

Transonic data memorandum

Method for predicting the pressure distribution on swept wings with subsonic attached flow

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METHOD FOR PREDICTING THE PRESSURE DISTRIBUTION ON SWEEPED WINGS WITH SUBSONIC ATTACHED FLOW

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METHOD FOR PREDICTING THE PRESSURE DISTRIBUTION ON SWEEPED WINGS WITH SUBSONIC ATTACHED FLOW

SUMMARY

This Memorandum explains in detail a formula that is widely used to predict the pressure distribution on swept wings at subsonic speeds. The basic theories underlying the formula have been published in a number of separate papers, and only a brief outline is given here, but references to the original work are quoted throughout.

The essential basis of the method lies in the linearised theory for certain special cases (Section 1) with the most important non-linear terms taken into account so that wings of moderate thickness can be dealt with. No detailed comparison with experimental results is given here, but the method is believed to be generally reliable for wings with effective aspect ratio (βA) greater than about $2\frac{1}{2}$ and with effective sweepback angles less than about 60° .

The basic formula is quoted first in Section 3 (Equation (3.1)) for compressible flow; Section 4 quotes the corresponding formula for incompressible flow, and explains in detail the significance of the various terms and the way in which they should be interpreted in a practical calculation. Section 5 develops the linearised version of the formula for compressible flow; this is not recommended for practical application, but forms the basis onto which the non-linear terms are added in Section 6. Section 7 gives some ancillary relations, including the formula for the evaluation of the pressure coefficient in compressible flow, and Section 8 outlines some practical limitations of the method, and some ways in which these may be circumvented. The application of the method to wings in combination with fuselages and other bodies is outlined. Section 9 presents a bibliography of papers that are relevant to the construction of the method outlined here, and of other papers dealing with cognate subjects. Section 10 indexes the tables and charts of the principal special functions used in the work, and all symbols used are listed in Section 2.

This Memorandum supersedes T.D.Memor. 6312 (December 1963) and T.D.Memor. 6312/1 (January 1967). The principal differences concern the definition of thickness sweep (Λ_t and Λ_x), the treatment of the non-linear effects of compressibility and the treatment of wing cranks. Many references issued since 1963 have been added and amongst these particular attention is drawn to developments in the subsonic linearised theory of thick finite wings at zero lift⁵¹⁻⁵⁵ and to procedures for calculating, by (linearised) lifting-surface theory or vortex lattice methods, the velocities due to lift on non-planar wings⁶⁵⁻⁶⁹.

A means of replacing the approximate calculation of the linear theory perturbation velocities due to thickness and lift by values obtained from the numerical methods in these quoted references is given by Equation (3.3).

1. INTRODUCTION

The basis on which the present work is founded is the use of planar distributions of singularities to represent the wing in potential flow. A source distribution is used for the thickness distributions, and vortices represent the effects of camber and incidence. It is implicit in this that the method can be applied only to calculate the velocities on wings with subcritical attached flow – separated flows and shocks of finite strength are both excluded.

To avoid extensive numerical integrations, the expressions for the velocities at any point of a complete swept wing are based on interpolation between the results for two especially simple cases – the infinite sheared wing of constant section, and the centre section of an infinite sweptback wing of constant section. This approach was evolved on the principle that the main three-dimensional effects of the flow are restricted to local regions near the centre and tips of the wing, and consequently it cannot be expected to succeed in a case when the whole flow has a three-dimensional character, as on a slender wing for example. The calculation methods available will become less reliable as the aspect ratio decreases and the angle of sweep increases; because an “analogous wing” method is used to deal with compressible flow, the reliability for a given wing also deteriorates as the Mach number increases. However, the general adequacy of the method for wings with effective aspect ratios (βA) greater than about $2\frac{1}{2}$ and effective sweepback angles less than about 60° has been demonstrated.

The present work has not been restricted to a purely linear theory, because the flow perturbations are not small everywhere. The mutual interactions between sources and vortices are taken into account, and the boundary conditions are effectively satisfied at the aerofoil surface rather than on the chord line. In addition the formulae are deliberately constructed so that the main effects of the boundary layer can be allowed for easily.

Although the formula presented here is founded on published work (mainly from the Royal Aircraft Establishment – see References 5 to 14), no detailed account had been published that explained the significance of the various terms until the present Memorandum was available in its original form (T.D.Memor. 6312). That Memorandum gave a detailed account of the structure of the formula whilst at the same time introducing certain modifications simplifying its practical application and improving its accuracy. In this revision still further improvements have been added whilst the detailed explanation of the formula is retained.

In order to justify the formula given here in a satisfactory manner, it would be necessary to present an exhaustive comparison between its predictions and experimental measurements on a variety of wings. This has not been done; a sufficiently reliable collection of such comparisons does not yet exist. However, experimental verification of most of the individual terms can be found in the original references.

