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Buckling of flat isotropic plates under uniaxial and biaxial loading

Associated software: ESDUpac A8147

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes Data Items Nos Struct 02.01.01, Struct 02.01.19 to 21 and Struct 02.01.38 to 45, was monitored and guided by the Aerospace Structures Committee, which first met in 1940 and now has the following membership:

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BUCKLING OF FLAT ISOTROPIC PLATES UNDER UNIAXIAL AND BIAXIAL LOADING

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BUCKLING OF FLAT ISOTROPIC PLATES UNDER UNIAXIAL AND BIAXIAL LOADING

1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

A_r	cross-sectional area of transverse member	m^2	in^2
a	plate length in x -direction	m	in
b	plate width in y -direction	m	in
E	Young's modulus in compression of plate material	N/m^2	lbf/in^2
f_n	stress at which $E_t = \frac{1}{2}E$, see Item No. 76016	N/m^2	lbf/in^2
f_x	stress in x -direction at which buckling occurs allowing for plasticity	N/m^2	lbf/in^2
f_{xe}, f_{ye}	combination of compressive stresses in x - and y -directions at which plate first buckles assuming linear elasticity	N/m^2	lbf/in^2
f_0	elastic buckling stress of a long simply-supported plate under uniaxial compression defined by $f_0 = \frac{\pi^2 E}{3(1 - \nu^2)} \left(\frac{t}{b}\right)^2$	N/m^2	lbf/in^2
M	elastic stiffness per unit length against rotation of edges parallel to x -axis	$(\text{N m/rad})/\text{m}$	$(\text{lbf in/rad})/\text{in}$
m	material characteristic, see Item No. 76016		
N	elastic stiffness per unit length against lateral movement of edges parallel to x -axis	$(\text{N/m})/\text{m}$	$(\text{lbf/in})/\text{in}$
s	number of half waves into which plate buckles along length		
t	plate thickness	m	in
η	plasticity reduction factor defined by $f_x = \eta f_{xe}$		
μ_L	elastic edge lateral restraint coefficient defined by $\mu_L = \frac{Nb}{Et}$		
μ_R	elastic edge rotational restraint coefficient defined by $\mu_R = \frac{12M(1 - \nu^2)b}{Et^3}$		
ν	Poisson's ratio for plate material		

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2. NOTES

This Item gives the elastic buckling stresses of flat isotropic plates under combinations of in-plane direct loadings. Data are given in non-dimensional form for several combinations of the basic edge conditions, namely clamped, simply-supported and free, and for longitudinally loaded plates with elastic restraint on the unloaded edges. Data are also provided to correct the buckling stress for the effects of plasticity.

Figures 1 and 2 provide data on the elastic buckling stresses of plates subjected to uniaxial loading when the unloaded sides have either lateral or rotational restraint, represented by the restraint coefficients μ_L and μ_R , respectively, acting along them. Figure 3 relates to long plates (a/b greater than about 3) having any combination of these restraints. The combination $\mu_L = 0$, $\mu_R = 0$ corresponds to simply-supported edges, whilst $\mu_L = 0$, $\mu_R = \infty$ corresponds to edges clamped against rotation. When a plate is elastically restrained in a lateral direction, the application of a longitudinal load induces loads in the direction of restraint. This effect is allowed for in Figures 1 and 3.

Figures 4 to 9 give elastic buckling data for plates with their shorter edges clamped or simply-supported and their longer edges clamped, free or simply-supported. Figures 4 and 5 are for uniaxial loading and Figures 6 to 9 for biaxial loading.

Table 2.1 explains the abbreviations used for the edge restraints and Table 2.2 summarises the edge and loading conditions treated and will enable the user to locate the required figure readily.

The plate buckles into s half waves in the x -direction and the value of s is indicated on the figures. The plate buckles into one half-wave across its width except for some biaxial loading cases where $a/b < 1$. In these cases it is advisable to interchange a and b to ascertain the number of transverse buckles.

Lateral restraint occurs when transverse members of cross section A_r are attached to the ends, of width b , of the plate. The restraint provided by such members is transmitted to the plate along the sides of length a and, when averaged over the plate length, it can be represented by $N = EA_r/ab$, where the value of E is appropriate to the transverse members.

For a uniaxially loaded plate, the plasticity reduction factor, η , may be obtained from Figure 10 and applied directly to the elastic buckling stress f_{xe} . The following approach is recommended when the plate is subjected to biaxial loading.

It is first necessary to determine from one of Figures 6 to 9 the stress f_{xe} necessary to buckle the plate for a given value of f_{ye} . Figure 10 must then be read, in the following three stages, using a value of stress ratio f_{ye}/f_{xe} calculated from the buckling curve results.

