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Aerodynamic characteristics of aerofoils in compressible inviscid airflow at subcritical Mach numbers

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AERODYNAMIC CHARACTERISTICS OF AEROFOILS IN COMPRESSIBLE
INVISCID AIRFLOW AT SUBCRITICAL MACH NUMBERS

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AERODYNAMIC CHARACTERISTICS OF AEROFOILS IN COMPRESSIBLE INVISCID AIRFLOW AT SUBCRITICAL MACH NUMBERS

1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
a_1	lift-curve slope at zero lift in compressible inviscid flow	deg^{-1}	deg^{-1}
C_L	lift coefficient in compressible inviscid flow		
C_{m0}	zero-lift pitching moment coefficient in compressible inviscid flow, based on aerofoil chord length		
c	aerofoil chord (see Section 2.1)	m	ft
F_1	ordinate in Figure 1		
F_2	ordinate in Figure 2	deg	deg
F_3	ordinate in Figure 4		
F_4	ordinate in Figure 5		
F_5	ordinate in Figure 7		
h	$h +$ for $h+ > h- $, $h -$ for $h+ < h- $	m	ft
$h+$	maximum positive camber line ordinate	m	ft
$h-$	maximum negative camber line ordinate	m	ft
M	free-stream Mach number		
t	aerofoil maximum thickness	m	ft
x	chordwise distance measured from leading edge of aerofoil	m	ft
x_a	chordwise position of aerodynamic centre in compressible inviscid flow	m	ft
x_h	chordwise position of h	m	ft
x_{h+}	chordwise position of $h+$	m	ft
x_{h-}	chordwise position of $h-$	m	ft

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x_t	chordwise position of maximum thickness	m	ft
z	vertical coordinate at station x (see Section 2.2)	m	ft
z_c	camber line ordinate at station x	m	ft
z_t	thickness ordinate at station x	m	ft
α	incidence	deg	deg
α_0	zero-lift incidence	deg	deg
β	compressibility parameter, $(1 - M^2)^{1/2}$		
γ	ratio of specific heat capacity at constant pressure to specific heat capacity at constant volume, taken as 1.4 for air		
ρ	aerofoil leading-edge radius	m	ft
τ	aerofoil trailing-edge angle	deg	deg

Subscripts

i denotes incompressible flow ($M = 0$)

The following suffixes relate to x and z only:–

- 1 pertaining to $x = 0.005c$
- 2 pertaining to $x = 0.05c$
- 3 pertaining to $x = 0.2c$
- 4 pertaining to $x = 0.5c$
- 5 pertaining to $x = 0.9c$
- 6 pertaining to $x = 0.92c$

2. AEROFOIL GEOMETRICAL PARAMETERS

2.1 Definition of Aerofoil Chord

In the context of this Item the aerofoil chord is defined as the length of the line joining the trailing-edge (assumed sharp) and that unique point at which a circle centred at the trailing-edge is tangential to the aerofoil.

2.2 Definition of Aerofoil Geometry

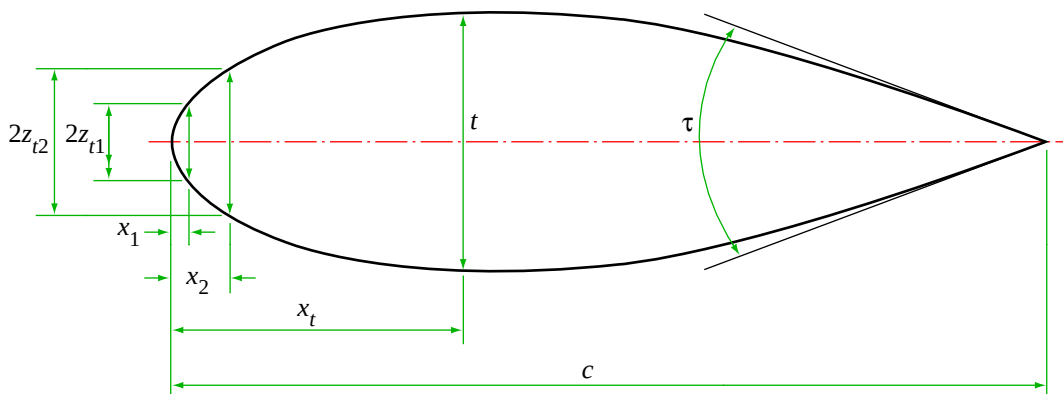
The coordinate z is measured perpendicular to the chord line. If the upper and lower surfaces of the aerofoil are defined by

