

## DRAG-RISE MACH NUMBER OF AEROFOILS HAVING A SPECIFIED FORM OF UPPER-SURFACE PRESSURE DISTRIBUTION: CHARTS AND COMMENTS ON DESIGN

### SUMMARY

Charts are presented in Sections 1 to 5 for estimating drag-rise Mach number ( $M_D$ ) as a function of lift coefficient ( $C_{LD}$ ) and thickness/chord ratio ( $t/c$ ) for an infinite family of aerofoils characterised in viscous flow by a particular form of upper-surface pressure distribution at the drag-rise condition (that condition beyond which a rapid increase in shock-wave drag may be anticipated) and a given form of thickness distribution.

The particular form of pressure distribution is specified at design values of Reynolds number and transition position. It has a level plateau ("flat roof-top") extending from the immediate vicinity of the leading edge to a given chordwise station ( $x_R$ ), followed by a linear rise to the trailing edge;  $x_R$  is varied from  $0.3c$  to  $0.6c$  through Figures 2 to 5. The corresponding thickness distributions are chosen from the RAE 101 to 104 series, with  $t/c$  ranging from 6 per cent to 18 per cent in each case. The design Reynolds number is  $10^7$  and the design transition position is at  $0.05c$  on both upper and lower surfaces.

The specification implies a camber line and incidence uniquely derived for each point on the charts. The loci on each chart were thus constructed by deriving aerofoils by an inverse process for a chosen range of  $M_D$  corresponding to a number of such discrete points and determining the corresponding value of  $C_{LD}$  from a knowledge of the pressure distribution derived in turn from a knowledge of the camber line, incidence, thickness distribution, design Reynolds number and transition position.

Further charts in Figures 6 to 9 indicate, according to Stratford's criterion, the threshold beyond which the linear pressure rise over the rear of the aerofoil would, at the drag-rise condition, cause the boundary layer to separate upstream of the trailing edge (*i.e.* in the form of separation often referred to as rear separation).

Section 6 of this Memorandum relates to the derivation of the charts, to their accuracy, to their inherent limitations and discusses the predictions of the principal charts in relation to those for alternative combinations of thickness and upper-surface pressure distributions. The means is then outlined whereby an aerofoil implicit in the charts may be developed to obtain improved drag-rise characteristics.

This Memorandum supersedes T.D. Memo. 67009 and the principal differences between the two Memoranda are commented upon in the Introduction.

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## 1. NOTATION

|          |                                                                                                                                            |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------|
| $a$      | see footnote <sup>†</sup> at end of Notation                                                                                               |
| $C_L$    | lift coefficient                                                                                                                           |
| $C_{LD}$ | lift coefficient when $M_0 = M_D$                                                                                                          |
| $C_{Li}$ | see footnote <sup>‡</sup> at end of Notation                                                                                               |
| $C_p$    | pressure coefficient                                                                                                                       |
| $c$      | aerofoil chord                                                                                                                             |
| $H$      | stagnation pressure in undisturbed stream                                                                                                  |
| $M_0$    | free-stream Mach number                                                                                                                    |
| $M_D$    | drag-rise Mach number as defined in Reference 1 (see Section 6.2, Item (a) here)                                                           |
| $M_{Ds}$ | <i>lowest</i> drag-rise Mach number that can be achieved without flow separation upstream of the trailing edge (see footnote to Section 2) |
| $N$      | number of chordwise stations in theory of Reference 5                                                                                      |
| $p$      | local static pressure                                                                                                                      |
| $R_{ed}$ | design Reynolds number based on chord length and $M_D$                                                                                     |
| $R_{e0}$ | operating Reynolds number based on chord length and $M_0$                                                                                  |
| $t$      | aerofoil maximum thickness                                                                                                                 |
| $x$      | chordwise distance measured from leading edge                                                                                              |
| $\alpha$ | angle of incidence                                                                                                                         |
| $v$      | specified stations in theory of Reference 5 (except in Appendix B)                                                                         |

### Suffixes

|       |                                                                            |
|-------|----------------------------------------------------------------------------|
| $cr$  | pertaining to crest position                                               |
| $l$   | denotes lower surface                                                      |
| $min$ | denotes minimum value                                                      |
| $R$   | identifies chordwise station defining end of the “roof-top” (see Figure 1) |
| $r$   | denotes local value on constant-pressure plateau                           |

*For footnotes refer to end of Notation Section*

|       |                                          |
|-------|------------------------------------------|
| $t$   | due to thickness distribution            |
| $te$  | pertaining to trailing edge              |
| $trd$ | identifies design transition position    |
| $tro$ | identifies operating transition position |
| $u$   | denotes upper surface                    |

Additional notation, first introduced in the Appendices, will be found in Sections A1 and B1.

<sup>†</sup> In Reference 14 (Chapter 4) the symbol  $a$  is used to denote, for mean lines (*i.e.* camber distributions) commonly associated with the NACA 6-series aerofoils, the fraction of the chord from the leading edge over which the loading in incompressible flow is uniform at the “ideal” angle of attack. The “ideal” angle of attack is simply the unique value at which, according to incompressible-flow thin-aerofoil theory, a finite velocity occurs at the leading edge (Reference 14, Chapter 4).

<sup>‡</sup> In Reference 14 (Chapter 4) the symbol  $C_{Li}$  is defined as the “ideal” lift coefficient, occurring at the “ideal” angle of attack.

## 2. INTRODUCTION

The main object of this Memorandum is to present charts from which a rapid but realistic assessment can be made of the dependence of section drag-rise Mach number on the principal design parameters, and which, at the same time, provide an acceptable starting point for the more detailed design of aerofoils to meet specified conditions.

The principal charts (Figures 2 to 5) are derived from the method given in Reference 1 for estimating the drag-rise Mach number of any arbitrary two-dimensional aerofoil with lift coefficient as variable. Here, however, each line on the charts is a locus of points ( $M_D$ ,  $C_{LD}$ ) for an infinite family of aerofoils of given thickness/chord ratio, with each point corresponding to a particular camber line and incidence uniquely determinable from known specifications.