- Construct a vertical line at the value of f_{xe}/f_n to intersect the line $f_{ye}/f_{xe} = 0$.
- Traverse horizontally until the required stress ratio is reached.
- Construct a second vertical line from this point to intersect the curve of the required value of m and read the plasticity reduction factor, η , directly.

Provided Figure 10 is used without extrapolation of the f_{ye}/f_{xe} construction, the errors in the plasticity reduction factor for biaxially loaded plates are not expected to exceed 5 per cent. These errors are conservative for negative ratios but non-conservative for positive ratios. Also for a positive ratio the error is likely to be reduced below the value for a similar negative ratio.

For the free edge cases of Figure 5, a value of ν of 0.3 has been assumed. Values of ν different from this are unlikely to result in errors in f_{xe} of greater than 5 per cent.

The elastic buckling loads provided by this Data Item may be computed using the program of ESDUpac A8147. The program is available in the form of a Fortran executable file. The executable file, along with example input and output files, can be found in ESDUscope (the CD version of the Series), on the internet and intranet versions of the Series, and on the CD that accompanies the paper version of the Series Software Volume. The program deals with any combination of in-plane loads acting on panels with symmetrical combinations of clamped and simply-supported edges. Up to 50 loading cases can be examined in a single run. ESDUpac A8147 is associated with Data Item No. 81047 "Buckling of flat rectangular plates (isotropic, orthotropic and laminated composite plates and sandwich panels)".

TABLE 2.1

Edge Restraints	Abbreviations
Simply-supported	S S
Clamped	C
Free	F
Elastic restraint in lateral direction (μ_L)	E R L
Elastic restraint in rotation (μ_R)	E R R
Elastic restraint in lateral direction and in rotation (μ_L and μ_R)	E R L R

TABLE 2.2

Applied Loading	Edge Restraint Conditions		Side Restraint Coefficients		Figure Number
	Ends (b)	Sides (a)	μ_L	μ_R	
Uniaxial	S S	E R L	0 to ∞	0	1
	S S	E R L R	0 to ∞	∞	1
	S S	S S	0	0	2
	S S	E R R	0	0 to ∞	2
	S S	C	0	∞	2
	S S	E R L R	0 to ∞	0 to ∞	3
	C	S S	—	—	4
	C	C	—	—	4
	S S	F, S S	—	—	5
	S S	F, C	—	—	5
Biaxial	S S	S S	—	—	6
	S S	C	—	—	7
	C	S S	—	—	8
	C	C	—	—	9

3. DERIVATION

This section lists selected sources that have assisted in the preparation of this Item.

1. COX, H.L. Buckling of a flat rectangular plate under axial compression and its behaviour after buckling. ARC R & M 2041, 1945.
2. LIBOVE, C.
STEIN, M. Charts for critical combinations of longitudinal and transverse direct stress for flat rectangular plates. NACA ARR L6A05, 1946.
3. STOWELL, E.Z. A unified theory of plastic buckling of columns and plates. NACA Rep. 898, 1948.
4. TIMOSHENKO, S. *Theory of elastic stability*. Second edition, McGraw-Hill, New York, 1961.
5. BULSON, P.S. *The stability of flat plates*. Chatto and Windus, London, 1970.

4. EXAMPLES

4.1 Elastic Restraint Against Rotation – Longitudinal Loading

It is required to find the elastic buckling stress of a $300 \times 200 \times 5$ mm plate, elastically restrained against rotation but free in translation along its longer edges and simply-supported on its shorter edges. The true restraint lies between the two limit cases of $M = 10\,000$ (N m/rad)/m and $M = 18\,000$ (N m/rad)/m and the material properties are $E = 69\,000$ MN/m² and $\nu = 0.33$.

$$\text{Case (i)} \quad \mu_R = \frac{12 \times 10\,000 \times (1 - 0.33^2) \times 0.2}{69\,000 \times 10^6 \times 5^3 \times 10^{-9}} = 2.48$$

$$\text{and case (ii)} \quad \mu_R = \frac{12 \times 18\,000 \times (1 - 0.33^2) \times 0.2}{69\,000 \times 10^6 \times 5^3 \times 10^{-9}} = 4.46.$$

Thus, for $a/b = 1.5$, interpolation from Figure 2 gives,

for case (i),

$$f_{xe}/f_0 = 1.22$$

and, for case (ii),

$$f_{xe}/f_0 = 1.28.$$

$$\text{Now,} \quad f_0 = \frac{\pi^2 \times 69\,000}{3(1 - 0.33^2)} \times \left(\frac{5}{200}\right)^2 = 159 \text{ MN/m}^2$$