



Principal stresses and strains in a two-dimensional stress field

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PRINCIPAL STRESSES AND STRAINS IN A TWO-DIMENSIONAL STRESS FIELD

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PRINCIPAL STRESSES AND STRAINS IN A TWO-DIMENSIONAL STRESS FIELD

1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

E	Young's modulus	N/m ²	lbf/in ²
f_x, f_y	orthogonal direct stresses; f_x is greater numerically	N/m ²	lbf/in ²
f_1, f_2	principal direct stresses	N/m ²	lbf/in ²
q_{max}	maximum shear stress	N/m ²	lbf/in ²
q_{xy}	shear stresses acting in line with f_x and f_y	N/m ²	lbf/in ²
γ_{max}	maximum shear strain		
ϵ_1, ϵ_2	principal direct strains		
θ	angle of principal stress f_1 measured from direct stress f_x	degree	degree
ν	Poisson's ratio		
Convention	Tensile stresses and strains are positive. Compressive stresses and strains are negative. Positive senses of the shear stresses and the direction of the angles are shown in each diagram.		

2. THE CURVES

Curves are given that enable the magnitudes and directions of the principal stresses in a plane (two-dimensional) stress field to be obtained from the components of the direct and shear stresses. In Tables 6.1 and 6.2 the relationships on which the curves are based are given together with simple supplementary formulae which allow the corresponding principal strains and the maximum shear stresses and strains to be calculated.

In Figure 1 f_1/q_{xy} and f_2/q_{xy} are plotted against f_x/q_{xy} for various values of f_y/f_x where the numerical value of q_{xy} is greater than the numerical values of f_x and f_y , i.e. $|q_{xy}| \geq |f_x| \geq |f_y|$. In Figure 2 values of θ plotted on the same basis are given. In Figure 3 f_1/f_x and f_2/f_x are plotted against q_{xy}/f_x for various values of f_y/f_x where the numerical value of f_x is greater than those of q_{xy} and f_y , i.e. $|f_x| \geq |q_{xy}|$ and $|f_x| \geq |f_y|$. In Figure 4 values of θ plotted on the same basis are given.

3. GENERAL NOTES

3.1 General Application of the Data

All the curves and the relationships in Table 6.1 are independent of the material or its condition. Relationships that interrelate stress and strain, Table 6.2, are valid for elastic isotropic homogeneous materials only.

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3.2 Application to Plane Stress Problems

For this condition the direct stress perpendicular to the reference plane is zero. In assuming plane stress it should be noted that if f_1 and f_2 are of the same sign the maximum shear stresses in a body *perpendicular* to the reference plane, being $f_1/2$ or $f_2/2$, can exceed q_{max} .

Expressions for the principal direct in-plane strains, ϵ_1 and ϵ_2 , are shown in Table 6.2.

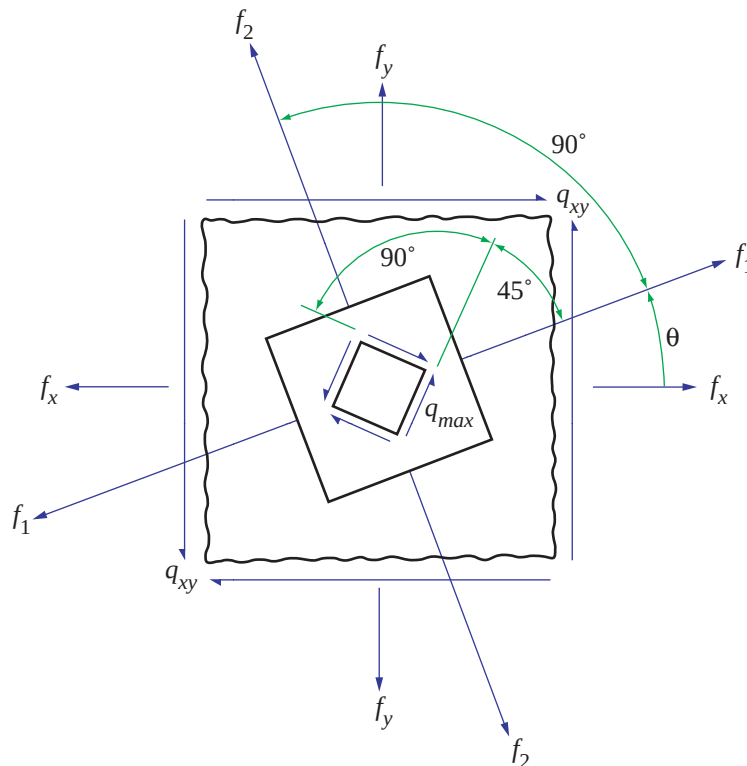
3.3 Application to Plane Strain Problems