The charts are thus supplementary to the original method itself. This original method (or an extension of it – see Section 6.4.1) still must be used for the calculation of the full  $M_D$ ,  $C_{LD}$  locus for a given aerofoil once its geometry, and hence pressure distributions for a range of values of lift coefficient, have been derived. If the given aerofoil is one corresponding to a point on the charts, the derivation of its  $M_D$ ,  $C_{LD}$  locus will often be the first step in an iterative process of optimising the design for a range of operating and “off-design” conditions.

Because the upper-surface pressure distribution plays such a dominant part in determining so many of the aerodynamic characteristics of an aerofoil section – for example, the drag-rise Mach number itself, the drag level and the onset of shock-induced and rear separation – the form of this distribution has been chosen as one of the parameters relating the whole family of aerofoils represented on each chart and as the main parameter relating the separate charts; this gives the charts an aerodynamic coherence that could not otherwise be achieved and adds realism to the assessment of the relative effects of the variables. The particular form chosen for the upper-surface pressure distribution has a flat roof-top of constant pressure extending from the immediate vicinity of the leading edge to a given chordwise point ( $x_R$ ), followed by a linear rise to a value at the trailing edge that is inherent in the specification (see Figure 1 and Section 6.2). This form was first discussed, in a similar context, by Bagley<sup>2</sup> whilst precisely the same specification was used in T.D. Memo. 67009 issued in 1967 which this present Memorandum supersedes. The specification has several merits from the points of view of directness in derivation of the charts and of the design of section shapes with desirable properties. The charts in T.D. Memo. 67009 were derived for inviscid flow using the compressibility correction factors believed at the time to be the best available but it has now been

possible to couple this specification with a design Reynolds number ( $R_{ed}$ ) and a design transition position  $[(x/c)_{trd}]$  to derive charts for viscous flow conditions, at the same time incorporating in the derivation more-recently derived compressibility correction factors which now have received substantial verification<sup>3</sup>.

For users of the charts in T.D. Memo. 67009 it is relevant to record that the principal difference between those and the present charts, for a given combination of  $M_D$ ,  $t/c$  and  $x_R/c$ , is some reduction in  $C_{LD}$ , maximum camber, angle of incidence at the design condition and  $C_{pte}$ . The camber distribution and the angle of incidence at the design condition are not presented here but values for a selection of the aerofoil shapes implicit in the charts, together with a discussion of the influence that the principal design variables have on the geometric shape of the aerofoils, are presented in Reference 4.

The reduction in  $C_{pte}$  is of particular significance. It is now apparent that the artifice<sup>†</sup> used in T.D. Memo. 67009 to define  $C_{pte}$  led to values of  $C_{pte}$  considerably higher than are to be anticipated in a real viscous flow. This led in turn, using Stratford's criterion<sup>7</sup>, to a considerable overestimate of the value of  $M_D$  below which (see footnote to Section 3) rear separation is to be anticipated. Consequently compared with the charts in T.D. Memo. 67009 the present charts, Figures 6 to 9, show a considerable increase in the portion of the charts that are estimated to be free from rear separation at a given combination of operating Reynolds number and upper-surface transition position.

The main body of this Memorandum falls naturally into two parts. Section 1 to 5 are concerned with the presentation of the material likely to be most frequently used – namely the charts and a description of their application in an aerofoil design process. Section 6 describes the basis of the derivation of the charts, discusses their accuracy and limitations and outlines some of the principles of aerofoil design pertinent to the high subsonic and low supersonic cruise régimes.

Appendices are added to present: (A) the equations used in deriving the charts and (B) the application of Stratford's criterion<sup>7</sup> for predicting the threshold beyond which a turbulent boundary layer would separate due to the linear pressure rise at the rear of the aerofoil.

<sup>†</sup> Namely, by defining  $C_{pte}$  as the trailing-edge pressure due to thickness, then calculating the complete pressure distribution due to thickness at  $M_0 = M_D$  according to the inviscid flow method of Reference 5 (modified to account for compressibility effects by the method of Reference 6) with  $N=16$ , but modifying the trailing-edge portion by extrapolating linearly from midway between the specified stations for  $v=4$ , ( $x/c=0.8536$ ), and  $v=3$ , ( $x/c=0.9157$ ), to the trailing edge.

### 3. PRESENTATION OF THE CHARTS

Each chart of drag-rise Mach number as a function of lift coefficient and thickness/chord ratio has been constructed for an infinite family of aerofoils related by having at the design Reynolds number and for the design transition position the form of upper-surface pressure distribution described in the Introduction at particular points ( $M_D$ ,  $C_{LD}$ ) and having also the same form of thickness distribution; it is the thickness distribution that determines the upper limit of drag-rise Mach number (see Section 6.3.4) and thus the upper Mach number limit of the charts. The particular points yield  $C_{LD} - v - M_D$  loci for aerofoils having fixed values of percentage thickness/chord ratio [6(2)18] and these loci are presented in Figures 2 to 5, each figure relating to a fixed value of  $x_R/c$  [0.3(0.1)0.6]. Interpolation between the curves, to obtain data for values of  $t/c$  and  $x_R/c$  other than those quoted, presents no difficulty.

A further set of charts repeating those in Figures 2 to 5 in outline only is presented in Figures 6 to 9. These have thresholds superimposed on them, corresponding to a range of values of Reynolds number and transition position (designated operating values to differentiate them from values used in the design process) to the left<sup>†</sup> of which a criterion, developed in Appendix B from that proposed by Stratford<sup>7</sup>, predicts that the pressure rise from the end of the roof-top will induce boundary-layer separation upstream of the trailing edge. Each figure again relates to a fixed value of  $x_R/c$ . These thresholds, together with others for additional operating transition positions, are also given in Table B9.1. In evaluating thresholds for a range of operating values of Reynolds number and transition position the assumption has been made that at these operating values the trailing edge pressure coefficient is unaltered from the values implicit in the design process which are themselves dependent upon the design values of Reynolds number and transition position. This assumption is not strictly true and so at other than design values of Reynolds number and transition position the thresholds should be regarded as no more than a guide. For combinations of operating Reynolds number and transition position other than those given in Table B9.1 the process described in Appendix B provides a rapid means of calculating the separation threshold.

