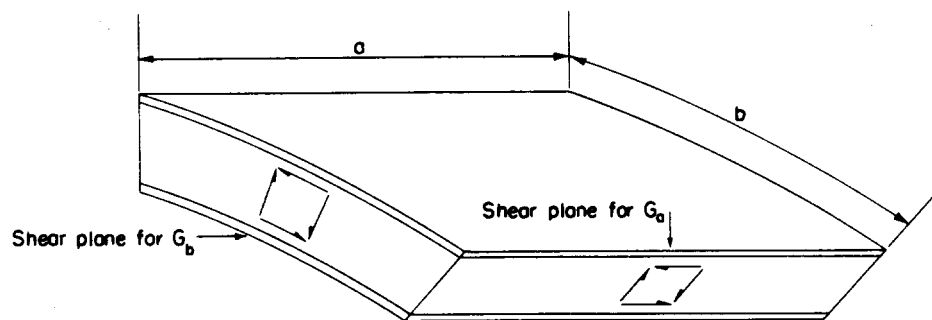


STRESS RESPONSE OF FLAT OR SINGLY-CURVED SANDWICH PANELS WITH CORES OF ZERO FLEXURAL STIFFNESS SUBJECTED TO RANDOM ACOUSTIC LOADING

1. NOTATION

a	longer dimension of flat panel, or length of curved panel straight edge	m	in
b	shorter dimension of flat panel, or arc length of curved panel	m	in
E	Young's modulus of face-plate material	N/m ²	lbf/in ²
F_1, F_2	non-dimensional parameters		
f	fundamental natural frequency of panel	Hz	Hz
G_a, G_b	transverse shear moduli of core in planes defined in sketch below	N/m ²	lbf/in ²



Sketch 1.1

$G_p(f)$	spectral density of acoustic pressure at frequency f	(N/m ²) ² /Hz	(lbf/in ²) ² /Hz
h	core thickness	m	in
$\left. \begin{array}{l} K_{1a}, K_{2a} \\ K_{1b}, K_{2b} \\ K_{1s}, K_{2s} \\ K_3 \end{array} \right\}$	non-dimensional parameters		
K_δ	damping ratio correction factor		
$L_{ps}(f)$	spectrum level of acoustic pressure at frequency f	dB*	dB*
p_{rms}	rms fluctuating pressure	N/m ²	lbf/in ²
R	radius of curvature of panel	m	in

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S_{rms}	rms stress at surface at centre of panel due to acoustic loading	N/m^2	lbf/in^2
t	face-plate thickness	m	in
δ	damping ratio in the fundamental mode		
ν	Poisson's ratio of face-plate material		
ρ_c	core density	kg/m^3	†
ρ_f	face-plate density	kg/m^3	†

* The reference pressure for sound pressure level is $20 \mu N/m^2$ (0.0002 dyn/cm^2).

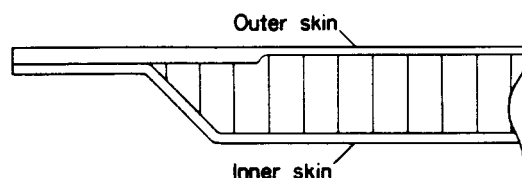
† A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before it can be used to calculate $(\rho_c h + 2\rho_f t)$. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2 .)

2. NOTES

This Data Item gives an approximate method of estimating the rms stress in the face plates of a sandwich panel subjected to random acoustic loading. The panels considered are initially unstressed, flat or singly-curved panels with identical face plates. The face-plate material is assumed to be isotropic. The data apply particularly to honeycomb sandwich panels but can be used for other sandwich panels provided that the core properties are such as to comply with the assumptions stated later in these notes.

The reference position for face-plate rms stress is the centre position of the panel. Stresses at other positions may be found by applying a suitable factor to the reference stress. When estimating honeycomb panel edge stresses it is necessary to consider the panel edge detail design. In obtaining a first estimate of panel edge stresses, for panels having reduced edge thickness of the type shown in Sketch 2.1, the panel surface stresses adjacent to the panel edge may be assumed to be of the same order as the corresponding stresses at the panel centre. For panel edge design, the edge surface stresses should first be estimated; then, either from test or from past experience, the design should be chosen to suit the estimated stress.