The curves may also be used for the condition known as plane strain. In this condition it is assumed that strain perpendicular to the reference plane is prevented. The direct stress normal to the plane, necessary to maintain plane strain, has the value $\nu(f_1 + f_2)$.

Expressions for the principal direct in-plane strains, ϵ_1 and ϵ_2 , are shown in Table 6.2.

3.4 Principal Stresses and Strains

In the plane of reference the principal direct stresses, f_1 and f_2 , at a point act along two orthogonal principal planes (or directions) on which the shear stresses are zero. The maximum shear stresses, q_{max} , act on planes at 45° to the principal direct stresses as shown in the sketch below.



Sketch 3.1

The principal direct strains, ϵ_1 and ϵ_2 , act along two orthogonal principal planes (or directions) on which the shear strains are zero and the maximum shear strains, γ_{max} , act on planes at 45° to the principal direct strains.

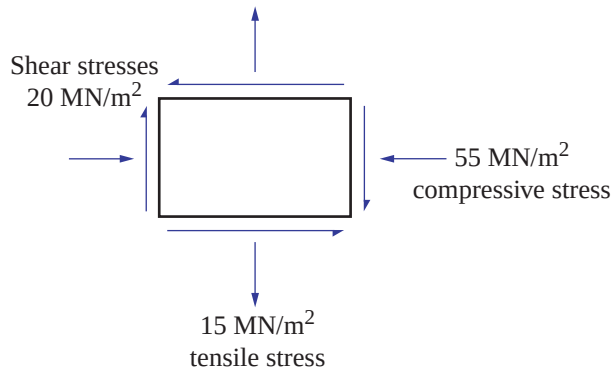
For isotropic materials the planes for principal stresses and principal strains are coincidental but for anisotropic materials this may not be so.

4. DERIVATION

1. TIMOSHENKO, S. *Theory of Elasticity*. Second edition. Chapter 2, pp. 11–27, GOODIER, J.N. McGraw-Hill, New York, 1957.

5. EXAMPLES

- 5.1 It is required to find the principal stresses and strains at the surface of a steel plate subjected to a system of direct and shear stresses as shown in the sketch below. Young's modulus for the plate material is $207\,000\text{ MN/m}^2$, $\nu = 0.29$ and the 0.2 per cent proof stress is 255 MN/m^2 .



Sketch 5.1

Since the numerical value of the shear stresses does not exceed that of the maximum direct stress, Figures 3 and 4 must be used. Using the sign convention shown on the figures and with the numerically larger stress designated f_x , the stresses are

$$f_x = -55\text{ MN/m}^2, f_y = 15\text{ MN/m}^2 \text{ and } q_{xy} = -20\text{ MN/m}^2.$$

Hence, $\frac{q_{xy}}{f_x} = 0.364$ and $\frac{f_y}{f_x} = -0.273$.

From Figure 3

$$\frac{f_1}{f_x} = 1.1 \text{ and } \frac{f_2}{f_x} = -0.37.$$

Therefore $f_1 = -60.5\text{ MN/m}^2$ (compression)

and $f_2 = 20.4\text{ MN/m}^2$ (tension).