For the form of upper-surface pressure distribution fundamental to this Memorandum the separation threshold – for specific values of operating Reynolds number and operating transition position – is dependent only on the pressure coefficient at the trailing edge ( $C_{pte}$ ). This value is not known *a priori*, but being inherent in the design process, is a function of all the design parameters, namely,  $t/c$ ,  $x_R/c$ ,  $M_D$ , thickness form,  $R_{ed}$  and  $(x/c)_{trd}$ . Values of  $C_{pte}$  implicit in the charts in Figures 2 to 5, are plotted in Figure 12 and have been superimposed on the thresholds in Figures 6 to 9. The value of  $C_{pte}$  adjacent to each broken vertical line occurs at the condition ( $M_D$ ,  $C_{LD}$ ) at which the broken vertical line intersects the threshold. A description is included in the next section of how these values may be used to assess what alleviation of a separation threshold would occur if  $C_{pte}$  were smaller than that implicit in the derivation of the charts – the difference arising in part, perhaps, from a change in thickness form from that used to derive the charts.

Here it is appropriate to note that although the charts in Figures 2 to 5 have been derived using the RAE 101 to 104 series of thickness forms<sup>8</sup> respectively (for the reason stated in Section 6.3.2) there is, in principle, no restriction on using alternative thickness forms in the design process, or associating with an upper-surface pressure distribution having a particular roof-top limit ( $x_R/c$ ) a thickness distribution having a shorter roof-top length.

When using an alternative thickness form some variation from the value of  $C_{LD}$  given by the charts is to be expected. The source of this variation can be appreciated from considering the changes that an alternative thickness form will introduce in the area between the curves of upper-surface pressure-distribution (LABT

<sup>†</sup> It may seem surprising at first reading that separation is predicted to occur in the lower of the two Mach number regions into which any separation threshold divides a chart. The reason for this is that the value of  $|C_p|$  along the roof-top, given by the condition in (a) of Section 6.2 and Figure 13, increases as  $M_D$  decreases, and that this in turn leads to an increase in the pressure rise and pressure gradient, expressed in coefficient form, between  $x_R/c$  and the trailing edge.

in Figure 1) and the pressure coefficient due to thickness (LCDT in Figure 1). Firstly, the curve LCDT will be modified and the consequent new value of  $C_{pte}$  at  $T$  will modify the pressure recovery BT. These changes, which are likely to be small, are illustrated in Figure 10.

A comprehensive evaluation of the effect of alternative thickness forms on  $C_{LD}$  and  $C_{pte}$  has not been attempted, but an example is presented in Figure 11.

In this case the use of an alternative thickness form produces advantages in the form of some alleviation of the rear pressure gradient on the upper surface and of some increase in  $C_{LD}$ . However there is no guarantee that these advantages will always occur and to examine the effects of changing thickness forms would, along with developments described in Section 6.6, constitute a refinement of the design process once the charts presented here had identified the parameter limits within which more detailed design studies should be made.

Attention is drawn to the rather extreme pressure gradient that occurs at  $x/c \approx 0.45$  on the lower surface of the derived aerofoil in Figure 11. In practice such a gradient might be undesirable and further iterations on the design would be required to eliminate it. One such process would be to smooth out the lower-surface pressure distribution in such a way that the maximum positive pressure gradient was reduced but  $C_{LD}$  was unaltered. Because the resulting pressure changes would usually be small, linearised theory<sup>9</sup> could be used to derive the corresponding changes in the thickness and camber distributions. This might involve a trial and error process to avoid reducing the maximum thickness of the aerofoil.

## 4. APPLICATION OF THE CHARTS IN AEROFOIL DESIGN

This section describes how the charts should be used in an aerofoil design problem, taking account of appropriate limiting conditions. The means of obtaining the camber line and incidence – and thus the geometry – corresponding to any point on the charts of Figures 2 to 5 forms part of Appendix A. Several examples of the complete process and descriptions of the effects of the major design variables are given in Reference 4. The accuracy and limitations of the charts are discussed in Section 6.4.

When used as a first step in aerofoil design the charts in Figures 2 to 5 will give the exchange rates between  $C_{LD}$ ,  $M_D$ ,  $t/c$  and  $x_R/c$  at the particular design conditions  $Re_d = 10^7$  and  $(x/c)_{trd} = 0.05$ . Knowledge of these exchange rates will facilitate the selection of values of  $M_D$ ,  $t/c$  and  $x_R/c$  compatible with specific design requirements. The upper-surface pressure distribution is then known and Stratford's criterion<sup>7</sup>, as developed in Appendix B and presented in the form of separation thresholds in Figures 6 to 9, will indicate whether the design conditions are compatible with attached flow.

If the design conditions and the separation threshold are clearly incompatible, further thought incorporating some radical change in approach is required and consideration of some form of boundary-layer control might even be appropriate.

If on the other hand the aerofoil implicit in the chosen design point on the charts is marginally unsatisfactory from the viewpoint of separation, some improvement may come from re-specifying the upper surface. This can be achieved in principle by a change in thickness form or by detailed changes in design of the trailing edge.

Again in principle, the solution of the design problem to achieve a suitable pressure gradient will be an iterative one following precisely the principles used to derive Figures 2 to 5 as set down in Appendix A.

Some guidance on the reduction in  $C_{pte}$  required to achieve a more suitable separation threshold, or the change in the separation threshold corresponding to an achieved reduction in  $C_{pte}$ , may be gained from

Figures 6 to 9 because the separation threshold for an upper-surface pressure distribution of the form basic to this Memorandum but specified independently of thickness-chord ratio takes the form of a vertical line on the figures. It intersects the threshold extracted from Table B9.1 (for the same conditions of  $x_R/c$ , operating Reynolds number and transition position) at the specified value of  $C_{pte}$ . Thus the short broken vertical lines, indicating the variation of  $C_{pte}$  along the separation thresholds that have been calculated for the upper-surface pressure distribution implicit in the charts, also serve to indicate where separation would occur for any independently specified value of  $C_{pte}$ .

