

STRESSES DUE TO INTERFERENCE-FIT PINS AND BUSHES IN PLATES, STRIPS OR LUGS

1. INTRODUCTION

This Item presents data on the elastic stresses arising in flat plates, bars, strips or lugs from the insertion of a circular interference-fit pin or bush of similar or different material. Section 3 deals with solid pins, Section 4 deals with bushes and Section 5 deals with the juxtaposition of two identical interference-fit pins or bushes.

The data are based upon two-dimensional analyses and therefore only give the average stress through the thickness of the plate or strip. Local deviations away from the average value are unlikely to be significant except perhaps where taper pins are used.

The data apply strictly to isotropic materials that obey Hooke's law, but may be used without significant error for most engineering materials. If during loading of the assembly the limit of proportionality of one (or both) of the materials is exceeded the residual stresses due to the interference-fit will be reduced. It should be noted that the degree of interference will be considerably reduced if significant creep takes place.

If the interference-fit pin or bush is additionally held, for example by means of a taper pin or a screw inserted from the side or inserted parallel to the centre line on the interface, the stress distribution will alter and the data of this Item will not be applicable.

Throughout this Item the term “bars or strips” applies to bodies having solid rectangular sections.

2. NOTATION

c	distance between centres of adjacent pins or bushes	m	in
d	diameter of unfilled hole	m	in
d_i	internal diameter of bush	m	in
e	ϵ/d	m/m	in/in
E	Young's modulus of lug, bar or strip or plate material	N/m ²	lbf/in ²
E_I	Young's modulus of pin or bush material	N/m ²	lbf/in ²
f	maximum tensile stress acting around boundary of hole, caused by interference	N/m ²	lbf/in ²
f_o	maximum tensile stress acting around boundary of single hole in infinite plate, caused by interference	N/m ²	lbf/in ²
k	shape factor (for bushes only, see Section 4)		
p	interface pressure	N/m ²	lbf/in ²

q	maximum shear stress at boundary of hole, caused by interference	N/m^2	lbf/in^2
W	width of lug, bar or strip or plate	m	in
ϵ	interference of pin or bush with hole (pin or bush diameter is $d + \epsilon$)	m	in
ν	Poisson's ratio of lug, bar or strip or plate material		
ν_I	Poisson's ratio of pin or bush material		

Both SI and British Units are quoted but any coherent system of units may be used.

3. SOLID CIRCULAR INTERFERENCE-FIT PINS IN BARS OR STRIPS OR PLATES

For the case of an interference-fit pin in a circular hole in an infinite plate, that is $d/W = 0$, the circumferential tensile stress, radial compressive and shear stresses are equal in magnitude and constant around the hole. For a plate of finite width the stress varies around the hole and the maximum value depends upon d/W .

Figure 1 shows q/Ee plotted against $E(1 - \nu_I)/E_I$ for various values of d/W and Figure 2 shows f/Ee plotted against $E(1 - \nu_I)/E_I$ for various values of d/W . At points B marked on the diagram accompanying Figure 1 the tensile stress f , the shear stress q and the interface pressure p approach the values for a pin in an infinite plate. The maximum tensile and shear stresses occur at the points A although the shear stress is only slightly above its value at points B. The interface pressure is at a minimum at points A and a maximum at points B. An estimate of the interface pressure at B is given by $p/Ee = [1 - (d/W)^2]q/Ee$.

The figures are constructed for $\nu = 0.3$. Variation of ν within the range 0.25 to 0.35 does not significantly alter the maximum stress. However, for significantly different values of ν it may be advisable to calculate the stresses from the following formulae which were used to construct the curves.

$$\frac{q}{Ee} = \frac{1}{2 + \left[\frac{E(1 - \nu_I)}{E_I} - (1 - \nu) \right] \left[1 - \left(\frac{d}{W} \right)^2 \right]}$$

and

$$\frac{f}{Ee} = \frac{q}{Ee} \left[1 - \left(\frac{d}{W} \right)^2 \right].$$

These formulae relate to a pin in a ring but they represent the conditions of a pin in a strip or lug with reasonable accuracy provided that d/W is less than about 0.8.

4. INTERFERENCE-FIT BUSHES IN LUGS, BARS OR STRIPS OR PLATES

Figure 3 gives the maximum tensile stress and the interface pressure in a lug, bar or strip or plate resulting from the insertion of an interference-fit bush.

The maximum tensile stress occurs at the points A marked on the diagrams accompanying Figure 3. The maximum shear stress is given by $(f + p)/2$. The chain dotted lines of Figure 3 indicate the order in which the curves should be used and relate to Example 2 in Section 7.

The form of the end of a bushed lug, or the proximity of a plate edge to a bushed hole in a plate, affects the interface pressure and the maximum stress. Figure 4 provides a correction to d/W for this effect for the four common geometries illustrated with the figure.

