

ADAPTATION OF DRAG-RISE CHARTS IN T.D. MEMOR. 71019 TO THE MID-SEMI-SPAN PORTION OF SWEEPED AND TAPERED PLANFORMS

1. NOTATION

C_{LD}	lift coefficients at $M_0 = M_D$, with specified form of upper-surface pressure distribution
c	local chord
M_D	drag-rise Mach number as defined in Reference 1
M_0	free-stream Mach number
n	fraction denoting specific chordwise location (<i>e.g.</i> , $n = \frac{1}{2}$ denotes mid-chord)
t	maximum thickness of two-dimensional section or streamwise section of swept wing
x	chordwise distance measured from leading edge
$(\bar{x}/c)_e$	chordwise location of line having a geometric sweep equal to $\bar{\Lambda}_e$
$\Delta(\bar{x}/c)_e$	contribution to $(\bar{x}/c)_e$ due to sweep and taper, measured from a datum condition
Λ_e	equivalent sweep angle at drag-rise condition of mid-semi-span portion of swept and tapered planform
$\bar{\Lambda}_e$	value of Λ_e averaged over range of streamwise thickness/chord ratios
Δ_n	sweepback of n^{th} chord line
Λ_0	sweepback of leading edge (<i>i.e.</i> , $n = 0$)
$\Lambda_{1/2}$	sweepback of mid-chord line (<i>i.e.</i> , $n = \frac{1}{2}$)
Λ_1	sweepback of trailing edge (<i>i.e.</i> , $n = 1$)
$\Delta\Lambda_e$	$(\Lambda_e - \bar{\Lambda}_e)$, correction dependent on x_R/c , streamwise thickness/chord ratio and taper

Suffixes

R	identifies chordwise station defining end of roof-top
<i>streamwise</i>	denotes the streamwise section
<i>tmax</i>	pertaining to maximum thickness
<i>2d</i>	pertaining to two-dimensional flow
<i>3d</i>	pertaining to three-dimensional flow

Issued November 1972

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2. INTRODUCTION

In the preliminary stages of aircraft wing design guidance is often needed on the combinations of values of, and exchange rates between, section and planform parameters that will allow the attainment of a certain cruise design point without the occurrence of shock wave drag, but beyond which point the onset of shock wave drag is to be anticipated. This design point is by definition the drag-rise condition (C_{LD} , M_D).

For two-dimensional flow such guidance is provided in Reference 2, for families of aerofoils that have a particular form of upper-surface pressure distribution at the drag-rise condition, by charts that relate the variables C_{LD} , M_D , t/c and x_R/c . These charts were derived by a method that constitutes a full aerofoil inverse design procedure for compressible viscous flow.

For three-dimensional flow these same charts may be adapted to provide similar guidance, for the mid-semi-span portion (away from root, crank and tip effects) of swept and tapered planforms having upper-surface pressure distributions in the streamwise direction equivalent⁴ to those used in Reference 2, by writing the simple sweep relations in the form

$$\left. \begin{aligned} M_{D3d} &= M_{D2d} \sec \Lambda_e \\ C_{LD3d} &= C_{LD2d} \cos^2 \Lambda_e \\ (t/c)_{streamwise} &= (t/c)_{2d} \cos \Lambda_e \end{aligned} \right\}, \quad (2.1)$$

where the values for two-dimensional flow are obtained from Reference 2 and Λ_e is the equivalent sweep angle at the drag-rise condition.

Λ_e is defined as the angle of sweep of an infinite constant-chord wing which gives the same drag-rise locus as the mid-semi-span portion of the swept and tapered planform in question, when both have the same streamwise thickness-chord ratio and the same thickness form, and may be determined using the equations and charts of Section 3.

3. APPLICATION

The equivalent sweep angle at the drag-rise condition, Λ_e , required in Equations (2.1) to adapt the drag-rise charts of Reference 2 (T.D. Memor. 71019) to the mid-semi-span portion of a swept and tapered planform (away from root, crank and tip effects) may be determined from the following equations

$$(\overline{x/c})_e = [(\overline{x/c})_e - \Delta(\overline{x/c})_e] + \Delta(\overline{x/c})_e \quad (3.1)$$

$$\tan \bar{\Lambda}_e = \tan \Lambda_0 - 2(\tan \Lambda_0 - \tan \Lambda_{1/2})(\overline{x/c})_e, \quad (3.2)$$

where

$$2(\tan \Lambda_0 - \tan \Lambda_{1/2}) = \frac{1}{n}(\tan \Lambda_0 - \tan \Lambda_n), \quad (3.3)$$

and

$$\Lambda_e = \bar{\Lambda}_e + \Delta\Lambda_e. \quad (3.4)$$

Charts of $[(\overline{x/c})_e - \Delta(\overline{x/c})_e]$, independent of taper, and of $\Delta(\overline{x/c})_e$, independent of x_R/c , are given in Figures 1 and 2 whilst curves of the correction term, $\Delta\Lambda_e$, to allow for the effect of streamwise thickness/chord ratio on Λ_e are given in Figures 3.

