



**Stiffness of loaded flat strips under
sinusoidally distributed bending couples
at their edges (for use in local buckling
calculations)
(Uniaxial or biaxial loading. Isotropic or
orthotropic materials)**

ESDU 73007 A 1983

Endorsed by
The Royal Aeronautical Society

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU Strut 02.01.29 to 33, was monitored and guided by the Structures Committee. This Committee first met in 1940 and now has the following membership:

Chairman	
Mr I.C. Taig	— British Aircraft Corporation Ltd, Preston
Vice-Chairman	
Prof. W.S. Hemp	— University of Oxford
Members	
Mr H.L. Cox	— Independent
	—
Mr R.S. Dabbs	— Hawker Siddeley Aviation Ltd, Kingston
Mr K.H. Griffin	— Cranfield Institute of Technology
Mr N.F. Harpur	— British Aircraft Corporation Ltd, Filton
Dr G.Z. Harris	— Royal Aircraft Establishment
Mr P.J. McKenzie	— Hawker Siddeley Aviation Ltd, Kingston
Mr J.G. ten Asbroek	— Fokker-VFW N.V., Schiphol-Oost, The Netherlands
Mr F. Tyson	— Scottish Aviation Ltd.

The construction and subsequent development of the Data Item was undertaken by

Mr M.E. Grayley	— Head of Strength Analysis Group.
-----------------	------------------------------------

**STIFFNESS OF LOADED FLAT STRIPS UNDER SINUSOIDALLY DISTRIBUTED
BENDING COUPLES AT THEIR EDGES (FOR USE IN LOCAL BUCKLING
CALCULATIONS)
(UNIAXIAL OR BIAXIAL LOADING. ISOTROPIC OR ORTHOTROPIC MATERIALS)**

CONTENTS

	Page
1. NOTATION	1
1.1 General Notation	1
1.2 Notation for Isotropic Materials	1
1.3 Notation for Orthotropic Materials	2
1.4 Sign Convention	3
2. NOTES	3
3. DERIVATION	5

STIFFNESS OF LOADED FLAT STRIPS UNDER SINUSOIDALLY DISTRIBUTED BENDING COUPLES AT THEIR EDGES (FOR USE IN LOCAL BUCKLING CALCULATIONS)

(UNIAXIAL OR BIAXIAL LOADING. ISOTROPIC OR ORTHOTROPIC MATERIALS)

1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used.

1.1 General Notation

b	width of strip	m	in
f_x	longitudinal compressive stress in strip	N/m ²	lbf/in ²
f_y	transverse stress in strip	N/m ²	lbf/in ²
t	thickness of strip	m	in
T_θ	applied bending couple per unit length of strip on edge whose rotation is θ (see Table 2.1)	N m/m	lbf in/in
x, y, z	strip coordinates (see Table 2.1)	m	in
θ	rotation of strip about one edge line (see Table 2.1)	rad	rad
λ	half-wavelength of applied bending couple distribution	m	in
μ_1, μ_2, μ_3	stiffnesses of strip, defined by T_θ/θ (see Table 2.1)	N m/m	lbf in/in

1.2 Notation for Isotropic Materials

E	modulus of elasticity of strip material	N/m ²	lbf/in ²
E'	$E/(1 - \nu^2)$	N/m ²	lbf/in ²
f_I	$(\pi^2 E'/3)(t/b)^2$	N/m ²	lbf/in ²
α_I	$\frac{f_x/f_I}{\beta_I} + \frac{1}{4}\left(\frac{b}{\lambda}\right)^2\left(\beta_I - \frac{1}{\beta_I}\right)$ which equals f_x/f_I when $f_y = 0$		
α_{I2}^*	α_I when β_I is positive or $(1 - \alpha_I)$ when β_I is negative		
α_{I3}^*	α_I when β_I is positive or $(4 - \alpha_I)$ when β_I is negative		
β_I	$1 - 2(f_y/f_I)/(b/\lambda)^2$		
ν	Poisson's ratio of strip material		

Issued July 1973
With Amendment A

1.3 Notation for Orthotropic Materials

D strip rigidity matrix* defined by

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2 \partial^2 w / \partial x \partial y \end{bmatrix}$$

D_o	$D_{12} + 2D_{33}$	N m	lbf in
D_{11}, D_{22}	flexural rigidity of strip per unit width for bending about y- and x-axes respectively	N m	lbf in
D_{12}, D_{21}	cross-flexural rigidities of strip per unit width ($D_{12} = D_{21}$)	N m	lbf in
D_{33}	twisting rigidity of strip about x- and y-axes	N m	lbf in
f_o	$4\pi^2 D_{22} / b^2 t$	N/m ²	lbf/in ²
M_x, M_y	bending moment per unit width acting about y- and x-axes respectively	N m/m	lbf in/in
M_{xy}	twisting moment per unit width acting about x- and y-axes	N m/m	lbf in/in
w	normal displacement of strip	m	in
α_o	$\frac{f_x/f_o}{\beta_o} + \frac{1}{4} \left(\frac{b}{\lambda} \right)^2 \left(\beta_o - \frac{D_{11}/D_{22}}{\beta_o} \right)$ which equals $\frac{f_x/f_o}{D_o/D_{22}} + \frac{1}{4} \left(\frac{b}{\lambda} \right)^2 \left(\frac{D_o}{D_{22}} - \frac{D_{11}}{D_o} \right)$ when $f_y = 0$		
α_{o2}^*	α_o when β_o is positive or $(1 - \alpha_o)$ when β_o is negative		
α_{o3}^*	α_o when β_o is positive or $(4 - \alpha_o)$ when β_o is negative		
β_o	$(D_o/D_{22}) - 2(f_y/f_o)/(b/\lambda)^2$		

