

NON-DIMENSIONAL APPROACH TO ENGINE THRUST AND AIRFRAME DRAG FOR THE ANALYSIS OF MEASURED PERFORMANCE DATA: AIRCRAFT WITH TURBO-JET AND TURBO-FAN ENGINES

1. NOTATION AND UNITS*

		SI	British
a	speed of sound	m/s, kn	ft/s, kn
B	air bleed flow	kg/s	lbf/s
C_L	lift coefficient, $\frac{L}{\gamma p_0 \delta M^2 S/2}$		
D	airframe drag	N	lbf
F_G, F_N	gross thrust, net thrust (see Item No. 69006 for thrust definitions)	N	lbf
g	acceleration due to gravity	m/s ²	ft/s ²
L	lift	N	lbf
M	Mach number, $V/a = V/(a_0 \sqrt{\theta}) = V_e/(a_0 \sqrt{\delta})$		
m	aircraft mass	kg	slug
N	engine speed (for multi-shaft engines it may be necessary to take account of more than one shaft speed)	rev/s	rev/s
P	engine shaft power offtake	W	lbf ft/s
p	static pressure	N/m ²	lbf/ft ²
Q	engine fuel consumption	kg/s	lbf/s
S	wing area	m ²	ft ²
T	ambient temperature	K	K
V	true airspeed	m/s, kn	ft/s, kn
V_e	equivalent airspeed, $V \sqrt{\sigma}$	m/s, kn	ft/s, kn
W	aircraft weight, mg	N	lbf
γ	ratio of specific heat capacities		

* As in previous Items, British units make use of a weight breakdown for the aircraft, but it should be noted that the SI units make use of a mass breakdown. The equations are written in British units but are equally applicable in SI units provided that the substitution $W = mg$ is made. The SI units chosen have not necessarily received universal acceptance for the particular applications, i.e. other multiples or sub-multiples may be used.

δ	relative pressure, p/p_0		
θ	relative temperature, T/T_0		
ρ	density	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0		
Suffix			
0	denotes value at sea level in International Standard Atmosphere (Item No. 68046 gives numerical values)		

2. INTRODUCTION

This Item outlines the method by which the engine and airframe characteristics used in the analysis and calculation of aircraft performance may be expressed in terms of a few compound parameters and indicates the limitations of such a method. Various additional factors which affect the analysis of measured performance are discussed.

The full theoretical treatment involves the derivation by dimensional analysis of non-dimensional quantities relating to thrust, fuel flow and drag and appears in a number of places in the literature (see, for example, Reference 1). These non-dimensional quantities include characteristic dimensions of the engine and aircraft (such as nozzle diameter and wing area). In treating the generalised performance of a given design, these dimensions are constant and are usually omitted from the parameters plotted. In addition, atmospheric temperatures, pressures, and densities are expressed in relative form by dividing by their sea level standard values. Because of this and the omission of the dimensions the plotted parameters are not truly non-dimensional but it is usual to refer to them as “non-dimensional” and the term is retained here.

3. ENGINE PERFORMANCE

For a turbo-jet or turbo-fan engine installation, using fuel of given calorific value, the thrust and fuel flow parameters may be expressed as

$$\frac{F_G}{\delta} = f_{1,1} \left(\frac{N}{\sqrt{\theta}}, M \right) \quad (3.1)$$

$$\text{or} \quad \frac{F_N}{\delta} = f_{1,2} \left(\frac{N}{\sqrt{\theta}}, M \right) \quad (3.2)$$

$$\text{and} \quad \frac{Q}{\delta \sqrt{\theta}} = f_2 \left(\frac{N}{\sqrt{\theta}}, M \right), \quad (3.3)$$

where $f_{1,1}$, $f_{1,2}$ and f_2 denote unique relationships for each particular design of engine subject to limitations that are discussed in Section 5.

4. AIRCRAFT DRAG

For an aircraft in a given configuration it is assumed that the drag parameter D/δ is a function of lift coefficient and Mach number only, so that

$$\frac{D}{\delta} = f_3(C_L, M), \quad (4.1)$$

where f_3 denotes a unique relationship for each particular design of aircraft subject to limitations which are discussed in Section 5. Since C_L is a function of W/δ and M , C_L can be replaced by W/δ in Equation (4.1) and since M is proportional to $V/\sqrt{\theta}$ this expression (or its equivalent) may be adopted in place of M in the equations, *i.e.*

$$\frac{D}{\delta} = f_{3,1}\left(\frac{W}{\delta}, M\right) = f_{3,2}\left(\frac{W}{\delta}, \frac{V}{\sqrt{\theta}}\right). \quad (4.2)$$

5. FACTORS AFFECTING THE ANALYSIS OF MEASURED PERFORMANCE DATA

The following effects which are not considered in Sections 3 and 4 may need to be corrected for in the analysis of measured performance data:

- (1) Reynolds number,
- (2) thrust angle,
- (3) centre of gravity variation,
- (4) power plant operation,
- (5) gravity variation,
- (6) flight path length,
- (7) buoyancy.

Some of these factors may be important in a particular case, and the notes which follow give some guidance on their significance and indicate how account may be taken of them in the analysis.

5.1 Reynolds Number

Because the possible Reynolds number variation for a particular combination of C_L and M is in general small (unless very large weight variation is possible), Reynolds number has not been included as a parameter in Equations (3.1) to (3.3), (4.1) and (4.2). However, it should be noted that the engine and airframe performance deduced from flight test will include the effects of a particular Reynolds number variation due to altitude which must be taken into account when making comparisons with other data.