The method presented here deals explicitly with an isolated wing; most practical applications will be concerned with wings in the presence of a fuselage and other bodies. The procedures available for allowing for these influence are outlined in Section 8.

2. NOTATION

		Principal Location	
<i>Symbol</i>	<i>Description</i>	<i>Section</i>	<i>Equation</i>
a_0	speed of sound in free-stream		
a_1	effective lift-slope of two-dimensional section	4.4 (ii)	(4.42), (4.44)
A	aspect ratio	1	
B	second-order compressibility factor for two-dimensional aerofoil	6.2	
B_n	second-order compressibility factor	6.2	(6.6)
B_s	second-order compressibility factor for sheared wing	6.2, 7.2	(6.5), (7.4)
c	local chord	2.1	
c_R	root chord	4.1(iii)	(4.9)
c_T	tip chord	4.1(iii), 4.2(iv)	(4.9)
c_c	crank chord	4.1(iii)	(4.9)
C_L	sectional lift coefficient	4.7, 8.2	(4.55), (4.56)
C_p	pressure coefficient	7.1	(7.1) to (7.3) Fig. 8
C_{pi}	pressure coefficient in incompressible flow	6.2	(6.5), (6.6)
$D\left(\frac{u}{l}\right)$	thick aerofoil factor, $1 + \left\{ \frac{S^{(2)}(x) \pm S^{(5)}(x)}{B_n \cos(\Lambda_x^*)} \right\}^2$	3, 4.6, 6.1	(3.1) to (3.3), (4.1), (4.52), (6.1)
$f(\Lambda)$	$= \frac{1}{\pi} \log_e \left\{ \frac{1 + \sin \Lambda}{1 - \sin \Lambda} \right\}$	4.2(ii)	
$\mathfrak{S}$	$= C_p \left/ \left\{ 1 - \left(\frac{V}{V_0} \right)^2 \right\} \right.$	7.1	(7.3), Fig. 8
H, H_a	factor in u_α and v_α	4.4(ii)	(4.34), (4.35), (4.46), (5.20),

Symbol	Description	Section	Equation
K_1, K_{1a} $K_2, \tilde{K}_2, K_{2a}, \tilde{K}_{2a}$	spanwise interpolation factors to allow for variation of root and tip effects on velocities due to thickness	4.1(iii) 4.1(iv), 4.2(iv)	(4.12), (4.13) (4.4), (4.14), (4.51), (5.14), (5.15), (6.4)
K_3, K_{3a}	spanwise interpolation factor to allow for effects of finite aspect ratio, planform taper and thickness taper on velocity at maximum thickness position due to thickness	4.1(iii)	(4.8)
M_0	$= V_0/a_0$; free-stream Mach number		
n	chordwise loading parameter (small aspect ratio wings)	4.4(ii), 4.4(iii)	(4.42), (4.47)
n^*	$= n$; but see Section 4.6 for exceptional cases	4.6	
n_a	$= n(y_a, \Lambda_a)$	5.1(iv)	(5.20),
n_s	value of n for sheared portion of finite wing	4.7(i)	
n_0	chordwise loading parameter (large aspect ratio wings)	4.3(i), 4.4(i)	(4.22)
N	number of chordwise points used in series approximation	4.1(i)	(4.7)
P	$= \left\{ 4 + \frac{8 \Lambda_m }{\pi} \right\}^{-1}$	4.4(ii)	(4.42), (4.43)
s	semi-span of wing	4.1(iii)	(4.9)
s_a	semi-span of analogous wing	5.1	(5.2)
s', s'_a	spanwise distance of point of intersection of projected leading and trailing edges of wing, or analogous wing, from root	4.1(iii)	(4.9)
$S^{(1)}(x)$	$= \frac{1}{\pi} \int_0^1 \left(\frac{dz_t}{dx} \right)_{x=x'} \frac{dx'}{x-x'}$	4.1(i)	(4.5), (4.6)
$S^{(2)}(x)$	$= dz_t/dx$	4.2(i), 4.6, 6.1	(4.16), (4.17), (6.3)
$S^{(3)}(x)$	$= \frac{1}{\pi} \int_0^1 \left\{ \left(\frac{dz_t}{dx} \right)_{x=x'} - \frac{2z_t(x')}{1-(1-2x')^2} \right\} \frac{dx'}{x-x'}$	4.4(iv), 6.1	(4.48), (4.49), (6.2)
$S^{(4)}(x)$	approximation for vortex distribution on two-dimensional wing	4.3(ii)	(4.29)