In the simple sweep relations of Equations (2.1), namely

$$\left. \begin{aligned} M_{D3d} &= M_{D2d} \sec \Lambda_e \\ C_{LD3d} &= C_{LD2d} \cos^2 \Lambda_e \\ (t/c)_{streamwise} &= (t/c)_{2d} \cos \Lambda_e \end{aligned} \right\}, \quad (3.5)$$

values of M_{D2d} and C_{LD2d} are given by charts in Figures 2 to 5 of Reference 2 for $0.06 \leq (t/c)_{2d} \leq 0.18$ and $0.3 \leq x_R/c \leq 0.6$.

The charts in Figures 1 to 3 here and in Figures 2 to 5 in Reference 2 were derived using the RAE 101 to 104 series thickness forms and by associating with a particular value of x_R a thickness form having the same roof-top limit. Results in Reference 3 suggest that removing this constraint or replacing an RAE series thickness form by another thickness form will not seriously modify the relationships of Figures 1 to 3 herein. However, a change in thickness form from that implied in the charts of Figures 2 to 5 of Reference 2 may involve some change in C_{LD2d} , see Part 2 Section 3.2 of Reference 2.

4. DERIVATION

The derivation of the charts in Figures 1 to 3 herein is described in detail in Reference 3. The first requirement was to specify for the mid-semi-span portion of a swept wing an upper-surface pressure distribution equivalent⁴ in its essential feature^{2,3} to that specified for the two-dimensional problem in Reference 2. This led to the upper-surface pressure distribution being characterised by a sloping roof-top having a constant local Mach number = 1.02 normal to the isobars (assumed to lie along lines of constant x/c), a variable roof-top extent designated x_R , a linear pressure recovery from the end of the roof-top to the trailing edge with the trailing-edge pressure coefficient chosen to be compatible with real (viscous) flow conditions and, to add further realism, some rounding of the pressure distribution in the vicinity of x_R (see Figure 1 of Reference 3).

With the upper-surface pressure distribution and thickness form specified, C_{LD3d} was calculated³ for a range of infinite sheared wings and a selection of six swept and tapered planforms with, in every case, streamwise thickness/chord ratio, roof-top extent and drag-rise Mach number as variables.

Drag-rise loci (C_{LD3d} - v - M_{D3d}) could then be plotted for constant values of streamwise thickness/chord ratio and roof-top extent. By comparing³ loci for the swept and tapered planforms with those for infinite sheared wings Λ_e could be determined because Λ_e is, by definition, the angle of sweep of the infinite sheared wing that gives the same drag-rise locus as the mid-semi-span portion of the swept and tapered planform in question when both have the same streamwise thickness/chord ratio and the same thickness form.

The analysis³ showed that Λ_e had some dependence on streamwise thickness/chord ratio but that an average value $\bar{\Lambda}_e$ could be determined that was dependent only on x_R/c , mid-chord sweep ($\Lambda_{1/2}$) and taper (expressed in the form $\Lambda_0 - \Lambda_{1/2}$). For each planform considered, $\bar{\Lambda}_e$ was expressed in terms of its equivalent chordwise location $(\bar{x}/c)_e$ through Equation (3.2). It was then found that $(\bar{x}/c)_e$ could be divided into two components $[(\bar{x}/c)_e - \Delta(\bar{x}/c)_e]$ and $\Delta(\bar{x}/c)_e$, the first independent of taper and the second independent of x_R/c . These components constitute Figures 1 and 2 whilst the dependence of Λ_e on streamwise thickness/chord ratio is represented by the curves of $\Delta\Lambda_e = (\Lambda_e - \bar{\Lambda}_e)$ in Figures 3.

5. REFERENCES

1. ESDU A method of estimating drag-rise Mach number for two-dimensional aerofoil sections. Royal Aeronautical Society Transonic Data Memor. No. 6407, 1964.
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