



Determination of the principal stresses and strains in a plane stress field from strain gauge data

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DETERMINATION OF THE PRINCIPAL STRESSES AND STRAINS IN A PLANE STRESS FIELD FROM STRAIN GAUGE DATA

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DETERMINATION OF THE PRINCIPAL STRESSES AND STRAINS IN A PLANE STRESS FIELD FROM STRAIN GAUGE DATA

1. NOTATION

1.1 General Notation

E	Young's modulus	N/m ²	lbf/in ²
f_1, f_2	principal direct stresses	N/m ²	lbf/in ²
q_{max}	maximum shear stress	N/m ²	lbf/in ²
γ_{max}	maximum shear strain		
ϵ_1, ϵ_2	principal direct strains		
ν	Poisson's ratio		

1.2 Equiangular Rosettes

$\epsilon_a, \epsilon_b, \epsilon_c$	strains of gauges chosen so that numerical value of ϵ_a is greater than that of ϵ_b and numerical value of ϵ_b is greater than that of ϵ_c		
ϕ	angle of principal stress f_1 or principal strain ϵ_1 to direct strain ϵ_a	degree	degree

1.3 Rectangular Rosettes

a	angle of principal stress f_1 or principal strain ϵ_1 to direct strain ϵ_e	degree	degree
ϵ_d	strain of diagonal gauge		
ϵ_e, ϵ_f	strains of outer (orthogonal) gauges chosen so that numerical value of ϵ_e is greater than that of ϵ_f		

Issued July 1972

1.4 Mohr's Circle

f_G, f_H, f_J	direct stresses	N/m ²	lbf/in ²
β	angle between direct and principal stresses or strains	degree	degree
$\varepsilon_G, \varepsilon_H, \varepsilon_J$	direct strains		
ψ, ω	angles of direct stresses or strains	degree	degree

1.5 Convention

Tensile stresses and strains are positive. Compressive stresses and strains are negative. Direction of angles of stresses and strains are positive as shown in sketches.

Both SI and British units are quoted but any coherent system of units may be used.

2. INTRODUCTION

This Item presents curves that enable the magnitudes and directions of the principal stresses and principal strains in a plane (two-dimensional) stress field to be obtained from the readings of strain gauges arranged as equiangular or rectangular rosettes. Simple supplementary formulae allow the corresponding maximum shear stresses and strains to be calculated and provide the relationships between principal stresses and strains. In addition the Mohr's circle method is given as an alternative solution for rosettes with gauges set at any angles.

The curves and relationships in this Item are all valid for the principal elastic stresses and strains in an isotropic homogeneous material. Curves and relationships that do not interrelate stress with strain are also valid for inelastic conditions and/or anisotropic homogeneous materials. This is indicated in Tables 8.1 and 8.2.

3. THE FIGURES

Table 8.1 indicates which figure should be used with a particular rosette form to obtain the principal stresses or strains or their angle from the measured direct strains. The curves for the principal stresses have been calculated for two values of Poisson's ratio, $\nu = 0.29$ and $\nu = 0.33$, which are representative of steels and aluminium alloys respectively. The principal stresses for materials having other values of ν may be obtained by using the general relationships given in Table 8.2.

4. SUPPLEMENTARY RELATIONSHIPS, TABLE 8.2

Relationships between the principal direct stresses or strains and the strains obtained from equiangular and rectangular rosettes are given in Table 8.2. These relationships, on which the curves are based, refer to the actual strain of the materials and it is assumed that practical factors to be considered in interpreting the readings obtained from particular types of gauges (*e.g.* cross-sensitivity effects in certain bonded wire gauges) have been taken into account. Also given in Table 8.2 are the general relationships between the principal direct stresses, the maximum shear stresses and their corresponding strains.

5. METHOD FOR STRAIN ROSETTES OF OTHER CONFIGURATIONS

Figures 13a to 13c and the accompanying notes describe a graphical method (Mohr's circle). This method gives the principal strains, ϵ_1 and ϵ_2 , the shear strains, γ_{max} , and the angle β for a rosette giving 3 direct strains ϵ_G , ϵ_H and ϵ_J set at any angles.

The principal elastic stresses, f_1 and f_2 , and the maximum shear stress, q_{max} , for isotropic materials may be found by using the values thus obtained for ϵ_1 , ϵ_2 and γ_{max} , together with the required value of σ in Equations (8.9), (8.10) and (8.12) of Table 8.2.

This method may also be used to obtain f_1 , f_2 and q_{max} directly if the three direct stresses f_G , f_H and f_J are available instead of the strains. It should be noted that while the same method may be applied to stress and strain separately, use of the same diagram for both will involve difficulties of scale.