$$z = z_u \text{ and } z = z_l, \text{ respectively,}$$

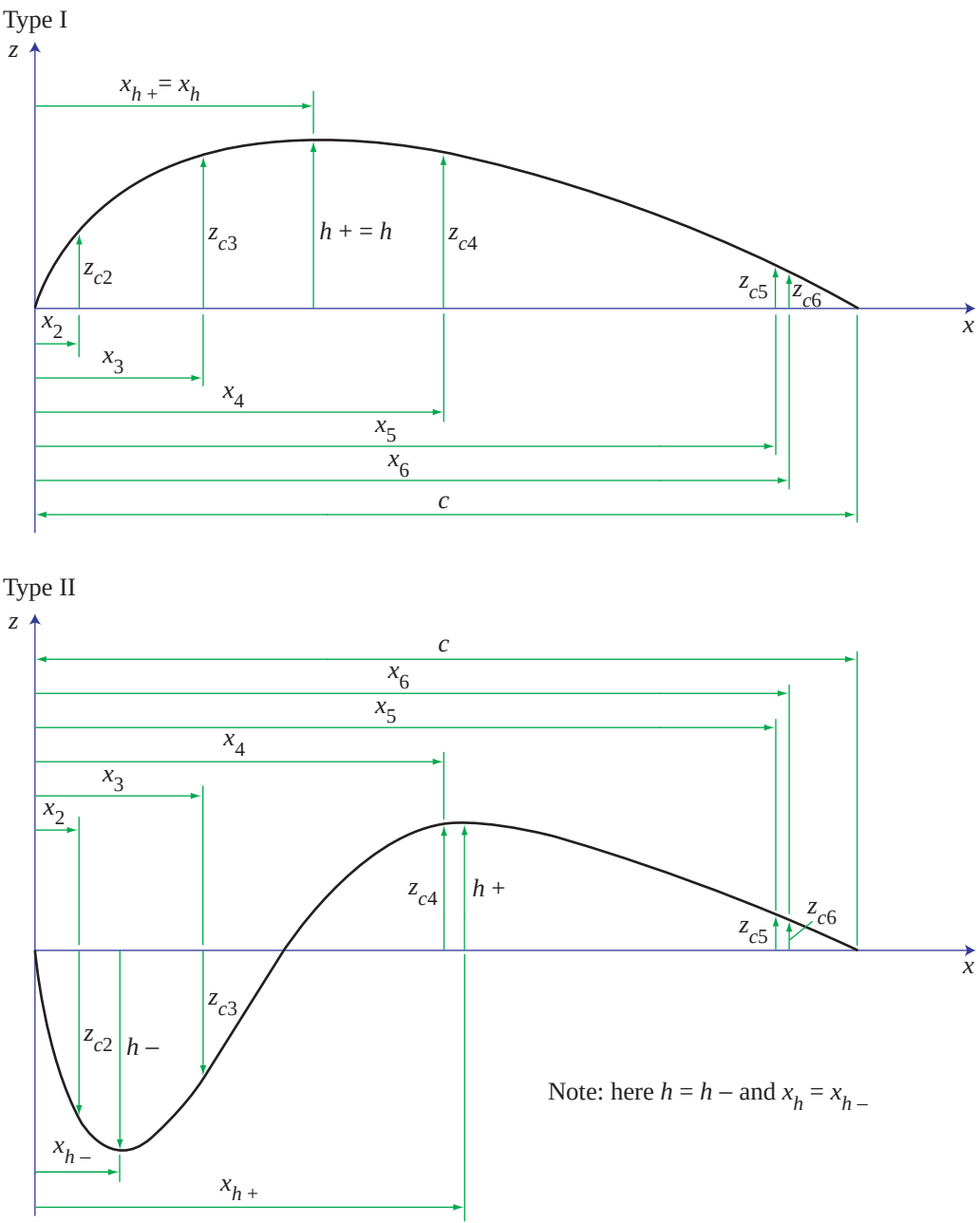
then the thickness and camber line ordinates are defined by

$$z_t = \frac{1}{2}(z_u - z_l) \text{ and } z_c = \frac{1}{2}(z_u + z_l), \text{ respectively.}$$

2.3 Thickness Distribution



2.4 Camber Distribution



3. INTRODUCTION

This Item provides a means of estimating a number of aerodynamic characteristics for a wide range of aerofoils in compressible inviscid airflow from a knowledge only of the thickness and camber line parameters shown in Sections 2.3 and 2.4.

It is stressed that the data given herein apply only to non-viscous flow and that in a real (viscous) flow the effects of viscosity must be taken into account*. These effects can be large at all Mach numbers, and increasingly so near the critical value where, for example, reductions of 25 per cent or more in lift coefficient are possible at wind-tunnel Reynolds numbers.

