

NATURAL FREQUENCIES OF BUILT-UP, FLAT, PERIODIC SKIN-STRINGER STRUCTURES

PART I: STRINGERS RIGID IN BENDING

1. NOTATION

a	width of skin plate or frame pitch	m	in
b	stringer pitch	m	in
D	flexural rigidity per unit width of skin plate, $Et^3/[12(1-\nu^2)]$	N m	lbf in
E	Young's modulus of skin material	N/m ²	lbf/in ²
E_s	Young's modulus of stringer material	N/m ²	lbf/in ²
f	frequency	Hz	c/s
G_s	shear modulus of stringer material	N/m ²	lbf/in ²
I_s	polar moment of inertia of stringer cross section about point on skin directly beneath shear centre of stringer	m ⁴	in ⁴
J_s	St Venant constant of uniform torsion for stringer cross section	m ⁴	in ⁴
K	frequency parameter	m/s	in/s
M_1	mode parameter for first group of natural frequencies		
M_2	mode parameter for second group of natural frequencies		
m	number of half waves across frame pitch		
N	number of spans		
r	mode number		
t	skin thickness	m	in
V	velocity parameter for skin material*		
Γ_s	warping constant of stringer cross section with respect to point on skin directly beneath shear centre of stringer	m ⁶	in ⁶
κ_R	non-dimensional torsional stiffness of stringer = actual rotational stiffness per unit length of stringer $\times b/D$		

*For footnote see end of Notation section.

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ρ	density of skin material	kg/m ³	†
ρ_s	density of stringer material	kg/m ³	†
ν	Poisson's ratio of skin material		

* The velocity parameter is defined in SI units as $V = (E/\rho)^{1/2}/5080$ and in British Units as $V = (E/\rho)^{1/2}/200\,000$. The value of V is approximately unity for all common structural metallic materials.

† A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before it can be used in the formula given here. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s².)

Both SI and British units are quoted but any coherent system of units may be used.

2. INTRODUCTION

This Data Item gives the natural frequencies of vibration of built-up, flat skin-stringer structures with equispaced and identical stringers as shown in Figures 1 and 2. The natural frequencies of such multi-span structures occur in groups, the number of frequencies in each group being equal to the number of spans. For a panel having a large number of spans the lowest natural frequency in any group is generally associated with a mode in which the stringers twist and the highest natural frequency in that group is associated with a mode in which the stringers tend to bend without twisting. For a panel having a small number of spans, typical mode shapes, as shown in Figure 3, are affected by the edge conditions. The mode shapes alternate between symmetric and anti-symmetric as frequency increases.

In the derivation the frame torsional stiffness has been neglected and the frame bending stiffness has been assumed to be infinite. The skin edges at the frames are thus assumed to be simply-supported. The natural frequencies are only slightly influenced by the stringer bending stiffness.

This Data Item can be used to predict the first two groups of natural frequencies of skin-stringer structures with four different end conditions. The results have been presented for varying degrees of stringer torsional stiffness, for three different aspect ratios (a/b) and for two different values of m , the number of half-waves across the frame pitch.

3. CALCULATION PROCEDURE

The natural frequencies are calculated from

$$f = VK \frac{t}{b^2},$$

where K appropriate to the mode under consideration is determined as follows.

- (i) Calculate κ_R from the equation

$$\kappa_R = \left\{ E_s \Gamma_s \left(\frac{m\pi}{a} \right)^4 + G_s J_s \left(\frac{m\pi}{a} \right)^2 - \rho_s I_s (2\pi f)^2 \right\} \frac{b}{D}.$$

Since f is unknown its value has to be estimated initially. Values of $f = 100$ Hz for the first group of frequencies and $f = 200$ Hz for the second group should give results of sufficient accuracy for commonly used skin-stringer structures. Alternatively, for the first group of natural frequencies, f may be taken as the fundamental natural frequency of any individual panel with fully-fixed edges. For the second group, f may be taken as the natural frequency of the same panel with fully-fixed edges in its second mode (*i.e.* with approximately two half waves across the stringer pitch). Both these frequencies can be calculated using Item No. 75030.

- (ii) Calculate M_1 or M_2 from the following table.

