

THE MEASUREMENT AND ANALYSIS OF CLIMB AND EXCESS POWER PERFORMANCE

1. NOTATION AND UNITS^{*†}

		SI	British
a	speed of sound	m/s	ft/s
D	airframe drag	N	lbf
D_M	intake momentum drag (ram drag)	N	lbf
F_G	engine gross thrust	N	lbf
F_N	net thrust	N	lbf
g	acceleration due to gravity	m/s ²	ft/s ²
H	geopotential height	m	ft
H_p	pressure height	m	ft
h_e	energy height, $H + (V^2/2g_0)$	m	ft
M	Mach number, $V/a (= V/(a_0\sqrt{\theta}) = V_e/(a_0\sqrt{\delta}))$		
m	aeroplane mass	kg	slug
m_F	mass of fuel consumed	kg	slug
N	engine speed	rev/s	rev/s
p	static pressure	N/m ²	lbf/ft ²
R	gas constant for air (287.052 87 N m/kg K, 3 089.811 4 lbf ft/slug K)	N m/kg K	lbf ft/slug K
SEP	specific excess power, dh_e/dt	m/s	ft/s
T	ambient temperature	K	K
t	time	s	s

* When using SI units it is usual to work in terms of aeroplane mass, m ; however, the customary practice with British units is to work in terms of aeroplane weight, W . In this Item the equations are presented for the most part in terms of m although they can, of course, be converted by making the substitution $W = mg$. The SI units here have not necessarily received universal acceptance for the particular applications, *i.e.* other multiples and submultiples may be used

† The units quoted are coherent sets and this avoids the necessity for introducing numerical constants into otherwise-algebraic equations in the Item. If required, conversion factors to other units can be found in the Conversion Factors Item in Volume 1 of the Performance Sub-Series. However, for purposes of presentation in Figure 2, limited use is made of the operational unit, the knot, kn.

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V	true airspeed* (resultant velocity with respect to air)	m/s	ft/s
V_c	calibrated airspeed* (CAS), see Item No. RG1/0	m/s	ft/s
V_e	equivalent airspeed* (EAS), $V\sqrt{\sigma}$	m/s	ft/s
V_K	flight-path velocity* (resultant velocity with respect to Earth)	m/s	ft/s
V_W	horizontal component of wind velocity (relative to Earth)	m/s	ft/s
W	aeroplane weight, mg	N	lbf
Z	geometric height	m	ft
α	angle of attack, <i>i.e.</i> angle between true airspeed vector and longitudinal datum or body axis	deg	deg
γ	angle of climb (flight-path inclination angle), $\sin\gamma = (dZ/dt)/V_K$	deg	deg
γ_a	angle of climb in still air (air-path angle of climb), $\sin\gamma_a = (dZ/dt)/V$	deg	deg
Δ	denotes an increment in the affixed parameter		
δ	relative pressure, p/p_0		
ε	angle between gross thrust vector and aeroplane plane of symmetry, positive when F_G acts to port	deg	deg
θ	relative temperature, T/T_0		
ρ	density of air	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0		
τ	angle between projection of gross-thrust vector on to aeroplane plane of symmetry and longitudinal datum or body axis; positive when F_G contributes to lift	deg	deg
ϕ	geographic latitude, positive in Northern hemisphere, negative in Southern hemisphere	deg	deg
χ	flight-path azimuth angle (angle of track), North corresponds to $\chi = 0$	deg	deg
χ_W	wind azimuth angle, North corresponds to $\chi_W = 0$ and χ_W differs by 180° from wind direction used in meteorology	deg	deg

For footnote refer to end of Notation Section

Ψ	aximuth angle (aircraft heading in horizontal plane), $\Psi = 0$ corresponds to North	deg	deg
ω_e	Earth's angular velocity ($= 0.072\,921\,2 \times 10^{-3}$ rad/s), see Item No. 79018	rad/s	rad/s

Subscripts

<i>std</i>	denotes conditions in International Standard Atmosphere, ISA, see Item Nos 68046 and 77022
0	denotes conditions at sea level in ISA
1, 2	denotes performance data in flight conditions "1" and "2" respectively

* The units quoted are coherent sets and this avoids the necessity for introducing numerical constants into otherwise-algebraic equations in the Item. If required, conversion factors to other units can be found in the Conversion Factors Item in Volume 1 of the Performance Sub-series. However, for purposes of presentation in Figure 2, limited use is made of the operational unit, the knot, kn.