## 5. ASSESSMENT AND FURTHER DEVELOPMENT OF AN AEROFOIL DESIGN

It is recommended that the next stage in any given problem of aerofoil design should involve evaluating the drag-rise characteristics of the derived aerofoil over its operating range of design and off-design conditions. This can be done either by the method of Reference 1 or by the extension of the method referred to in Section 6.4.1.

It is appropriate here to draw attention to the need to examine the leading-edge suction loop generated by the aerofoil when evaluating its off-design performance.

Because the particular form of pressure distribution implicit in the charts and depicted in Figure 1 has zero pressure gradient ahead of the crest, the suction loop in the incompressible flow corresponding to this condition cannot be of the triangular form<sup>1</sup> that leads to a premature onset of shock-wave drag, but it should be noted that as the incidence is increased above the value corresponding to the flat roof-top condition a leading-edge suction peak will develop. Initially, this suction peak, from the point of view of drag-rise Mach number, may be favourable or unfavourable depending on the shape of the suction loop produced by the particular combination of aerofoil shape and pressure distribution. However, the criterion developed in Reference 1 to identify those features of an aerofoil and its pressure distribution which contribute to the full triangular form of the suction loop implies that, as the incidence is increased, *all* aerofoils will *eventually* develop a pressure distribution of the form that leads to a supercritical drag-creep. Thus, in assessing the off-design performance of an aerofoil implicit in the charts, examining the suction loop is no less important than for any other aerofoil and will establish the incidence beyond which a supercritical drag-creep would be expected (see also further remarks below).

Once the drag-rise characteristics have been obtained the development of the shock waves on the upper surface of the aerofoil should be studied to ensure that there is an adequate margin in terms of  $C_L$  and  $M_0$  between drag rise and the onset of shock-induced separation effects at both design and off-design conditions. The divergence of trailing-edge pressure<sup>10</sup> may be used as a criterion for the onset of significant separation effects on the magnitude and steadiness of the loads on the aerofoil. Correlations of critical values of shock-upstream pressure for trailing-edge pressure divergence have been developed<sup>10</sup> and these form a reasonable basis for prediction within the framework of Sinnott's transonic aerofoil theory<sup>11, 12</sup>. Here again, in considering the onset of shock-induced separation, examination of the suction loop generated by the aerofoil in incompressible flow as incidence is increased from the design value can provide a useful indication of whether any particular aerofoil is likely to experience the weakened shocks associated with favourable development of local supersonic flow and a peaky suction loop<sup>1</sup>, or to suffer from the opposite trends associated with suction loops of triangular form<sup>1</sup>.

A study of the aerofoil's characteristics could then be extended to calculate profile drag and pitching moment over the operating range and, finally, the further means outlined in Section 6.6 of developing the aerofoil to improve its drag-rise characteristics could be investigated. If the margins of  $C_L$  between drag-rise and the onset of shock-induced separation are important then, in this process of developing a design, the tendency for the aerofoil to generate a suction loop of peaky form would be sought, if necessary by adjustments to the thickness form.

## 6. DERIVATION, ACCURACY AND LIMITATIONS OF THE CHARTS AND SOME PRINCIPLES OF AEROFOIL DESIGN

### 6.1 PRESENTATION OF MATERIAL

The underlying principles that form the basis of the charts in Figures 2 to 5 are described in Section 6.2 and the procedure used to derive them is explained in Section 6.3. Attention is drawn in Section 6.4 to the accuracy of the charts, to certain inherent limitations and to the fact that they go beyond – in terms of lift coefficient – what has been established as plausible through experimental work, but in doing so point to important advances that could be exploited by further research work.

Section 6.5 indicates the merit of using an aerofoil implicit in the charts rather than one from a standard family<sup>13, 14</sup> and discusses how a locus from the charts compares with those for standard aerofoils with varying amounts of conventional camber<sup>14</sup>.

Section 6.6 indicates how still greater improvements in aerofoil design are already being achieved and discusses some of the principles that are fundamental to modern concepts of aerofoil designs suitable for high subsonic and low supersonic cruise conditions.

### 6.2 BASIS OF THE CHARTS; The choice of upper-surface pressure distribution – features and merits

The essential feature of the particular form of upper-surface pressure distributions basic to the charts of Figures 2 to 5 is the flat roof-top of constant pressure referred to in the Introduction, with the value of this pressure chosen to ensure that the free-stream Mach number is  $M_D$ . The other principal features are the extent of the roof-top, the pressure recovery from the end of the roof-top to the trailing edge and the treatment of the pressure distribution in the vicinity of the end of the roof-top. Each of these features is discussed below.

#### (a) *The essential feature*

The method of estimating drag-rise Mach number employed to establish the charts has evolved from that of Reference 1. The method makes use of an empirical criterion that the drag-rise Mach number coincides with the Mach number for shock-at-crest. Then, from the prediction of the shock-at-crest condition in Sinnott's transonic aerofoil theory<sup>11</sup>,  $M_D$  is empirically predicted to occur when the crest Mach number reaches 1.02 or, in the terminology used by Sinnott, when the equivalent shock-free<sup>†</sup> crest pressure-ratio  $(p/H)_{cr}$  equals 0.515.

It follows from this, assuming that the empiricism can be generalised, that an aerofoil will be free of shock-wave drag if the crest pressure equals (or exceeds)  $0.515H$ , provided the incompressible-flow suction loop does not transgress the criterion developed<sup>1</sup> to identify the incidence above which aerofoils are likely to exhibit a supercritical drag-creep before the onset of the rapid growth of shock-wave drag. The foregoing conditions are met when one imposes the constraint that  $p$  equals  $0.515H$  over the region of the aerofoil upper surface in which the crest must lie. The corresponding value of pressure coefficient ( $C_{pr}$ ) as a function of Mach number ( $M_D$ ) is shown in Figure 13.