In this Data Item no allowance is made for the stress relief in the outside skin of a sandwich panel having crushed, or cut away, core such that the whole of the outer skin lies in the same plane. The stress relief is due to the supports restraining the panel skin in-plane extensions. Some guidance in assessing the effect of reducing the panel edge thickness on the calculated stress may be found in References 5 and 6. In these References, ratios of inner and outer face-plate stresses are given for a panel mounted on rigid supports. In practical structures the compliance of the supports will reduce the stress ratios.



Sketch 2.1 Typical reduced thickness panel edge

It is assumed that only the fundamental mode is excited and that the panels are of shallow curvature and simply-supported. The shallow curvature condition is satisfied when b/R is much less than 1.0. In the fundamental mode the panel is assumed to vibrate with one half wave in each of the principal directions. Experimental evidence shows that the assumption of unimodal response gives good results for stresses in flat, square honeycomb panels. As the curvature and panel aspect ratio (a/b) are increased it is likely that the effect of higher order modes will be increased. The range of panel aspect ratios over which the simple-response theory has been validated is given in Section 4. The assumption that panels have simply-supported edges is justified for conventional sandwich panel construction where the core material is crushed, or cut away, to bring the face plates together at the panel edge: tests show that for panels of this type the fundamental natural frequency is close to the simply-supported panel value.

This Data Item may be used for panels having fixed edges if an effective value is used for panel dimension a . As a first approximation it is recommended that the effective length should be the length of the simply-supported panel having the same fundamental natural frequency as the fixed-edge panel. It should be noted that experimental data are not available for comparison with calculated fixed-edge panel rms stresses so stresses calculated in this way should be treated with caution.

The core is assumed to have zero flexural stiffness and to be isotropic, although a moderate degree of orthotropy, *i.e.* $1/3 < G_a/G_b < 3$, can be taken into account. It is assumed that the orthotropic core carries no in-plane loading, but carries all the transverse shear. The shear moduli of the core can be obtained from manufacturers' data, test or, in the case of honeycomb cores, by calculation. A suitable test method is described in Item No. 66025 and a calculation for hexagonal honeycomb cores is given in Item No. 67022.

In producing this Data Item the values of ν and δ have been assumed to be 0.3 and 0.019 respectively. This value of the damping ratio in the fundamental mode is typical for bonded aluminium alloy honeycomb panels. However, rms stresses at different value of damping ratio may be found using the correction factor K_δ which is plotted against δ in Figure 14.

In Item No. 86024* a computer program is provided for the estimation of the rms surface strains in laminated face plates of sandwich panels subject to random acoustic loading. That Item may also be used to estimate the response of sandwich panels having isotropic face plates. The theory on which the computer program is based is the same as that used in this Item. However, the simplifying assumptions needed to provide the graphical solution for this Item are not included in the computer program. The graphical procedure in this Item gives the same results as the computer program for flat panels having isotropic face plates and $G_a = G_b$. As the curvature of the panel increases, the responses estimated by the graphical and computer procedures diverge but for shallow curvature the differences are not great. For sandwich panels having isotropic face plates, Appendix A provides simple relationships to allow the surface strains, predicted using the computer program in Item No. 86024, to be converted into surface stresses.

3. CALCULATION PROCEDURE

3.1 The procedure of estimating S_{rms} in a general case is as follows.

- (i) Estimate the fundamental natural frequency of the panel using Item No. 72016.
- (ii) Obtain the value of spectrum level of acoustic pressure $L_{ps}(f)$ at the calculated frequency. If only the band pressure level is known, it is first corrected to pressure spectrum level (unit bandwidth) using Item No. 66016.

* Estimation of rms strain in the laminated face plates of simply-supported sandwich panels subjected to random acoustic loading. Including a simplified natural frequency prediction method.

- (iii) Evaluate a/b and, from Figure 1, read the value of K_1 appropriate to the direction in which the rms stress is required (see Derivation).
- (iv) Evaluate G_b/G_a and $G_a a^2 h / \{Et(t+h)^2\}$ and from Figures 2-6 or Figures 7-11, depending on the direction in which the stress is required (see Derivation), read the appropriate value of K_2 . The Figure used within these two groups depends on the value of G_b/G_a .
- (v) Evaluate a/R and $[(h/2) + t]/a$ and calculate K_3 from:

$$K_3 = \left| \frac{a}{R} K_{1s} \mp \left(\frac{h}{2} + t \right) \frac{K_{2s}}{a} \right|.$$

The sign chosen for the product including K_{2s} is dependent on the panel surface for which the rms stress is required (see Derivation, Section 5).