2. INTRODUCTION

This Item gives methods for analysing flight-measured data to determine an aeroplane's climb or descent performance[†]. The Item provides a background to the subject as a whole but gives particular emphasis in Section 3 to the development and use of energy methods and the associated measurement of specific excess power which has applications to acceleration and manoeuvring as well as climb performance. The use of these methods does not, however, preclude the interpretation of results in simple, rate-of-climb terms. Section 4 describes test techniques used for determining the flight conditions for point-performance optima with particular attention given to the use of level accelerations. Corrections to measured data and the use of "reduction" formulae and non-dimensional methods to predict performance in different conditions are discussed in Sections 5, 6 and 7. The synthesis of practical climb techniques and the scheduling of performance for operational use are discussed in Section 8. Appendix A gives a limited amount of guidance on sources of data for, and typical values of, wind and wind gradients.

2.1 Use of Other Related Data Items

Because the subject is so extensive, considerable use is made of cross references to Items dealing with related topics. The more significant of these are as follows.

The present Item and these related Items permit a wide range of test and analysis methods to be considered and, while some advice is given on their relative merits, the final choice rests with the user and depends, for example, on the class of aeroplane, the available flight time and instrumentation and the scope and purposes of the investigation.

[†] Much of the text is written in terms of climbing flight but the methods are valid for both climb and descent.

<i>Subject area</i>		<i>Item No.</i>
Relationships between pressure and geopotential heights	off-standard atmospheres	77022
	non-standard atmospheres	78012
Acceleration factors		81046
“Reduction” formulae for jet aeroplanes		RJ 2/1
“Reduction” formulae for propeller-driven aeroplanes		RP 2/0
“Non-dimensional” methods for jet aeroplanes		RJ 1/2
“Non-dimensional” methods for turbo-props		RT 1/2
Use of accelerometers to measure accelerations		79018
Corrections to noise data: introduction	ESDU	81035
Atmospheric attenuation of noise	NOISE	78002
Correction of noise data for ground reflections	SUB-SERIES	80032

2.2 Climb and Descent Performance

The point-performance properties of an aircraft which characterise climb and descent are the rate of climb and the climb gradient or flight path angle to the horizontal. As with all performance characteristics, these are functions of aircraft configuration, mass and powerplant settings as well as ambient conditions. However, for climb and descent it is usual to establish and schedule performance for only a limited number of powerplant settings or “ratings”.

The performance equations in Section 3 show that aeroplane climb and acceleration performances are closely linked and, to an extent, interchangeable so that the rate of climb at any instant depends on the rate of change of true airspeed. Consequently, the “best” climb performance, as expressed by an optimum energy change rate, requires an appropriate operating technique, usually in the form of a speed-height schedule. For airworthiness purposes, on the other hand, it is necessary to obtain the best climb gradient subject to such constraints on airspeed as are specified in the appropriate airworthiness requirements as, for example, for climb immediately after take-off or during a missed or discontinued approach – see References 12 and 13.

An alternative method to the direct measurement of climb performance in flight is to utilise reliable (preferably flight-confirmed) thrust and drag data to solve the equations of motion to predict the aeroplane's point-performance capabilities, see Item Nos 70021 and 79018. In this way the point performance can be computed for any flight conditions and the path performance determined by integration. Examples of the use of this approach are given in Item Nos EG3/3 to 3/5 and Item No. 70021. Comparisons of predicted and actual performance have to be made in such cases but these may be limited in number and directed primarily to confirming or, if necessary, modifying the predicted operating techniques. Much of the material in the present Item is also applicable to this “synthesis” approach to climb performance.