It is this constraint that constitutes the essential feature of the flat roof-top form of pressure distribution referred to in the Introduction and illustrated in Figure 1. It follows that the charts of Figures 2 to 5 represent nearly optimum drag-rise conditions because the criterion on which they are based identifies a stage in the

<sup>†</sup> The term equivalent shock-free flow is used to identify a pressure (or velocity) distribution for just-supercritical free-stream Mach numbers calculated by a method valid, strictly, only for subcritical flow conditions. Here the method of Reference 3 is used. In supercritical conditions the calculated distribution will differ from the real flow when shock waves are present in the latter. Nevertheless the equivalent shock-free flow can be used in an empirical way to predict the appearance of these shock waves.

development of local supersonic flow beyond which a rapid rise in drag can be expected. Furthermore, a flat roof-top pressure distribution cannot transgress the criterion already referred to that identifies the threshold of the supercritical drag-creep.

Improvement on the loci should be possible, as described in Section 6.6, by adding further loading near the leading edge and over the rear portion of the aerofoil. In doing so, however, it is important to bear in mind the “off-design” performance of the aerofoil and attention is drawn to the remarks in Section 5 on this point.

(b) *The extent of the plateau*

The preceding sub-section established the concept that has led to incorporating a plateau of constant pressure ratio into the generalised upper-surface pressure distributions that form the basis of the charts.

To ease the presentation of the charts it was desirable to limit the number of parameters defining the overall pressure distributions and so it was convenient to specify a plateau always starting from the leading edge<sup>†</sup> but extending rearward to a variable chordwise station. This station is designated  $x_R$  and the charts presented in Sections 3 and 4 are for  $x_R/c = 0.3(0.1)0.6$ .

(c) *The pressure recovery*

For the purpose of constructing the charts it was convenient to assume a linear pressure recovery from  $x_R$  to the trailing edge.

The trailing-edge pressure ratio is implicit in the design process and is compatible with real (*i.e.* viscous) flow conditions.

(d) *The pressure distributions in the vicinity of  $x = x_R$*

The conditions described in sub-sections (b) and (c) above lead to a particular form of upper-surface pressure distribution that has a discontinuity in pressure gradient at  $x_R$ . Such a discontinuity in gradient is undesirable in practice because it leads to a rapid increase in shock-upstream Mach number in just-supercritical flow, which in turn leads to a rapid onset of shock-induced separation and buffeting. Thus, to add further realism some rounding-off of the upper-surface pressure distributions in the vicinity of  $x_R$  was incorporated in the method of deriving Figures 2 to 5. Such rounding-off also helps to avoid rear separation in shock-free flow; further comment on rear separation will be found in Section 6.4.2. The extent of the rounding-off for each value of  $x_R$  is detailed in Section 6.3.5.

From the foregoing it will be appreciated that the chosen form of upper-surface pressure distribution has the merit of being one which is approaching an optimum from the point of view of drag-rise Mach number but which allows  $M_D$  to be presented in chart form without the need to define the corresponding aerofoils in explicit terms. The extent to which this leaves the designer free to develop an aerofoil approaching an optimum in terms of drag-rise Mach number and to meet other requirements specific to his purpose is the subject of further reference in Section 6.3.2. Furthermore, it should be noted that since the profile drag is dependent on high powers of the ratio of the local flow velocity to the free-stream velocity the particular form of flat roof-top pressure distribution used here may be approaching an optimum in respect of this quantity also<sup>‡</sup>.

<sup>†</sup> This also ensures that the loading near the leading edge is as high as it can be without having a peak velocity at the design conditions.

<sup>‡</sup> The profile drag coefficients for a selection of the aerofoils implicit in the charts have been evaluated for a range of operating Reynolds number and transition position at the drag-rise condition ( $M_D, C_{LD}$ ) using the method of Nash, Osborne and Macdonald<sup>15</sup>, modified in the light of unpublished work at NPL. The results are presented in Reference 16.

A further feature of choosing the one form of upper-surface pressure distribution is that, with the possible exception of a region near the trailing edge, the distributions at a chosen value of  $M_D$  are sensibly independent of the thickness/chord ratio. Lock in unpublished work and Pearcey<sup>10</sup> have demonstrated that at a given value of  $M_D$  this leads to upper-surface geometries that are nearly the same irrespective of  $t/c$ . It follows, and has been demonstrated by Pearcey<sup>10</sup>, that the development of the local supersonic flow on the upper surface with increasing free-stream Mach number above a given value of  $M_D$  – and thus the margin in terms of Mach number between the drag-rise and buffet thresholds – will be insensitive to  $t/c$  ratio. Hence an aerofoil thickness and lower surface may be optimised from considerations distinct from those that govern the choice of the upper surface.

Finally, other merits of the particular form of upper-surface pressure distribution used here lie in the aerodynamic coherence, referred to in the Introduction, which it gives to the charts and which, in turn, is reflected in their accuracy (see Section 6.4.1) in a relative sense. This high order of relative accuracy between points on the charts will also feature in estimates of profile drag and of the Mach number ( $M_{Ds}$ ) corresponding to rear separation (see Section 6.4.2 and Appendix B).

## 6.3 DERIVATION OF THE CHARTS

### 6.3.1 Method

The method of deriving the charts constitutes a full aerofoil design procedure and is an inversion of the direct method given in Reference 3. A brief statement of the equations used is given in Appendix A.

The value of  $M_D$  is known directly from the specification of the upper-surface pressure distribution. Thus, to derive the charts it was simply necessary to determine at each  $M_D$  the values of  $C_{LD}$  resulting from the particular forms specified for the upper-surface pressure distributions and thickness distributions.

Referring to Figure 1, the pressure distribution due to thickness alone at zero incidence is represented by the curve *LCDT* while the prescribed upper-surface pressure distribution is represented by the curve *LABT*. The procedure was to derive the ordinates of the camber line and the value of the associated incidence, which, at the known drag-rise Mach number, would generate the required upper-surface pressure distribution when combined with the chosen thickness form and thickness/chord ratio at the design Reynolds number and with the design upper- and lower-surface transition positions. An outline of this particular part of the procedure constitutes part of Appendix A. The camber ordinates added to the thickness ordinates gave the complete aerofoil; the application of the direct Lock, Wilby, Powell method<sup>3</sup> to this aerofoil at  $M_0 = M_D$  and the appropriate incidence yielded the lower-surface pressure distribution – curve *LEFT* in Figure 1.