Figure 3 has been constructed for $\nu = \nu_I = 0.3$. Variation of ν or ν_I within the range 0.25 to 0.35 does not significantly alter the maximum stress. However, for significantly different values of ν and ν_I the following expressions, from which the curves were constructed, can be used.

$$\frac{p}{Ee} = \frac{1}{\left[\frac{1 + (kd/W)^2}{1 - (kd/W)^2} \right] + \nu + \frac{E}{E_I} \left[\frac{1 + (d_i/d)^2}{1 - (d_i/d)^2} - \nu_I \right]}$$

and

$$\frac{f}{Ee} = \frac{p}{Ee} \left[\frac{1 + (d/W)^2}{1 - (d/W)^2} \right].$$

5. TWO ADJACENT IDENTICAL INTERFERENCE-FIT PINS OR BUSHES IN A FLAT PLATE

When two identical interference-fit pins or bushes in a wide flat plate are close to each other the maximum tensile stress f is increased. Figure 5 gives f/f_o plotted against c/d for various ratios of E/E_I . The value of f_o should be taken from Figure 2 at $d/W = 0$ for pins or from Figure 3 at $d/W = 0$ for bushes.

The maximum stress occurs on the boundary of each hole at the points A in the diagram with Figure 5. The effects of the proximity of the holes dies away rapidly around the boundary of each hole. At points B values of f/f_o are below 1.0 and at points D they return to within about ± 5 per cent of 1.0.

The data apply strictly to infinite plates. However, provided that the edge of the plate is more than a distance of about $3d/2$ from the edges of both holes the data are sufficiently accurate.

6. DERIVATION

1. WANG, C.T. *Applied elasticity*. Chapter 4 article 4, McGraw-Hill, New York, 1953.
2. JESSOP, H.T.
SNELL, C.
HOLISTER, G.S. Photoelastic investigation on plates with single interference-fit pins with load applied to the plate only. *Aeronaut.Q.*, Vol. VII, pp. 297-314, November 1956.
3. LAMBERT, T.H. Elastic stresses due to an interference fit pin in a rectangular plate of finite width. *J. mech. Engng Sci.*, Vol. 3, No. 3, pp. 236-240, July 1961.
4. MORGAN, F.G. Static stress analysis and fatigue tests of interference-fit bushes. RAE. tech. Note Structures 316, August 1962.
5. TAKAGI, S. Shrinkage fitting of circular disks into two similar circular holes in infinite plate. *Bull. J.S.M.E.*, Vol. 73, No. 59, pp. 650-659, May 1970.

7. EXAMPLES

1. It is required to find the maximum circumferential tensile stress at the boundaries of two adjacent equal diameter holes in a wide plate when the holes are filled by identical interference-fit pins. The material properties and dimensions are,

plate: material properties	$E = 69\,000 \text{ MN/m}^2$ and $\nu = 0.3$
diameter of holes	$d = 22.00 \text{ mm}$
distance between hole centers	$c = 35.20 \text{ mm},$

pins: material properties	$E_I = 190\,000 \text{ MN/m}^2$ and $\nu_I = 0.24$
diameter	$d + \epsilon = 22.05 \text{ mm}.$

Then,

$$\frac{E(1 - \nu_I)}{E_I} = \frac{69\,000(1 - 0.24)}{190\,000} = 0.276$$

and, using Figure 2 at $d/W = 0$, $f_o/Ee = 0.635$.

Hence,

$$f_o = \frac{0.635 \times 69\,000 \times 0.05 \times 10^{-3}}{22.00 \times 10^{-3}} = 99.6 \text{ MN/m}^2.$$

Now, $\frac{c}{d} = \frac{35.20}{22.00} = 1.6$ and $\frac{E}{E_I} = \frac{69\,000}{190\,000} = 0.363$; therefore, from Figure 5, $f/f_o = 1.44$

giving $f = 1.44 \times 99.6 = 143 \text{ MN/m}^2$.

2. It is required to find the maximum circumferential tensile stress and the maximum shear stress in an aluminium square-ended lug, caused by the interference-fit of a steel bush. The material properties and dimensions are,

lug: material properties	$E = 10 \times 10^6 \text{ lbf/in}^2$ and $\nu = 0.34$
diameter of hole	$d = 1.0 \text{ in}$
width of lug	$W = 1.5 \text{ in},$

bush: material properties	$E_I = 30 \times 10^6 \text{ lbf/in}^2$ and $\nu_I = 0.29$
outside diameter of bush	$d + \epsilon = 1.005 \text{ in}$
inside diameter of bush	$d_i = 0.9 \text{ in}.$

The values of ν and ν_I are within the quoted range so that Figure 3 can be used. The following ratios are required.

$$\frac{d}{W} = 0.667, \frac{d_i}{d} = 0.9 \text{ and } \frac{E}{E_I} = 0.333.$$

From Figure 4, using the curve for a square-ended lug, $kd/W = 0.60$. Then, from Figure 3, entering with