* The isotropic plate values of the terms of **D** are as follows.

$$\begin{aligned} D_{11} &= D_{22} = Et^3/12(1 - \nu^2), \\ D_{12} &= D_{21} = \nu Et^3/12(1 - \nu^2) \\ \text{and } D_{33} &= Et^3/24(1 + \nu). \text{ (Thus } D_o = D_{11} = D_{22}.) \end{aligned}$$

A factor of 2 has been introduced as a coefficient of $\partial^2 w / \partial x \partial y$ instead of being absorbed into D_{33} .

1.4 Sign Convention

Compressive stresses are positive, tensile stresses are negative.

2. NOTES

This Item provides curves for determining the stiffness of long flat strips initially subjected to in-plane uniform longitudinal and transverse direct stresses. The stiffnesses relate to sinusoidally distributed edge rotations and the corresponding edge couples. The application of these data to the estimation of the local buckling stresses of structural members is described in Item No. Struct 02.01.28. Items Nos Struct 02.01.34 to 37 provide example calculations.

The three modes of deformation of flat strips for which data are provided are illustrated and described in Table 2.1. In all cases the strips are taken to be infinitely long, with, excepting the one free edge of mode 1, all edges simply-supported.

The figures provide values of μ_1 , μ_2 and μ_3 in non-dimensional form where the scale factor is $4D_{22}/b$, which reduces to $E't^3/3b$ for isotropic materials. These are plotted against $\beta_o^{1/2} b/\lambda$ or $\beta_I^{1/2} b/\lambda$ for several values of α_o or α_I . Mode 1 is, as a result of its free edge condition, also dependent upon the ratio D_{12}/D_o (which reduces to ν for isotropic materials). Accordingly, sets of curves are provided for mode 1 for values of D_{12}/D_o of 0.25, 0.3 and 0.35. Owing to the range of parameters involved two sets of curves are provided for every case. Table 2.1 provides a guide to the figures and the modes to which they apply.

Figures 7 to 10 provide values of μ_2 and μ_3 for β_o or β_I positive. Figures 7 and 9 also provide sufficiently accurate values of μ_2 and μ_3 for negative values of β_o or β_I .

For isotropic strips under biaxial loading Figures 11 and 12 are provided for rapid calculation of α_I and β_I . An example calculation is illustrated on Figure 11. It begins with values of f_y/f_I and b/λ in the left-hand side of the figure. The other parts of the figure are used in the order indicated by the arrowed chain-dotted line, shown on Figure 11, using values of f_x/f_I and b/λ where required. Figure 12 is used in a similar fashion.

The data are based upon thin plate theory and apply to thin strips of isotropic materials or to thin strips of orthotropic materials whose moment-curvature relationships are given by $\mathbf{D}$. The matrix $\mathbf{D}$ relates to strips whose principal axes of orthotropy are aligned with their edges and whose neutral axes for bending in the principal directions are coplanar. No account is taken of the out-of-plane shear stiffnesses of the strip which are often so large that the deflections associated with them are too small to affect significantly the values of μ_1 , μ_2 or μ_3 .

TABLE 2.1

Diagram of mode form	Description								
Mode 1, μ_1 <table><tr><th>$\frac{D_{12}}{D_o}$</th><th>Figure number</th></tr><tr><td>0.25</td><td>1 and 2</td></tr><tr><td>0.30</td><td>3 and 4</td></tr><tr><td>0.35</td><td>5 and 6</td></tr></table>	$\frac{D_{12}}{D_o}$	Figure number	0.25	1 and 2	0.30	3 and 4	0.35	5 and 6	One side free, one side simply-supported. Uniform direct stress applied to ends only. Couple applied to simply-supported side only. Both couple and rotation vary sinusoidally along strip.
$\frac{D_{12}}{D_o}$	Figure number								
0.25	1 and 2								
0.30	3 and 4								
0.35	5 and 6								
Mode 2, μ_2, Figures 7 and 8	Both sides simply-supported. Uniform direct stress applied to ends and sides. Opposed couples applied to sides. Couples and rotations vary sinusoidally, in phase, along strip.								
Mode 3, μ_3, Figures 9 and 10	Both sides simply-supported. Uniform direct stresses applied to ends and sides. Couples of the same sense applied to sides. Couples and rotations vary sinusoidally, in phase, along strip.								

ESDU Copyright material. For current status contact ESDU.