- (vi) Evaluate the parameter $E/\{a(\rho_c h + 2\rho_f t)\}$ and from the nomograph (Figure 12 or 13) obtain the value of S_{rms} . The nomographs are entered at a value of $L_{ps}(f)$, each quadrant being used in turn in the direction indicated through ranges of $E/\{a(\rho_c h + 2\rho_f t)\}$, K_3 and f . Figure 13 is an extension of the range of $L_{ps}(f)$ given in Figure 12.
- (vii) For values of δ other than 0.019, factor the estimated value of S_{rms} by K_δ obtained from Figure 14. The value of $\delta = 0.019$ is typical for bonded aluminium honeycomb panels.

3.2 Note on the Units of Spectral Density of Acoustic Pressure

Within the nomograph the spectrum sound pressure level is converted into the spectral density of acoustic pressure. The spectrum sound pressure level is converted into the root mean square fluctuating pressure in units of $(\text{N/m}^2)/\text{Hz}$ (see expression below or Item No. 66018) and then squared giving a value in units of $(\text{N/m}^2)^2/\text{Hz}^2$. Since unit bandwidth is used this is numerically equal to the spectral density of acoustic pressure $G_p(f)$ in units of $(\text{N/m}^2)^2/\text{Hz}$.

$$L_{ps}(f) = 20(\log_{10} p_{rms} + 4.70).$$

If $L_{ps}(f)$ is required in British units of $(\text{lbf/in}^2)^2/\text{Hz}$ it is given by

$$L_{ps}(f) = 20(\log_{10} p_{rms} + 8.54).$$

4. COMPARISON WITH MEASURED DATA

Figures 15 and 16 show a comparison of estimated and measured face-plate rms stress at the centre of sandwich panels. In estimating stresses for Figure 15 the fundamental natural frequency calculated using Item No. 72016 was used, and the value of damping ratio was taken to be 0.019. For estimated stresses in Figure 16 measured values of fundamental natural frequency and corresponding damping ratio were used.

All data used in these comparisons are for flat panels having crushed or cutaway cores at the edge attachment positions. No allowance has been made for stress relief due to the rigidity of the supports. In all cases the highest measured panel centre stress is compared with the equivalent highest calculated principal stress. Data presented in Figures 15 and 16 are for panels having a/b ratios in the range 0.6 to 2.0.

In Derivation 1, surface strains are given on the concave surfaces of four curved sandwich panels subjected

to jet noise. The estimated rms strains for those panels were much lower than measured values. However, for these panels the response was multimodal so the discrepancy with the simple unimodal response theory is to be expected. Until additional experimental data are available for curved panels, the range of curvature for which the simple prediction theory is valid cannot be checked.

5. DERIVATION AND REFERENCES

The expression for rms stress is

$$S_{rms} = \frac{0.29K_3EK_8}{a(\rho_ch + 2\rho_ft)f^{3/2}} \{G_p(f)\}^{1/2},$$

where
$$K_3 = \left| \frac{a}{R}K_{1s} \mp \left(\frac{h}{2} + t\right) \frac{K_{2s}}{a} \right|.$$

(i) For surface stresses parallel to the panel side of length a , $K_{1s} = K_{1a}$ and $K_{2s} = K_{2a}$ where

$$K_{1a} = -\frac{2}{F_1}(1-\nu) \left\{ \left(\frac{a}{b}\right)^2 - \nu \right\} + \nu - \nu \left(\frac{a}{b}\right) \frac{2}{F_1} \left\{ \nu(1+\nu) - 2 - \left(\frac{a}{b}\right)^2(1-\nu) \right\}$$

and
$$K_{2a} = \frac{2\pi^3}{F_2} \left\{ 2\pi \left(\frac{G_b}{G_a}\right) \left[1 + \left(\frac{a}{b}\right)^2 \nu \right] + \frac{\pi^3 Et(t+h)^2}{2G_a a^2 h(1-\nu^2)} \times \right. \\ \left. \left\{ 2\left(\frac{a}{b}\right)^2 \left[1 + \nu \left(\frac{G_b}{G_a}\right) \right] + (1-\nu) \left[1 + \left(\frac{a}{b}\right)^4 \left(\frac{G_b}{G_a}\right) \nu \right] - \left(\frac{a}{b}\right)^2 (1+\nu) \left[\left(\frac{G_b}{G_a}\right) + \nu \right] \right\} \right\}.$$