6. DERIVATION

1. TIMOSHENKO, S. *Theory of elasticity*. Second edition. Chapter 2, pp.11–27, GOODIER, J.N. McGraw-Hill, New York, 1957.

7. EXAMPLE

It is required to find the principal stresses on the surface of a steel plate under plane stress from the readings provided by a rectangular strain gauge rosette, where the strain from the diagonal gauge is -165×10^{-6} and the strains from the two orthogonal gauges are 190×10^{-6} and 550×10^{-6} . Young's modulus for the plate material is 207 000 MN/m² and $\nu = 0.29$.

The numerically larger orthogonal strain will be designated ϵ_e in accordance with the notation of the Item. Hence $\epsilon_e = 550 \times 10^{-6}$, $\epsilon_f = 190 \times 10^{-6}$ and $\epsilon_d = -165 \times 10^{-6}$. Now

$$\frac{\epsilon_d}{\epsilon_e} = -0.300 \quad \text{and} \quad \frac{\epsilon_f}{\epsilon_e} = 0.345.$$

From Table 8.1 it can be seen that for a rectangular rosette, when $|\epsilon_e| > |\epsilon_d|$ and $\nu = 0.29$, Figures 5 and 7 should be used to obtain the principal stresses and their orientation.

From Figure 7

$$\frac{f_1}{E\epsilon_e} = 1.74 \quad \text{and} \quad \frac{f_2}{E\epsilon_e} = 0.15$$

so that $f_1 = 198 \text{ MN/m}^2$ and $f_2 = 17.1 \text{ MN/m}^2$.

From Figure 7

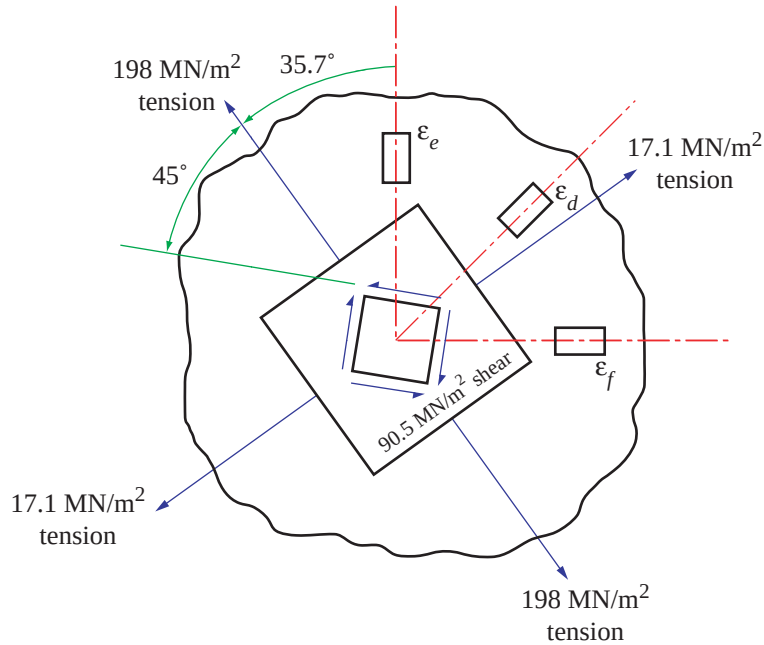
$$\alpha = 35.7^\circ.$$

This is the angle of f_1 from ϵ_e ; f_2 is displaced 90° from this.

From Table 8.2 Equation (8.11)

$$q_{max} = 1/2(f_1 - f_2) = 90.5 \text{ MN/m}^2.$$

The angles between the maximum shear stresses, q_{max} , and the principal direct stresses, f_1 and f_2 , are 45° as shown below.



Sketch 7.1

8. TABLES

TABLE 8.1
All the Figures are Valid for Elastic Materials;
the Validity for Inelastic or Anisotropic Materials is Indicated

Rosette type	Required values	Material ν	Figure No	Validity for inelastic and/or anisotropic materials
Equiangular rosettes	Principal stresses (steels)	0.29	1	NO
	Principal stresses (aluminium alloy)	0.33	2	NO
	Orientation	ANY	3	YES
	Principal strains	ANY	4	YES
Rectangular rosettes	<i>Diagonal strain not greatest</i>			
	Principal stresses (steels)	0.29	5	NO
	Principal stresses (aluminium alloy)	0.33	6	NO
	Orientation	ANY	7	YES
	Principal strains	ANY	8	YES
	<i>Diagonal strain greatest</i>			
	Principal stresses (steels)	0.29	9	NO
	Principal stresses (aluminium alloy)	0.33	10	NO
	Orientation	ANY	11	YES
	Principal strains	ANY	12	YES
Other forms	Mohr's circle for principal strains and orientation	ANY	13c	YES