3. PERFORMANCE EQUATIONS (STILL-AIR)

The performance equations are here derived from energy considerations for *still-air conditions*; the effects of winds on climb performance are considered separately in Section 5.4.

The sum of the kinetic and potential energies of an aircraft of weight mg flying at true airspeed V and geometric height Z is

$$m \int g dZ + mV^2/2. \quad (3.1)$$

If the concept of geopotential height is introduced (see Item No. 77022), from which

$$H = \frac{1}{g_0} \int g dZ, \quad (3.2)$$

then Expression (3.1) becomes

$$mg_0 H + mV^2/2 = mg_0 \left(H + \frac{V^2}{2g_0} \right) = mg_0 h_e. \quad (3.3)$$

The quantity $h_e (= H + (V^2/2g_0))$ has the dimension of length and is referred to as energy height*. In this Item h_e is based on true airspeed, V , and not on an inertial or flight-path speed so that no account is taken of energy acquired or lost due to the motion of the atmosphere.

3.1 Specific Excess Power

The derivative of h_e with respect to time is referred to as specific excess power, SEP , and is widely used in the analysis of climb, acceleration and manoeuvring performance. From Equation (3.3),

$$SEP = \frac{dh_e}{dt} = \frac{dH}{dt} + \frac{V}{g_0} \frac{dV}{dt} = \frac{dH}{dt} \left[1 + \frac{V}{g_0} \frac{dV}{dH} \right]. \quad (3.4)$$

An alternative form, of value in climb performance analysis, is

$$SEP = \frac{dh_e}{dt} = \frac{dH}{dt} + \frac{1}{2g_0} \frac{d(V^2)}{dt} = \frac{dH}{dt} \left[1 + \frac{1}{2g_0} \frac{d(V^2)}{dH} \right]. \quad (3.5)$$

The relevant aeroplane properties are introduced by equating SEP to the rate at which work is done on the aeroplane by thrust and drag to give, for example,

$$mg_0(SEP) = mg_0 \frac{dh_e}{dt} = [F_G \cos \epsilon \cos(\alpha + \tau) - D_M - D] V. \quad (3.6)$$

* In Reference 1 and Item Nos EG3/3 to 3/5 (on estimation of performance of aeroplanes climbing at high subsonic and supersonic flight speeds) h_e is defined in terms of geometric height and $V^2/2g$. In this Item, as in most current practice, h_e is defined in terms of geopotential height and $V^2/2g_0$, in which case the variation of acceleration due to gravity with height is taken into account automatically.

In Equation (3.6) the component of thrust acting in the direction of V is expressed in the form appropriate to turbo-jet and turbo-fan engines; for propeller-driven aeroplanes the propeller thrust would of course be included in the term in square brackets. For cases where different values of F_G , ε , τ , are considered for each engine, see Section 4.1.1 of Item No. 79018.

From Equations (3.5) and (3.6) the geopotential rate of climb is

$$\frac{dH}{dt} = \frac{\frac{dh_e}{dt}}{\left[1 + \frac{1}{2g_0} \frac{d(V^2)}{dH}\right]} = \frac{\{F_G \cos \varepsilon \cos(\alpha + \tau) - D_M - D\}V}{mg_0 \left[1 + \frac{1}{2g_0} \frac{d(V^2)}{dH}\right]} \quad (3.7)$$

3.1.1 Acceleration factor

The term in square brackets in Equations (3.4), (3.5) and (3.7) is referred to as the “acceleration factor” (= 1.0 for climb at constant true airspeed) and provides the relationship whereby climb rates may be adjusted to take account of the acceleration along the flight path corresponding to different climb techniques, see Section 5.3.

