

CRITERIA OF YIELDING FOR DUCTILE MATERIALS UNDER STATIC LOADING

1. NOTATION

f_e	equivalent uniaxial tensile stress	N/m ²	lbf/in ²
f_y	uniaxial tensile yield stress	N/m ²	lbf/in ²
f_1, f_2, f_3	principal stresses ($f_1 > f_2 > f_3$)	N/m ²	lbf/in ²

Convention: Tensile stresses positive. Compressive stresses negative.

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The strength of a homogeneous isotropic material is generally obtained from simple uniaxial tensile tests. In such tests a noticeable departure from the linear stress-strain relationship often occurs well before the tensile strength is reached. The stress at which this non-linearity reaches a prescribed value is termed the yield stress*. This stress is often used as a design criterion for components that have to maintain their form. However, the stresses to which engineering components are subjected are usually more complex than simple tension. It is therefore necessary to predict the point of yielding under multiaxial stresses from a knowledge of the material yield stress provided by the simple uniaxial tensile test. This is done by expressing any combination of the principal stresses acting at a point as an equivalent uniaxial tensile stress, f_e . Yielding occurs when this value of f_e reaches the uniaxial yield stress. Thus, f_e may be termed the yield criterion.

3. DUCTILE MATERIAL CRITERIA

Since yielding is in essence a shear phenomenon, it is appropriate that the criteria should be based on either the shear stresses or shear strains present in the material. Of the various yield criteria developed only two, termed the maximum shear stress criterion and the shear strain energy criterion, follow this principle and correspond closely to the material behaviour. Table 3.1 provides a synopsis of these two criteria for systems of both two- and three-dimensional principal stresses. The merits of these criteria are discussed in the following section.

For two-dimensional stress systems Figure 1 provides the yield boundary represented by the two criteria in terms of the ratios of the principal stresses to the uniaxial yield stress, f_y . Provided that a point defined by these ratios lies within the bounds of a yield criterion curve, the material is below the state of yield according to that criterion. Also shown are the regions in which experimental results typically fall.

3.1 Maximum Shear Stress Criterion (Tresca)

The criterion, formulated by Tresca and Guest, states that yielding occurs when the maximum shear stress reaches the same value as in the case of uniaxial tension at yield, that is, half the tensile yield stress. This

* The general behaviour of a material as it approaches and passes this stress is discussed in Section 4.

criterion is widely used since f_e , which is simply given by the largest difference between the principal stresses, is usually conservative with respect to experimental results.

3.2 Shear Strain Energy Criterion (Von Mises and Hencky)

This criterion, developed by Von Mises and Hencky, is widely used. It states that yielding occurs when the value of the strain energy of shear deformation reaches the same value as in the case of uniaxial tension at yield. In this case, f_e is proportional to the root mean square of all three principal stress differences. A similar expression to that given in Table 3.1, differing only by a constant of proportionality, also gives the shear stress on the octahedral plane (the plane equally inclined to all three principal stress axes) thus, the criterion is sometimes called the octahedral shear stress criterion.

3.3 General Comments

As can be seen from Figure 1, the Tresca criterion is conservative for all stress combinations. The Von Mises and Hencky criterion, while following more closely the trend of the experimental values, can over-estimate the allowable stresses at yield for some combinations. According to either criterion, yielding is not affected by the mean hydrostatic stress (that is, $(f_1 + f_2 + f_3)/3$, which also represents the normal stress on the octahedral plane) and this is broadly confirmed by experiment.

The mechanics of material behaviour beyond yield in a multiaxial situation may be controlled in a different manner from that governing initial yield. However, the equivalent stress, f_e , is a fundamental and useful characterising parameter for any shear controlled situation such as elastic-plastic deformation, creep deformation and fatigue crack initiation predictions.