(ii) For surface stresses parallel to the panel side of length b , $K_{1s} = K_{1b}$ and $K_{2s} = K_{2b}$ where

$$K_{1b} = -\frac{2\nu}{F_1}(1-\nu) \left\{ \left(\frac{a}{b}\right)^2 - \nu \right\} + 1 - \frac{2}{F_1} \left(\frac{a}{b}\right) \left\{ \nu(1+\nu) - 2 - \left(\frac{a}{b}\right)^2(1-\nu) \right\}$$

and
$$K_{2b} = \frac{2\pi^3}{F_2} \left\{ 2\pi \left(\frac{G_b}{G_a}\right) \left[\nu + \left(\frac{a}{b}\right)^2 \right] + \frac{\pi^3 Et(t+h)^2}{2G_a a^2 h(1-\nu^2)} \times \right. \\ \left. \left\{ 2\left(\frac{a}{b}\right)^2 \left[\nu + \left(\frac{G_b}{G_a}\right) \right] + (1-\nu) \left[\nu + \left(\frac{a}{b}\right)^4 \left(\frac{G_b}{G_a}\right) \right] - \left(\frac{a}{b}\right)^2 (1+\nu) \left[1 + \nu \left(\frac{G_b}{G_a}\right) \right] \right\} \right\}.$$

where
$$F_1 = \left(\frac{a}{b}\right)^2 (1 + \nu)^2 - \left[2\left(\frac{a}{b}\right)^2 + 1 - \nu\right] \left[2 + \left(\frac{a}{b}\right)^2 (1 - \nu)\right]$$

and
$$F_2 = 4\pi^2 \left(\frac{G_b}{G_a}\right) + \frac{\pi^4 E t (t + h)^2}{G_a a^2 h (1 - \nu^2)} \left\{ 2\left(\frac{a}{b}\right)^2 + 2\left(\frac{G_b}{G_a}\right) + (1 - \nu) \left[1 + \left(\frac{a}{b}\right)^2 \left(\frac{G_b}{G_a}\right) \right] \right\} - F_1 \left\{ \frac{\pi^3 E t (t + h)^2}{2 G_a a^2 h (1 - \nu^2)} \right\}^2.$$

In the equation for K_3 the plus sign, in front of the final product, is used for surface stress on the convex side of the panel and the minus sign for the surface stress on the concave side of the panel.

1. BALLENTINE, J.R. *et al.* Sonic fatigue in combined environment. Air Force Flight Dynamics Lab., tech. Rep. AFFDL-TR-66-7, May 1966.
2. BALLENTINE, J.R. *et al.* Refinement of sonic fatigue structural design criteria. Air Force Flight Dynamics Lab., tech. Rep. AFFDL-TR-67-156, November 1967.
3. JACOBSON, M.J. Stress and deflection of honeycomb panels loaded by spatially uniform white noise. *AIAA J.*, Vol. 6, No. 8, pp. 1503-1510, November 1968.
4. THOMSON, A.G.R. LAMBERT, R.F. Acoustic fatigue design data, Part 2. AGARD-AG-162-Part 2, 1972.

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5. SWEERS, J.R. Prediction of response and fatigue life of honeycomb sandwich panels subjected to acoustic excitation. *Acoustic fatigue in aerospace structures*. Syracuse University Press, 1965.
6. SOOVERE, J. Random vibration analysis of stiffened honeycomb panels with bevelled edges. *J. Aircraft*, Vol. 23, No. 6, pp. 537-544, June 1986.

6. EXAMPLE

It is required to estimate the rms stress on the face plates at the centre of a simply-supported honeycomb sandwich panel subjected to jet noise. The variation of sound pressure level over a range of frequencies is given in the table, sound pressure level being 1/3 octave band levels.