The lift coefficient ( $C_{LD}$ ) was obtained as  $\int_0^1 (C_{pl} - C_{pu}) dx/c$ .

Finally, loci for particular values of  $t/c$  were generated by repeating the process for a range of  $t/c$  at each of several values of  $M_D$ . It is these loci that are presented in Figures 2 to 5.

### 6.3.2 Basic Thickness Distribution

The thickness distribution used in deriving the charts represented to some extent an arbitrary choice, but it seemed sensible (from the viewpoint of simplifying the calculations) to associate with an upper-surface pressure distribution having a particular roof-top limit ( $x_R/c$ ) a thickness distribution having the same property. For this reason the RAE series<sup>8</sup> was chosen for the present calculations, since sections RAE 101,

102, 103 and 104 are designed to have  $x_R/c = 0.3, 0.4, 0.5$  and  $0.6$  respectively.<sup>†</sup>

It is known that the integral of the pressure distributions referred to above differ little from each other for any given thickness/chord ratio; this is illustrated in Figure 10 by sketches of typical pressure distributions.

It follows that the charts of Figures 2 to 5 are comparatively insensitive to the exact form of thickness distribution chosen and so maximum simplicity of presentation is obtained by associating with a particular upper-surface pressure distribution the thickness distribution with the same roof-top limit.

In the design of an aerofoil it should be borne in mind that the charts may be used for a thickness distribution within the RAE 101 to 104 series other than that used to derive any particular chart provided the roof-top limit for the thickness distribution does not exceed that for the upper-surface pressure distribution. This simply requires a small correction to be made to  $C_{LD}$ , the reason for the revised value being described in Section 3.

Furthermore, the RAE series thickness distribution can be replaced by any other thickness distribution provided the above correction is made. Indeed the integral  $\int_0^1 C_{pt} dx/c$  is not necessarily a minimum for the RAE series and for this and other reasons it must be emphasised that an aerofoil from this series is not necessarily the best thickness distribution to use. These other reasons involve the small leading-edge radius of the series and the discontinuity in surface curvature at  $x = x_R$  which, in some cases, can lead to undesirable lower-surface shapes and velocity distributions, more particularly with the form of upper-surface pressure distribution chosen here, for the reasons set out in (d) of Section 6.2. Thus, although the charts are based on the RAE series thickness distributions, there remains considerable freedom of choice in this respect which can be exploited in many aspects of aerofoil design; for instance, in choosing the leading-edge radius, in locating the centre-of-pressure position, or in optimising the lower-surface pressure distribution in some other way.

### 6.3.3 Pressure Coefficient at the Trailing Edge

For an aerofoil with a non-zero trailing-edge angle, in inviscid flow full stagnation pressure occurs at the trailing edge, but for a real viscous flow (even in the absence of boundary-layer separation) the trailing-edge pressure is normally only slightly above the undisturbed-stream pressure.

Values of  $C_{pte}$  are implicit in the charts of Figures 2 to 5 and derive directly from the design procedure. They are presented in Figure 12.

### 6.3.4 Definition of the Mach Number Limits

Consideration of Figure 1 shows that, for any thickness distribution and  $t/c$  ratio, there is an upper limiting value of  $M_D$ . This is the Mach number at which the velocities on the *lower* surface become sufficiently supersonic to provoke drag-rise. This limit condition, associated with finite values of  $C_{LD}$ , is indicated on the charts in Figures 2 to 5 by the broken lines at the foot of each chart. Each chart contains these limit lines for combinations of the given upper-surface pressure distribution with those alternative thickness distributions for which the roof-top limit is at, or forward of, that for the given upper-surface pressure distribution. It will be noted that replacing the thickness distribution implicit in the charts by an alternative thickness distribution in the RAE series slightly restricts the maximum value of  $M_D$  attainable with any particular value of thickness/chord ratio.

<sup>†</sup> In fact the RAE series at zero incidence has a flat roof-top from the leading edge to the station  $x_R/c$  only according to linear theory for incompressible flow. According to exact theory the station  $x_R/c$  corresponds to the point of minimum pressure in incompressible flow. The effect of increasing Mach number can be such as to relocate the minimum pressure ahead of  $x_R/c$  (see, for example, Figure 4 of Reference 4).

In this Memorandum this upper limiting value of  $M_D$  has been approximated to by the Mach number at which the minimum value of  $p/H$  on the lower surface is equal to 0.515. The curve of these limiting values was established simply by plotting the minimum value of  $C_p$  on the lower surface against  $C_{LD}$  at constant  $M_D$  and interpolating or extrapolating to find the lift coefficient at which  $C_p$  equalled the required value. The loading associated with this limit will be concentrated near the leading edge and over the rear portion of the aerofoil – the magnitude of the loading at the rear depending on the concavity of the pressure distribution due to thickness.

The charts were not extended below  $M_D = 0.6$ ; this was an arbitrary choice.

### 6.3.5 Calculations

The procedure outlined in Section 6.3.1 was repeated for increments of 0.05 in  $M_D$  between 0.6 and the upper limiting value.

The upper-surface pressure distribution was generated by the process outlined in Section 6.2, the discontinuity in pressure gradient at  $x_R$  being rounded out by increasing the pressure coefficient at the Weber point nearest to  $x_R$  (with  $N = 16$ ) to 0.03 above the roof-top value.

The pressure distribution due to thickness for the RAE 101 to 104 series of aerofoils was evaluated by the methods of References 5 and 6 as modified by Powell<sup>17</sup> for use in viscous flow for values of thickness/chord ratio of 6, 10, 14 and 16 per cent.

The complete procedure was programmed in ALGOL for use on the K.D.F.9 computer. Loci in Figures 2 to 5 for values of thickness/chord ratio of 8, 12 and 18 per cent were obtained by interpolation or extrapolation.

