

Elastic direct stresses and deflections for flat rectangular plates under uniformly distributed normal pressure

Associated software: ESDUpac A9311

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Aerospace Structures Committee. This Committee first met in 1940 and now has the following membership:

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The Item was accepted for inclusion in the Stress and Strength Series by the Stress Analysis and Strength of Components Committee which first met in August 1964 and now has the following membership:

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(Continued inside back cover)

ELASTIC DIRECT STRESSES AND DEFLECTIONS FOR FLAT RECTANGULAR PLATES UNDER UNIFORMLY DISTRIBUTED NORMAL PRESSURE

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1. NOTATION

a	length of plate	m	in
b	width of plate	m	in
E	Young's modulus	N/m ²	lbf/in ²
f_C	maximum total stress at centre	N/m ²	lbf/in ²
f_c	stress at middle surface corresponding to f_C	N/m ²	lbf/in ²
f_D	maximum total stress on diagonals	N/m ²	lbf/in ²
f_d	stress at middle surface corresponding to f_D	N/m ²	lbf/in ²
f_E	total stress at mid-point of long edge	N/m ²	lbf/in ²
f_e	stress at middle surface corresponding to f_E	N/m ²	lbf/in ²
p	normal pressure on plate	N/m ²	lbf/in ²
r	ratio $(1 - \nu^2)/0.91$ (when $\nu = 0.3$, $r = 1$)		
t	plate thickness	m	in
δ	maximum deflection of plate	m	in
ν	Poisson's ratio		

Suffixes

- 1 denotes stress in direction of long edge or diagonal
- 2 denotes stress normal to long edge or diagonal

Convention: Tensile stresses are positive. Compressive stresses are negative.

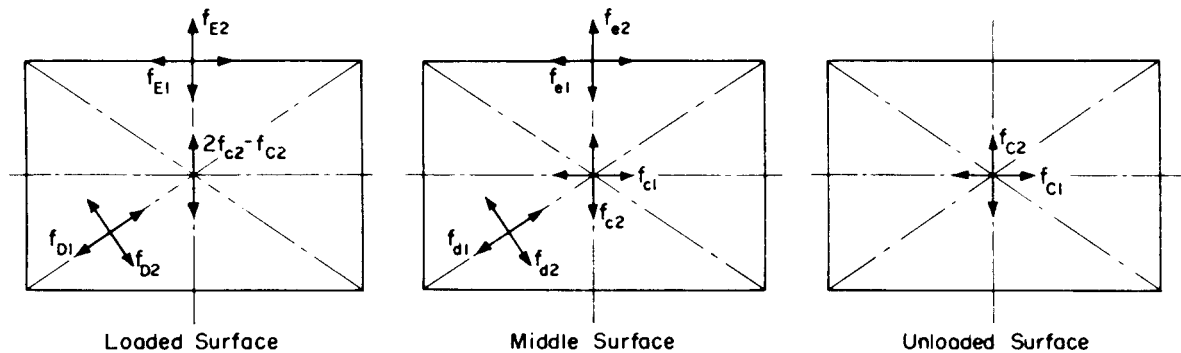
Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

Curves are given that enable the direct stresses and deflections to be determined for initially flat rectangular plates of uniform thickness and various length to width ratios under uniformly distributed normal pressure. The data given can also be obtained using the computer programs ESDUpacs A9311 and A9433 associated with Data Items Nos 93011* and 94033† respectively.

In Figures 1 to 4, according to the form of edge restraint, values of δ/t are plotted against $(b/t)(rp/E)^{1/4}$ for various values of a/b . Values of $(f/p)(t/b)^2$ plotted on the same basis are given in Figures 5 to 11. The curves in Figures 1 to 11 are based on large deflection theory. If the deflection is comparatively small (say $\delta/t < 0.5$) a more rapid and convenient interpolation for a/b may be obtained by using Figure 12 in which $(\delta/t)(E/rp)(t/b)^4$ and $(f/p)(t/b)^2$ are plotted against a/b . This figure is based on linear small deflection (bending) theory. If Figure 12 is used for larger deflections ($\delta/t > 0.5$) the values obtained for the deflections and stresses will increasingly overestimate the true values.