Sound pressure level dB	152	155	154	150
Frequency Hz	200	500	1000	2000

The panel has the following dimensions and material properties:

$$\begin{aligned}
 a &= 450 \text{ mm}, & b &= 300 \text{ mm}, & R &= 1600 \text{ mm}, \\
 h &= 10 \text{ mm}, & t &= 0.5 \text{ mm}, & \nu &= 0.3, \\
 E &= 70\,400 \text{ MN/m}^2, & G_a &= 141 \text{ MN/m}^2, & G_b &= 212 \text{ MN/m}^2, \\
 \rho_f &= 2660 \text{ kg/m}^3, & \rho_c &= 64 \text{ kg/m}^3.
 \end{aligned}$$

From Item No. 72016 the fundamental natural frequency of the panel is 662 Hz and the damping ratio for that mode of vibration is known to be 0.03.

By interpolation from the table the 1/3 octave band sound pressure level at 662 Hz is 154.9 dB.

From Item No. 66016, $L_{ps}(f) = 154.9 - 21.7 = 133 \text{ dB}$.

With $a/b = 1.5$, from Figure 1, $K_{1a} = 0.290$ and $K_{1b} = 0.408$.

Now $G_b/G_a = 1.5$

$$\text{and } \frac{G_a a^2 h}{Et(t+h)^2} = \frac{141 \times 10^6 \times 0.45^2 \times 0.01}{70\,400 \times 10^6 \times 0.0005(0.0005 + 0.01)^2} = 72.6;$$

therefore from Figure 5, $K_{2a} = 13.5$ and from Figure 10, $K_{2b} = 21.2$.

$$\text{Also } \frac{a}{R} = \frac{0.45}{1.60} = 0.281 \text{ and } \frac{\left(\frac{h}{2} + t\right)}{a} = \frac{\frac{0.01}{2} + 0.0005}{0.45} = 0.0122.$$

From the equations in the Derivation, Section 5, the values of K_3 required to determine the rms stress parallel to side a are given by

(i) on the concave face-plate surface

$$K_3 = \left| \frac{a}{R} K_{1a} - \left(\frac{h}{2} + t\right) \frac{K_{2a}}{a} \right| = |0.281 \times 0.290 - 0.0122 \times 13.5| = 0.083.$$

(ii) on the convex face-plate surface

$$K_3 = \left| \frac{a}{R} K_{1a} + \left(\frac{h}{2} + t \right) \frac{K_{2a}}{a} \right| = |0.281 \times 0.290 + 0.0122 \times 13.5| = 0.246.$$

Similarly the values of K_3 required to determine the rms stress parallel to side b are given by

(i) on the concave face-plate surface

$$K_3 = \left| \frac{a}{R} K_{1b} - \left(\frac{h}{2} + t \right) \frac{K_{2b}}{a} \right| = |0.281 \times 0.408 - 0.0122 \times 21.1| = 0.144.$$

(ii) on the convex face-plate surface

$$K_3 = \left| \frac{a}{R} K_{1b} + \left(\frac{h}{2} + t \right) \frac{K_{2b}}{a} \right| = |0.281 \times 0.408 + 0.0122 \times 21.2| = 0.373.$$

Also
$$\frac{E}{a(\rho_c h + 2\rho_f t)} = \frac{70\,400 \times 10^6}{0.450 \times (64 \times 0.01 + 2 \times 2660 \times 0.0005)} = 47.4 \times 10^9 \text{ s}^{-2}.$$

From Figure 13, entering the nomograph at 133 dB, the stresses in the face-plate are as shown in the table.

	S_{rms} parallel to side a (MN/m ²)	S_{rms} parallel to side b (MN/m ²)
<i>convex surface</i>	17.9	27.1
<i>concave surface</i>	6.0	10.5

Therefore the resultant stress on the panel convex surface ($\delta = 0.019$) is

$$\sqrt{17.9^2 + 27.1^2} = 32.5 \text{ MN/m}^2,$$

and the resultant stress on the panel concave surface ($\delta = 0.019$) is

$$\sqrt{6.0^2 + 10.5^2} = 12.1 \text{ MN/m}^2.$$

From Figure 14, for $\delta = 0.03$, $K_\delta = 0.796$.

Hence the rms stress at the centre of the panel on the convex surface is $32.5 \times 0.796 = 25.9 \text{ MN/m}^2$ and on the concave surface is $12.1 \times 0.796 = 9.6 \text{ MN/m}^2$.