## 6.4 ACCURACY AND LIMITATIONS OF THE CHARTS

### 6.4.1 Accuracy

The method of Reference 1 on which the charts are based was found, after extensive comparisons with experimental data, to predict the onset of wave drag to within  $\pm 0.015$  in Mach number for the majority of cases evaluated. It is relevant to recall that this method derives a locus  $M_D$ ,  $C_{LD}$  from a knowledge of the inviscid incompressible-flow pressure distribution and makes recourse to the Kármán-Tsien and Prandtl-Glauert compressibility correction factors to account for the effect of Mach number on pressure coefficient and lift coefficient respectively. Because the pressure coefficient near the crest for a given lift coefficient and Mach number is relatively insensitive to viscous effects (provided, of course, attention is confined to attached flow) the method of Reference 1 eliminated the major influence of viscosity by deriving a drag-rise boundary directly in terms of Mach number and lift coefficient. However, in determining the crest position and thus the crest pressure it was important to know the angle of incidence for a given value of  $C_L$  in incompressible flow more accurately than was given by taking the incompressible inviscid-flow lift-curve slope. A procedure utilising ESDU Aerodynamics Data Sheet Wings 01.01.05 was thus employed for estimating the lift-curve slope in incompressible viscous flow.

In the method outlined in Section 6.3.1 and described in more detail in Appendix A the compressibility factors inherent in the Lock, Wilby, Powell<sup>3</sup> method represent the effect of Mach number on the velocity distribution and, through the use of exact relationships, on  $C_p$  and  $C_L$  as well, whilst the effect of viscosity is accounted for by Powell's contribution<sup>17</sup> to the method of Reference 3. Inasmuch as both methods employ the same criterion for drag-rise in terms of local velocity – with the method of Section 6.3.1 employing a more complex description of the compressibility and viscous effects that produce this velocity – the method of Section 6.3.1 may be regarded as an extension of the method in Reference 1. Extensive comparisons

have not been made between the method of Reference 1 and the method in Section 6.3.1 but a very limited comparison suggests that the latter, at given values of  $C_{LD}$ , may yield values of  $M_D$  lower than the former by 0 to 0.01 but within the limit of accuracy for the former method.

The experimental data examined in Reference 1 that fell within the limit of  $\pm 0.015$  all corresponded to values of lift coefficient less than 0.9. It follows that the charts in Figures 2 to 5 would be expected to have the same accuracy of  $\pm 0.015$  in Mach number, in absolute terms, below  $C_{LD} = 0.9$ . However, due to the coherence introduced by the use of a particular form of upper-surface pressure distribution, it is to be expected that the accuracy with which the charts predict the “exchange rates” between the various basic parameters will be significantly better than their accuracy in absolute terms.

Above  $C_{LD} = 0.9$  the lack of relevant experimental data prevents an assessment being made of the adequacy of the empiricism in the criterion of (a) in Section 6.2 and hence of the absolute accuracy of the charts<sup>†</sup>.

## 6.4.2 Limitations

The practical possibility of attaining in viscous flow exactly the particular form of upper-surface pressure distribution depicted in Figure 1 and of maintaining attached flow over the whole of the aerofoil surface will need to be considered in developing an aerofoil corresponding to a chosen point on the charts.

The latter constitutes the principal limitation of the charts. Some guidance on the onset of rear separation on the upper surface can be gained from the use of a criterion developed by Stratford<sup>7</sup> for predicting the pressure rise that will cause separation of a turbulent boundary layer. The application of this criterion to the form of upper-surface pressure distribution depicted in Figure 1 is described in Appendix B. It is shown that the Mach number regime in which one can expect to maintain attached flow is, according to Stratford’s criterion, extremely sensitive to the value of  $C_{pte}$  to be expected in viscous flow.

Here it must be emphasised that the trailing edge pressure will depend in a complicated way on the design and operating Reynolds numbers, lift coefficient and detailed shape of the section near the trailing edge. Clearly in a practical aerofoil design it would be desirable if possible to treat  $C_{pte}$  as one of the variables in an optimising process rather than to accept a value resulting from other requirements.

Separation thresholds calculated in Appendix B for various combinations of operating Reynolds number and transition position for the range of values of  $x_R/c$  and  $t/c$  covered by the charts are presented in Table B9.1. To indicate the dependence of these thresholds on  $Re_0$  and  $(x/c)_{tro}$  a selection is shown in Figures 6 to 9 superimposed on outlines of the drag-rise charts. Particular attention is drawn to the fact that for low Reynolds numbers there are appreciable regions of the charts for which Stratford’s criterion predicts that the flow will separate upstream of the trailing edge.

Attention is also drawn to the comment in Section 3 that in deriving these thresholds the trailing-edge pressure coefficient has been assumed independent of the operating Reynolds number and transition position.

A further limitation may arise in respect of the values of  $C_{LD}$  for which the method in Section 3.1 is justifiable. Here an arbitrary upper limit of 1.6 has been placed on  $C_{LD}$  but the region above  $C_{LD} = 1.0$  must be regarded as tentative until it has been explored experimentally.

<sup>†</sup> Furthermore the method of deriving the charts (see Appendix A) ignores certain second-order terms (in particular those involving camber) which may also affect the absolute accuracy of the charts at the higher values of  $C_{LD}$ .

## 6.5 COMPARISON OF THE CHARTS WITH RESULTS FOR A STANDARD FAMILY OF AEROFOILS

To obtain the aerofoil with the drag-rise characteristic, in terms of  $M_D$  and  $C_{LD}$ , represented by any point on the charts (Figures 2 to 5) presented here involves deriving the camber line and incidence which, when combined with the chosen thickness distribution, will generate the required upper-surface pressure distribution at the design Reynolds number with the design transition position. This process has the merit of producing, directly, the design that will exploit to the full the lift that can be obtained within the constraints that must be built into the upper-surface pressure distribution (*e.g.* those ensuring that the design is free of shock-wave drag and separated flow). It is relatively simple, with computer programs available, to derive the geometry corresponding to a chosen point on the charts and such a process is rapidly superseding the more arbitrary process of combining standard thickness distributions with standard camber lines – the latter derived on the basis of their load distribution according to incompressible, thin aerofoil theory. Nevertheless, it is appropriate to describe with a few illustrative sketches how the drag-rise loci presented in the charts compare with the corresponding envelope of drag-rise curves for a family of aerofoils obtained by combining a standard thickness distribution with readily available standard camber lines<sup>13, 14</sup>. For this purpose a thickness form from the NACA 6-series<sup>14</sup> combined with varying degrees of camber, conforming to the designation<sup>†</sup>  $a = x_R/c$  constitutes a valid standard for comparison.

