medium. These phenomena, which largely affect only the leading-edge region, are discussed more fully in Reference 15.

The general field of hypersonic flow is treated in a number of textbooks, References 15 and 16 for example.

5. ACCURACY

Theoretical results have been compared with the more reliable data of those tabulated in Reference 12, in addition to some later data. The data relate almost entirely to rectangular or delta wings, together with a few isolated data concerning trapezoidal wings with unswept mid-chord lines. The range of values of βA covered extends from about 1 to 10. The survey has indicated that, provided βA is greater than about 2 and provided the wing thickness/chord ratio is small (less than about 0.1) and the leading edges are either sharp or only slightly rounded, the likely accuracy involved in the use of this Item is $\pm 15\%$ on lift-curve slope and $\pm 0.05\bar{c}$ on aerodynamic centre position.

6. DERIVATION AND REFERENCES

Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

Lift-curve slope

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Aerodynamic centre position

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