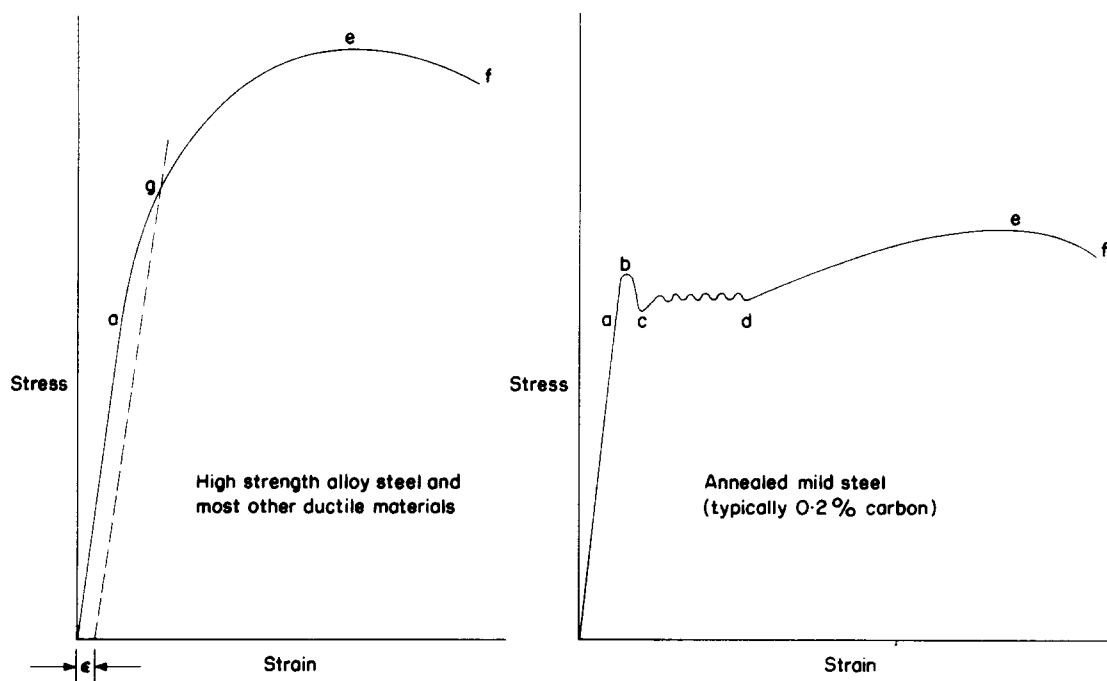
TABLE 3.1

Criterion	Two-dimensional stress system	Three-dimensional stress system
Maximum Shear Stress (Tresca)	$f_e = \text{maximum of}$ $ f_1 \text{ or } f_2 \text{ or }$ $ f_1 - f_2 $	$f_e = \text{maximum of}$ $ f_1 - f_2 \text{ or }$ $ f_2 - f_3 \text{ or }$ $ f_3 - f_1 $
Von Mises and Hencky	$f_e = \sqrt{\{f_1^2 - f_1 f_2 + f_2^2\}}$	$f_e = \sqrt{\frac{1}{2}\{(f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2\}}$

For materials in a plane (two-dimensional) stress field, the principal stresses (f_1, f_2) may be obtained from Data Item No. 72013, "Principal stresses and strains in a two-dimensional stress field".

4. CHOICE OF YIELD STRESS AND MATERIAL BEHAVIOUR

In most metals the initial relationship between stress and strain is essentially linear. The maximum stress of this relationship is known as the limit of proportionality and is indicated by *a* on Sketch 4.1. On further straining the relationship between stress and strain is no longer linear and, for most metals and alloys, the change from purely elastic to elastic-plastic deformation is gradual. It is usual to take the proof stress* at, typically, 0.1, 0.2 or 0.5 per cent permanent strain, as the yield stress. Mild steels do not exhibit this gradual change but demonstrate the particular characteristic during the yield region shown in Sketch 4.1. Mild steels remain elastic, although non-linear, until they approach the upper yield point, *b*, which is normally taken as the material yield stress. This is usually followed by a fall in stress and a definite plastic deformation, *c-d*, until the material begins to strain-harden at *d*. The subsequent behaviour is continuous and similar to that of most other metals.



Sketch 4.1 NOMINAL TENSILE STRESS-STRAIN CURVES (not to scale)

- a* limit of proportionality
- b, c* upper and lower yield points respectively
- d* commencement of strain-hardening
- e* ultimate tensile strength, test piece already necking
- f* nominal fracture stress
- g* proof (stress associated with percentage strain ϵ)

* Proof stress is defined as the stress (load divided by original area of cross section) that will cause a permanent extension to a specified percentage of the original gauge length.