Drag-rise curves (representing the condition  $p/H = 0.515$  at the crest) for individual NACA aerofoils having a common thickness/chord ratio,  $t/c$ , and for a range of incidences embracing that which produces a roof-top in the upper-surface pressure distribution are sketched in Figure 14. These drag-rise loci correspond to a NACA 6-series thickness form combined with camber lines corresponding to, say, the designation<sup>‡</sup>  $C_{Li} = -0.1, 0$  (*i.e.* uncambered), 0.1 and 0.2. It is implicit in this figure that both the NACA 6-series thickness form and the camber line are those designed to produce pressure distributions with roof-tops back to  $a = x_R/c$ . The appropriate locus from the present charts with which these results should be compared is that having the same value of  $x_R/c$  and  $t/c$ . This, reproduced in Figure 14, would take the form of the curve *AB*.

The envelope formed by the family of curves for the individual NACA sections lies quite close to the locus taken from the charts. It is clear, however, that for any particular aerofoil of the NACA series there is an “optimum” incidence (or lift coefficient) below and above which the drag-rise curve departs progressively from the locus taken from the charts. This incidence is in all cases a great deal higher than the ideal<sup>†</sup> incidence for the NACA sections.

The development of the upper-surface pressure distribution for a member of the NACA 6-series corresponding to the points *Q*, *R*, *S* on the drag-rise curves of Figure 14 (so that in each case the upper surface is critical in the sense that  $p/H = 0.515$  at the crest) is sketched in Figure 15. At the point *Q* the roof-top in the upper-surface pressure distribution has a marked negative gradient ahead of the peak suction. As the incidence increases the forward part of the upper-surface pressure distribution fills out and becomes gradually flatter until between points *Q* and *R* it does in fact become of flat roof-top form. The locus corresponding to the roof-top condition for the NACA series is sketched in Figure 14 and is the line *CD*. At the point *R* a small leading-edge peak is developing (Figure 15) but the pressure distribution still corresponds very closely to the flat roof-top fundamental to the present method. The discrepancy between the point *R* and the point *R'* on the locus *AB* is due largely to differences in loading over the rear parts of the aerofoils.

<sup>†</sup> In Reference 14 (Chapter 4) the symbol  $a$  is used to denote, for mean lines (*i.e.* camber distributions) commonly associated with the NACA 6-series aerofoils, the fraction of the chord from the leading edge over which the loading in incompressible flow is uniform at the “ideal” angle of attack. The “ideal” angle of attack is simply the unique value at which, according to incompressible-flow thin-aerofoil theory, a finite velocity occurs at the leading edge (Reference 14, Chapter 4).

<sup>‡</sup> In Reference 14 (Chapter 4) the symbol  $C_{Li}$  is defined as the “ideal” lift coefficient, occurring at the “ideal” angle of attack.

Further improvement in the drag-rise curves for the NACA series of aerofoils can be achieved in principle by a further increase in incidence as typified by the point *S*. This further improvement is achieved through the development of a marked leading-edge suction peak – as shown in Figure 15 and calculated on the assumption of an equivalent shock-free flow. It must be emphasised that such leading-edge suction peaks must always occur on the NACA series of aerofoils lying between the boundaries *CD* and *AB*. Above a certain value of incidence, that may be determined from the criterion in Reference 1, these aerofoils will exhibit a supercritical drag-creep before the onset of the rapid growth in shock-wave drag.

## 6.6 DEVELOPMENT OF AN AEROFOIL WITH IMPROVED DRAG-RISE CHARACTERISTICS

It will be appreciated that an aerofoil developing a roof-top pressure distribution does not yield the optimum value of  $C_{LD}$  for a given value of  $M_D$  because additional loading can be added to the upper surface – in the forms of a leading-edge suction peak and/or further rear loading – by modifying the camber line in such a way that the crest pressure is not altered. Thus in developing an optimum aerofoil from the aerofoil implicit in a chosen point on the charts the first step would be to re-specify the upper-surface pressure distribution to create a leading-edge peak (see Figure 16a) and to develop this into one that leads to a suction loop of peaky form<sup>1</sup>, because not only will additional lift be generated but a gain in drag-rise Mach number will accrue. A formal design method for an aerofoil generating a peaky suction loop has not been developed yet, although the principles have been set out by Pearcey<sup>10</sup>, whilst Pearcey and Osborne<sup>18</sup>, using the charts herein as a “standard”, have demonstrated gains in  $M_D$  that have been achieved by aerofoils of given thicknesses; these gains being measured as displacements along lines of constant  $M_0^2 C_L$  from the “standard” for the same thicknesses.

Care must be taken in developing this peak to avoid transgressing the criterion developed in Reference 1 to identify the full triangular form of the suction loop when the aerofoil is operating at “off-design” conditions. In this respect attention is drawn to the remarks in Section 5.

By modifying the camber line and/or thickness to increase the rear loading, again in such a way that the crest pressure is not altered, the aerofoil can be further improved. The resulting upper-surface pressure distribution is illustrated in Figure 16b.

It will be noted that the additional rear loading depicted in Figure 16b decreases the upper-surface pressure gradient immediately downstream of the station  $x_R$  but leads to a significantly increased pressure gradient near the trailing edge. This increased gradient may provoke rear separation more readily and lead to regions of attached flow smaller than those achievable with a constant gradient. The general form of Stratford’s criterion<sup>7</sup> would still be applicable to this modified pressure distribution provided there was no leading-edge suction peak.

It should also be noted that the increase in rear loading depicted in Figure 16b leads to a significantly increased pressure gradient downstream of the station  $x_R$  on the *lower* surface. It may be that this pressure gradient also will impose some constraint on the magnitude and form of rear loading that is tolerable.

The above design principles clearly cannot be formulated as an itemised design schedule, but leave room for extensive research work at appropriate Establishments and development by the individual aerofoil designer.

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|-----|------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
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