3.2 Angle of Climb in Still Air

The air-path angle of climb, γ_a , i.e. the angle of climb in still air, is derived from the rate of climb using

$$\sin \gamma_a = \frac{1}{V} \frac{dZ}{dt} = \frac{1}{V} \frac{g_0}{g} \frac{dH}{dt} \quad (3.8)$$

The ratio $g_0/g = 1$ for most purposes but, if required, values can be obtained from Appendix A of Item No. 73018.

3.3 Application of Performance Equations to Measurement and Analysis Work

Equations (3.4) to (3.7) provide the basis for several approaches to the measurement and analysis of climb, and other aspects of performance, as follows:

Simple rate of climb measurements may be adjusted to conditions of zero acceleration along the flight path by use of the acceleration factor.

The energy approach to climb performance measurement utilises dh_e/dt or *SEP* rather than dH/dt as the point-performance property of greatest interest and offers a method of wider applicability than the use of dH/dt alone. From Equations (3.6) and (3.7) it is apparent that values of dh_e/dt measured during climbs or *level accelerations* (as in Section 4.2) may be used to derive values of dH/dt and dV/dt corresponding to any desired speed-height schedule.

Manoeuvring and acceleration performance can be analysed in a manner similar to the energy approach to climb performance. For example, instantaneous values of *SEP* are measured during level accelerations or turns, “pull-up” or “push-over” manoeuvres and, if necessary, corrected to “standard” conditions as indicated in Section 5. Typically, the results for each manoeuvre are presented separately or in combination as plots of *SEP* against M or V for constant values of normal load factor.

3.3.1 Normal load factor

All manipulations of climb rates and specific excess power are valid only for flight conditions when the normal load factor, n , is essentially the same as in the test conditions. For the case considered in Section 3.1, n is defined by

$$n = \frac{F_G \cos \varepsilon \sin(\alpha + \tau) + L}{mg_0} . \quad (3.9)$$

The relationship between load factors and the readings of accelerometers mounted with their sensitive axes aligned with each of the aircraft's body axes are given in Section 5.3 of Item No. 79018. That Item also considers the adjustment of such readings to air-path axes and the corrections for any displacement of the accelerometers from the aircraft's centre of gravity.

4. ESTABLISHING FLIGHT CONDITIONS FOR OPTIMUM POINT PERFORMANCE IN CLIMB

The determination of the flight conditions for optimum point performance in climb is a major step in the construction of practical speed-height schedules for operational use (see Section 8). This Section introduces four definitions of "optimum" point performance and describes the two principal flight* techniques used to determine the true airspeed at which each definition is satisfied. All relationships here are expressed in terms of "true" quantities, *i.e.* V and H ; Section 5 gives details of correction procedures that may be needed to process measured data into this form.

For any given aeroplane configuration the following optima are defined:

- (i) The speed for maximum rate of change of potential energy (maximum rate of climb) is obtained when,

$$\left[\frac{\partial}{\partial V} \left(mg_0 \frac{dH}{dt} \right) \right]_{\text{const } H} = 0 . \quad (4.1)$$

See Section 4.1 for principal flight technique. A climb schedule based on speeds established using Equation (4.1) gives the minimum time between two (geopotential) heights provided the changes in V are small.

- (ii) The speed for maximum angle of climb in still air is obtained when,

$$\left[\frac{\partial}{\partial V} \left(\frac{mg_0}{V} \frac{dH}{dt} \right) \right]_{\text{const } H} = 0 . \quad (4.2)$$

See Section 4.1 for principal flight technique.

- (iii) The speed for maximum rate of change of total energy is obtained when,

$$\left[\frac{\partial}{\partial V} \left(mg_0 \frac{dh_e}{dt} \right) \right]_{\text{const } h_e} = 0 . \quad (4.3)$$

* The alternative approach of synthesising optimum point and path performance from known aerodynamic and powerplant data is touched upon in Section 2.2 and demonstrated in Item Nos EG 3/3 to 3/5.