Curves are given for the maximum deflection, the total stress at the centre and at the edge, and for the maximum total stress on the diagonal where this may be greater. The middle surface stresses corresponding to the total stresses are also given. The centre diagram below shows the locations of the middle surface stresses in the plate and the side diagrams show the locations of the total stresses.



The total stresses f_C , f_D and f_E result from the combined effects of bending and middle surface tension or compression. The bending stresses can be found by subtracting the middle surface stresses f_c , f_d or f_e from the corresponding total stresses.

For plates with the edge condition free in rotation and translation certain local compressive stresses can exceed the tensile stresses at the centre or the edge. Accordingly the stress f_{e1} is given in Figure 5 as the maximum stress at the long edge and f_{D2} is given in Figure 6 as the maximum total stress on the diagonal. Depending on the values of a/b and $(b/t)(rp/E)^{1/4}$, f_{D2} may be replaced by, or exceeded by, the compressive stress at the centre of the loaded surface. Therefore $2f_{c2} - f_{C2}$, the transverse compressive total stress at the centre of the loaded surface, replaces f_{D2} where applicable in Figure 6.

The curves of f_{D2} given here are based on values of stress on the diagonals of the plate which are believed to be a close approximation to the maximum principal compressive total stress in the plate occurring in the vicinity of the diagonals.

* Data Item No. 93011 "Flat rectangular orthotropic plates under uniformly distributed normal pressure. Elastic stresses and deflections for various forms of edge restraint." (Structures Sub-series).

† Data Item No. 94033 "Flat rectangular isotropic plates under uniformly distributed normal pressure. Elastic deflections, stresses and strains, moments and forces, for various forms of edge restraint." (Stress and Strength Sub-series).

In Figure 9 the maximum total stress f_{C2} may be exceeded by the stress at a point between the centre and the centre of the long edge. This effect is small and may be estimated from the curve for this stress given for $a/b = 1$ in Figure 4 of Engineering Sciences Data Item No. 70001. As a/b approaches 1, f_{C2} becomes marginally less than f_{e2} ; this effect can also be seen in Figure 4 of Engineering Sciences Data Item No. 70001.

Table 2.1 shows the figures to be used to obtain the deflection and particular stresses for each form of edge restraint.

TABLE 2.1

Applicability	Type of edge restraint		Number of figure	Deflection δ	Stress f (denoted by its suffixes)											
					Total						Middle surface					
	Rotation	In-plane translation			C1	C2	D1	D2	E1	E2	c1	c2	d1	d2	e1	e2
Small and large deflections	Free	Free	1	x												
			5		<	x			x	0	<	x			x	0
			6				<	x					<	x		
	Fixed	Free	2	x												
			7		<	x	<	<			<	x	<	<		
			8						<	x					<	0
	Free	Fixed	3	x												
			9		<	x	<	<	<	x	<	x	<	<	<	x
			10		<	x	<	<			<	x	<	<		
	Fixed	Fixed	4	x												
			11						<	x					<	x
			12													
Small deflections only ($\delta/t < 0.5$)	Free	Free to Fixed	12	x	<	x	<	<	<	0	0	0	0	0	0	0
	Fixed	Free to Fixed	12	x	<	x	<	<	<	x	0	0	0	0	0	0

x denotes that a curve is given

< denotes that no curve is given because the values are lower than the maximum in the plate

0 denotes that the stress is zero.

It is assumed that the plate is initially flat with no irregularities and that the edges are constrained to remain in the original plane of the undeflected middle surface. It is also assumed that when the edges are free in translation they are free to curve (in plane) along their length.