End Conditions as in	M_1 ($1 \leq r \leq N$)	M_2 ($N+1 \leq r \leq 2N$)
Figure 2a	$\frac{r-1}{N}$	$2 - \frac{r-1}{N}$
Figure 2b	$\frac{r}{N}$	$2 - \frac{r}{N}$
Figure 2c	$\frac{2r-1}{2N}$	$2 - \frac{2r-1}{2N}$
Figure 2d	See Note in Step (iii)	

- (iii) Read K from Figures 4 to 9 for appropriate values of m , aspect ratio (a/b), κ_R and M_1 or M_2 .

Note: For structures with end conditions as in Figure 2d, K is approximately equal to the arithmetic average of the values of K from Figures 2a and 2b, respectively.

4. NOTES

The effect of including the frame torsional stiffness would be to increase the natural frequencies slightly and this would be more pronounced for structures with smaller aspect ratios. The effect of including the frame bending stiffness would be to decrease the natural frequencies slightly.

For structures with a low stringer torsional stiffness, the bounding frequencies of the first group (with $m = 1$ or 2) can also be found from Item No. 75030.

Derivations 2 to 4 describe the transfer-matrix method of analysis of skin-stringer structures which can be used to calculate the natural frequencies of non-periodic structures, *i.e.* structures with non-uniform skin/stringer characteristics.

5. DERIVATION

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3. MERCER, C.A.
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5. SEN GUPTA, G. Natural flexural waves and the normal modes of periodically supported beams and plates. *J. Sound Vib.*, Vol. 13, No. 1, September 1970.
6. SEN GUPTA, G. Natural frequencies of periodic skin stringer structures using a wave approach. *J. Sound Vib.*, Vol. 16, No. 4, June 1971.
7. Acoustic fatigue design data. Part 1. AGARD-AG-162-72, 1972.

6. EXAMPLE

It is required to determine the first group of natural frequencies of a six-span skin-stringer structure having the following dimensions *etc.* with end conditions as in Figure 2d and with a single half wave across the skin width.

$$\begin{aligned}
 a &= 508 \text{ mm}, & J_s &= 94.4 \times 10^{-12} \text{ m}^4, \\
 b &= 254 \text{ mm}, & \Gamma_s &= 3.1 \times 10^{-12} \text{ m}^6, \\
 t &= 1 \text{ mm}, & I_s &= 105 \times 10^{-9} \text{ m}^4, \\
 \nu &= 0.3, & E &= E_s = 72\,300 \text{ MN/m}^2, \\
 \rho &= \rho_s = 2800 \text{ kg/m}^3, & G_s &= 27\,800 \text{ MN/m}^2.
 \end{aligned}$$

From this

$$D = \frac{Et^3}{12(1-\nu^2)} = 6.62 \text{ N m}, \quad V = \frac{(E/\rho)^{1/2}}{5080} = 1.00$$

and for the first group of natural frequencies with a single half wave across the skin width ($m = 1$ and assuming $f = 100 \text{ Hz}$),

$$\kappa_R = \left\{ E_s \Gamma_s \left(\frac{\pi}{a} \right)^4 + G_s J_s \left(\frac{\pi}{a} \right)^2 - \rho_s I_s (2\pi \times 100)^2 \right\} \frac{b}{D}$$

$$= (328 + 100 - 116) \times 0.0384$$

$$= 12.0.$$

For this group of frequencies, with $m = 1$, $a/b = 2.0$, $\kappa_R = 12.0$ and using Figure 6, the results can be calculated and presented in the form of the following table.

Mode number r	End conditions as in Figure 2a		End conditions as in Figure 2b		End conditions as in Figure 2d	$f = VK \frac{t}{b^2}$ (using K from column VI) Hz
	$M_1 = \frac{r-1}{N}$	K (from Figure 6) m/s	$M_1 = \frac{r}{N}$	K (from Figure 6) m/s	K (average of results in columns III and V) m/s	
I	II	III	IV	V	VI	VII
1	0	4270	1/6	4360	4315	66.9
2	1/6	4360	1/3	4610	4485	69.5
3	1/3	4610	1/2	4970	4790	74.2
4	1/2	4970	2/3	5370	5170	80.1
5	2/3	5370	5/6	5690	5530	85.7
6	5/6	5690	1	5830	5760	89.3

It may be noted that the frequencies obtained in column VII are less than 100 Hz and therefore the value of κ_R has been slightly under-estimated. This means the actual frequencies are slightly higher in this particular problem. Further calculations show that the frequencies calculated taking $f = 100$ Hz in the expression for κ_R are within about 3.5 per cent of those obtained taking a further iteration using the frequencies from the table.