The particular aerodynamic characteristics considered are lift-curve slope at zero lift, a_1 , zero-lift incidence, α_0 , zero-lift pitching moment coefficient, C_{m0} , and chordwise position of the aerodynamic centre, x_a/c . The geometrical parameters (see Sections 2.3 and 2.4) required to obtain estimates for the aforementioned characteristics from this Item are given below, together with the numbers of the relevant Figures to be used.

For camber lines outside the ranges of Figures 2 and 3 see Addendum A.

Aerodynamic Characteristic	Geometrical Parameters Required	Figure Numbers
a_1	$\frac{t}{c}, \tau, \frac{2z_{t2}}{t}, \frac{h+}{c}$	1 and 5
α_0	$\frac{z_{c3}}{z_{c4}}, \frac{z_{c6}}{z_{c4}}, \frac{z_{c4}}{c}$	2
C_{m0}	$\frac{z_{c2}}{h+}, \frac{z_{c5}}{h+}, \frac{h+}{c}$	3 and 6
$\frac{x_a}{c}$	$\frac{t}{c}, \tau, \frac{2z_{t1}}{t}, \frac{x_t}{c}$	4 and 7

4. INCOMPRESSIBLE FLOW

Figures 1 to 4 present data by means of which the lift-curve slope at zero lift, zero-lift incidence, zero-lift pitching moment coefficient and the chordwise position of the aerodynamic centre of an aerofoil in incompressible ($M = 0$) inviscid flow may be estimated. The Figures were obtained by correlating (Derivation 2) these particular aerodynamic characteristics for a large number of aerofoils designed using the nominally-exact method for incompressible flow due to Lighthill (Reference 4). These aerofoils† were designed (Derivation 1) to have low-drag characteristics at low speeds by virtue of the attainment of a roof-top pressure distribution on the upper and lower surfaces at the extremes of a specified incidence range.

Subsequent calculations for a number of standard aerofoils (including some from the NACA 6 and 6A series⁶, the RAE 100 to 104 series⁵) and others of more recent design¹², using the nominally-exact method of Reference 10 further substantiate the correlations.

* See Item Nos 97020 (Reference 13) and 98011 (Reference 14), if the flow is wholly subsonic.

† Details of the ordinates, pressure distributions and aerodynamic characteristics of any of the aerofoils tabulated in Derivation 1 may be obtained from ESDU.

The data used to establish Figure 1 relate to the zero-lift condition. The lift coefficient is obtained, using values of a_{1i} from Figure 1, via the expression

$$C_{Li} = 57.296 a_{1i} \sin(\alpha - \alpha_{0i}). \quad (4.1)$$

However, the error in using the approximation

$$C_{Li} = a_{1i}(\alpha - \alpha_{0i}) \quad (4.2)$$

is small at low incidences, amounting, for example, to about +0.5 per cent at $(\alpha - \alpha_{0i}) = 10$ degrees.

In view of their empirical nature it is advisable that the correlations should not be applied outside the ranges of the relevant aerofoil geometrical parameters for which they were derived. These parameters are given in Tables 10.1a and 10.1b for the symmetrical and cambered aerofoils, respectively. Caution should be exercised in using Figures 1 to 4 for any aerofoil with a geometry that lies outside the ranges given in those Tables.

Figures 1 and 4 for lift-curve slope and aerodynamic centre involve only parameters due to thickness distribution since no significant effect of camber was found. Figures 2 and 3 were derived from data relating to the two basic camber line shapes shown in Section 2.4 and designated Types I and II. On the basis of some calculations for five aerofoils^{11,12} with “double-hump” camber lines* and two aerofoils¹² with Type I camber lines giving values of $z_{c6}/z_{c4} > 0.5$, a tentative extension to Figure 2 is given. Caution would be necessary should Figures 2 and 3 be used for any other camber line shape. For such cases, and for camber lines outside the ranges of Figures 2 and 3, see Addendum A.

The accuracy to be expected from Figures 1 to 4 is tabulated in Section 7. With regard to Figure 4, the accuracy quoted in Section 7 applies to the range $0 \leq \tau \leq 30^\circ$ using linear interpolation and extrapolation.

5. COMPRESSIBLE FLOW

Figures 5 to 7 present data for estimating the effect of air compressibility on, respectively, the lift-curve slope at zero lift, zero-lift pitching moment coefficient and the chordwise position of the aerodynamic centre of an aerofoil in inviscid flow. These Figures must be used in conjunction with Figures 1, 3 and 4.