See Section 4.2 for principal flight technique and associated analysis. A climb schedule based on speeds established using Equation (4.3) gives the minimum time between pairs of end conditions of speed and (geopotential) height.

- (iv) The speed for maximum change in total energy per unit of fuel consumed is obtained when

$$\left[\frac{\partial}{\partial V} \left(mg_0 \frac{dh_e}{dm_F} \right) \right]_{\text{const } h_e} = 0 . \quad (4.4)$$

The method of analysis in this case is similar to that indicated in Section 4.2. A climb schedule based on speeds established using Equation (4.4) requires the minimum fuel between pairs of end conditions of speed and (geopotential) height.

4.1 Partial or Sawtooth Climbs

The method of partial climbs is used primarily for aeroplanes which climb at low forward speeds and/or in cases where the expected climb schedules lead to small or negligible values of dV/dH or dV/dt . The technique is also used when establishing climb performance with one or more engines inoperative.

The aeroplane is climbed at a series of constant indicated airspeeds through a small height band around the height under consideration. In graphical terms, the speed for maximum rate of climb (Equation (4.1)) is obtained at the point where a horizontal tangent meets the resulting plot of dH/dt (or $mg_0 dH/dt$) against V .

In practice, a plot of rate of climb in terms of pressure height, dH_p/dt , against indicated airspeed is usually sufficient. Also in graphical terms, the speed for maximum angle of climb in still air (Equation (4.2)) is obtained at the point where a tangent from the origin meets the curve of dH/dt against V .

This technique is of course particularly susceptible to the effects of wind gradients and becomes increasingly difficult to apply at high rates of climb.

4.2 Use of Accelerations in Level Flight to Determine Climb Optima

For aeroplanes which climb at high true airspeed the changes in kinetic energy due to acceleration along the flight path can be a substantial proportion of the total energy changes during climb. It is then of greater importance to maximise dh_e/dt or dh_e/dm_F than dH/dt , i.e. to establish the speeds defined by Equations (4.3) and (4.4) respectively.

The case defined by Equation (4.3) is considered here in detail. Since it is impracticable to carry out the tests at constant h_e – as would be required by Equation (4.3) – the following procedure* is usually adopted. The aircraft is accelerated at approximately constant pressure height over a speed range chosen so as to include the likely optimum value at the required engine control setting. (A much wider speed range is used if the aeroplane's acceleration performance is to be investigated at the same time, see Section 3.3.) Tests are repeated at a series of heights, H_1, H_2, H_3 , say, and readings taken from which curves of $mg_0(dh_e/dt)$ against V can be drawn, as in Sketch 4.1a. The subsequent analysis of these data to determine the speed for maximum dh_e/dt is based on the premise that, for constant weight, the condition of Equation* (4.3) is met

* In Equation (4.3) dh_e/dt is expressed as a function of h_e and V but for flight-test work a version of the equation is required with dh_e/dt expressed as a function of H and V . The method is to consider the following equivalent forms of a small change, $\Delta(dh_e/dt)$, in dh_e/dt ,

$$\Delta \left(\frac{dh_e}{dt} \right) = \left[\frac{\partial}{\partial h_e} \left(\frac{dh_e}{dt} \right) \Delta h_e + \frac{\partial}{\partial V} \left(\frac{dh_e}{dt} \right) \Delta V \right] \equiv \left[\frac{\partial}{\partial H} \left(\frac{dh_e}{dt} \right) \Delta H + \frac{\partial}{\partial V} \left(\frac{dh_e}{dt} \right) \Delta V \right] .$$

From the definition of h_e , Δh_e can be expressed as $(\Delta H + V \Delta V / g_0)$ and the required relationship obtained by equating the coefficients of dH and dV from the equivalent expressions in square brackets.