The curves are derived on the basis of thin plate theory which is certainly valid for b/t greater than 20. Strictly they apply only to stresses within the elastic range of the material but are reasonably accurate beyond this provided the stresses do not exceed the 0.2 per cent proof stress.

The stresses and deflections for long flat rectangular plates under uniform and linearly varying pressure and square plates under uniform pressure are given in Engineering Sciences Data Items Nos 69018 and 70001 respectively.

3. DERIVATION

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4. BROWN, J.C. Large deflections of rectangular plates subjected to uniform lateral pressure and compressive edge loading. *J. mech. Engng Sci.*, Vol. 11, No. 3, pp. 305-317, June 1969.
5. HARVEY, J.M. Bending of normally loaded simply supported rectangular plates in the large deflection range. *J. Strain Anal.*, Vol. 4, No. 3, pp. 190-198, July 1969.
6. SCHOLES, A. Application of large-deflection theory to normally loaded rectangular plates with clamped edges. *J. Strain Anal.*, Vol. 5, No. 2, pp. 140-144, April 1970.
7. SCHOLES, A. Unpublished work at Pilkington Brothers Ltd, Research and Development Laboratories, 1970.

4. EXAMPLES

4.1 Example 1

A flat rectangular plate of length $a = 172$ mm, width $b = 95.3$ mm and thickness $t = 4.07$ mm with its edges free to rotate but fixed against movement in the plane of the plate, is subjected to a pressure of 0.469 MN/m². Young's modulus for the plate material is $207\,000$ MN/m² and Poisson's ratio is 0.29 . It is required to find the maximum deflection and the total, bending and middle surface stresses.

For this material $r = 1.0065$.

The parameters required for use with this Item are:

$$\frac{b}{t} \left(r \frac{p}{E} \right)^{1/4} = 0.91 \quad \text{and} \quad \frac{a}{b} = 1.8.$$

For the edge condition given, Table 2.1 shows that Figures 3 and 9 or Figure 12 should be used. Inspection of Figure 3 shows the deflection will be small ($\delta/t < 0.5$). A similar inspection of Figure 9 shows the stresses will deviate little from the values at the origin. There is therefore, in this case, advantage to be gained in using the increased accuracy of interpolation for a/b offered by Figure 12.

From Figure 12,

$$\frac{\delta}{t} \left(\frac{E}{rp} \right) \left(\frac{t}{b} \right)^4 = 0.102;$$

also,

$$\frac{f_{C2}}{p} \left(\frac{t}{b} \right)^2 = 0.57 \quad \text{and} \quad \frac{f_{E2}}{p} \left(\frac{t}{b} \right)^2 = 0.$$

Now,

$$r \frac{p}{E} \left(\frac{b}{t} \right)^4 = 0.685 \quad \text{and} \quad p \left(\frac{b}{t} \right)^2 = 257 \text{ MN/m}^2.$$

Therefore,

$$\delta = 0.102 \times 0.685 \times 4.07 = 0.28 \text{ mm};$$

also,

$$f_{C2} = 146 \text{ MN/m}^2 \quad \text{and} \quad f_{E2} = 0 \text{ MN/m}^2.$$

Inspection of the curves for f_{C2} on Figure 9 will show the value obtained to be within 2 per cent of that to be expected from large deflection analysis. According to the linear theory on which Figure 12 is based f_{C2} is due to bending only; however, Figure 9 shows that there is in reality a small middle surface stress.

From Figure 9,

$$\frac{f_{c2}}{p} \left(\frac{t}{b} \right)^2 = 0.02,$$

hence,

$$f_{c2} = 5.14 \text{ MN/m}^2,$$

and the corresponding bending stress,

$$\begin{aligned} f_{C2} - f_{c2} &= 146 - 5.14 \\ &= 141 \text{ MN/m}^2. \end{aligned}$$

Also

$$f_{E2} = f_{e2} \approx f_{c2} = 5.14 \text{ MN/m}^2.$$