Figures 5 to 7 were obtained from correlations (Derivation 3) of the aerodynamic characteristics of 14 symmetrical and 18 cambered aerofoils using the nominally-exact method of Reference 7 in which air is assumed to behave as a perfect gas ($\gamma = 1.4$). The calculations were made using a 60×10 mesh, in general, for incidences to cover the design range; e.g. in the case of “roof-top” aerofoils the incidence range spanned those incidences for which a substantially flat pressure distribution over part of the upper and/or lower surface is obtained. The actual incidences for which the calculations were made all fell within the range $(\alpha - \alpha_0) < 8$ degrees, and over this range the lift curve, for a given Mach number, was found to be essentially linear. The Mach number range of the calculations extended to, or near to, the critical value (i.e. that for which the maximum local Mach number is unity) in each case. No calculations were made at incidences sufficiently high to establish whether the departure from linearity differs in compressible flow from the sine relationship of Equation (4.1). This is not, however, a point of any great practical importance because as the Mach number is increased the range of incidence above the zero-lift value for which the flow over the aerofoil is shock-free decreases rapidly, and this Item is not applicable to cases in which shock waves are present (see Section 7). Hence, within the practical range of application of this Item, the lift coefficient

* See sketch on Figure 2.

may be written in a form equivalent to Equation (4.2), namely,

$$C_L = a_1(\alpha - \alpha_0). \quad (5.1)$$

It will be noted from Figure 5 that, in direct contrast to the incompressible flow case, camber is shown to influence the lift-curve slope in compressible flow. The bulk of the data used to establish the effect of camber in Figure 5 were for aerofoils with camber lines of Type I illustrated in Section 2.4. Data derived from three aerofoils with “double-hump” camber lines (of less than 2 per cent maximum camber) fitted the correlation well. Data from two aerofoils with reflex camber lines of Type II illustrated in Section 2.4 (for which $h(= h+) < 0.01c$) also correlated well. With these comparisons in mind it is anticipated that Figure 5, in addition to being generally applicable to aerofoils with the Type I camber line, will also be applicable to any aerofoil having small maximum camber, less than about 1 per cent, say.

The calculations for the cambered aerofoils showed that zero-lift incidence varied from its incompressible flow value by less than 3 per cent in the most extreme case (Derivation 3), so that Figure 2 may be assumed to apply also to compressible flow.

The zero-lift pitching moment coefficient in incompressible flow (Figure 3) requires a correction factor in compressible flow dependent on Mach number only. This correction factor is $1/\beta$, and may be obtained either from Figure 6 or by means of Table 10.3.

The effect of camber on the ratio C_{m0}/C_{m0i} and on the aerodynamic centre position in compressible flow (Figure 7) was found to be negligible, provided that $t/c < 2|h/c|$ (see later). The calculations showed that the movements of the aerodynamic centre with increase in Mach number up to the critical were quite small (less than about $0.02c$ in the most extreme case) for the particular aerofoils considered in Derivation 3, so that for many purposes the influence of F_5 in Figure 7 would be considered negligible.

Again, since the correlations used to derive Figures 5 to 7 are empirical in nature, it is appropriate to tabulate the ranges of the relevant geometrical parameters of the aerofoils from which the correlations were obtained. These are given in Tables 10.2a and 10.2b for the symmetrical and cambered aerofoils, respectively. The warning in Section 4 with regard to the use of the data for aerofoils outside the tabulated ranges also applies here.

Calculations for aerofoils with low thickness and high camber indicate that Figures 5 to 7 fail to correlate data for aerofoils with thickness less than about twice the maximum camber ordinate. Caution should therefore be exercised in using Figures 5 to 7 for any aerofoil for which $t/c < 2|h/c|$.

Three functions of Mach number used in Figures 5 to 7 are $1/\beta$, M^5 and M^8 , and values of these functions are given in Table 10.3 for Mach numbers up to 0.95.

A tabulation of the accuracy to be expected from the use of Figures 5 to 7, and Figure 2 for compressible flow, is given in Section 7.

6. DERIVATION OF AEROFOIL GEOMETRICAL PARAMETERS

Use of this Item is dependent solely on the knowledge of the geometry of the aerofoil whose aerodynamic characteristics are required. Many tabulations of aerofoil ordinates will include most of the values required here. Those ordinates that are often not tabulated, *e.g.* those for $0.92c$, may generally be satisfactorily estimated by plotting out the profile to a large scale, or even by linear interpolation. This procedure is not satisfactory for ordinates very close to the leading edge and the determination of $2z_{t1}/t$ may prove difficult. Many tabulations do, however, often include a value for the leading-edge radius, and where this is known