<i>Symbol</i>	<i>Description</i>	<i>Section</i>	<i>Equation</i>
$S^{(5)}(x)$	$= dz_s/dx$	4.6	(4.53)
$S^{(15)}(x)$	approximation for vortex distribution at centre of sweptback wing	4.3(ii)	(4.33)
$s_{\mu\nu}^{(1)}$	coefficient in series approximation to $S^{(1)}(x)$	Table	(4.6)
$s_{\mu\nu}^{(2)}$	coefficient in series approximation to $S^{(2)}(x)$	Table	(4.17)
$s_{\mu\nu}^{(3)}$	coefficient in series approximation to $S^{(3)}(x)$	Table	(4.49)
$s_{\mu\nu}^{(4)}$	coefficient in series approximation for $\frac{\gamma}{2V_0}$ on unswept wing	4.3(ii), Table	(4.29), (5.18)
$s_{\mu\nu}^{(5)}$	coefficient in series approximation to $S^{(5)}(x)$	4.6, Table	(4.53)
$s_{\mu\nu}^{(15)}$	coefficient for $\frac{\gamma}{2V_0}$ at centre of sweptback wing	4.3(ii), Table	(4.33)
t	max. thickness $= 2cz_{t\max}$	} (dimensional quantities)	
t_R	max. thickness, wing root		
t_T	max. thickness, wing tip		
u, v, w	velocity increments in x, y, z -direction divided by V_0	4.1 to 4.4, 5.2, 5.3	(5.10) to (5.12), (5.22), (5.23)
u_{t1}	} streamwise components of velocity perturbation in linearised theory (non-dimensional with free-stream velocity)	3, 4.1, 5.1(i)	(3.2), (4.2), (4.3), (5.13)
u_{t2}		3, 4.2, 5.1(ii)	(3.2), (4.2), (4.14), (5.15)
u_c		3, 4.3, 5.1(iii)	(3.2), (4.2), (4.19), (5.16)
u_α		3, 4.4, 5.1(iv)	(3.2), (4.2), (4.34), (5.20)

<i>Symbol</i>	<i>Description</i>	<i>Section</i>	<i>Equation</i>
v_t	spanwise components of velocity perturbation in linearised theory (non-dimensional with free-stream velocity)	3, 4.1, 5.1(i)	(3.2), (4.2), (4.4), (5.14)
v_c		3, 4.3, 5.1(iii)	(3.2), (4.2), (4.20), (5.17)
v_α		3, 4.4, 5.1(iv)	(3.2), (4.2), (4.35),
V	local flow velocity	3, 7.2	(3.1), (7.4)
V_0	free-stream velocity		
x	non-dimensional streamwise co-ordinate measured from leading edge in terms of local chord	2.1	
x_μ	$\frac{1}{2} \left(1 + \cos \frac{\mu \pi}{N} \right)$	stations used in series approximations	(4.7)
x_v	$\frac{1}{2} \left(1 + \cos \frac{v \pi}{N} \right)$		
x_{ac}	aerodynamic centre position	4.4(iii)	(4.47)
x_{tmax}	chordwise position of maximum thickness	4.1(iii)	(4.8), (4.9), (4.11), (4.12)
X_{tmax}	$\begin{cases} = x_{tmax} & \text{for sweptback wings} \\ = (1 - x_{tmax}) & \text{for sweptforward wings} \end{cases}$	4.1(iii)	(4.9), (4.10)
y	spanwise co-ordinate (dimensional quantity)	2.1	(5.1)
y_c	spanwise location of a crank station (dimensional quantity)	4.1(iii)	(4.9)
z	vertical co-ordinate, non-dimensional with local chord	2.1	
z_s	aerofoil or wing camber-line ordinate, non-dimensional with local chord	2.1	(5.8)
z_t	aerofoil or wing thickness ordinate, non-dimensional with local chord	2.1	(5.7)
α	geometric wing incidence	4.7	(4.54), (5.9)
α_e	sectional effective incidence	4.7	(4.54)
α_i	sectional induced incidence	4.7	(4.54), (4.55), (4.57)

<i>Symbol</i>	<i>Description</i>	<i>Section</i>	<i>Equation</i>
β	$(1 - M_0^2)^{1/2}$	1, 5.1	(5.1) to (5.2)
β_n	first-order compressibility factor	6.2	(6.7)
γ	= 1.4, ratio of specific heat capacities of air	7.1	(7.1) only
$\gamma, \gamma(x), \gamma(x,y)$	vortex distribution, representing lift on aerofoil or wing	4.3(ii), 4.4, 4.4(i), 4.4(ii)	(4.29), (4.31), (4.36), (4.39), (4.45)
δ	change of spanwise slope of wing maximum thickness	4.1(ii)(c)	(4.9)
δ_R	spanwise slope of wing surface at wing root discontinuity	4.1(iii)(c)	(4.9)
δ_T	spanwise slope of wing surface at wing tip discontinuity	4.1(iii)(c)	(4.9)
ΔC_p	difference between pressure coefficients on upper and lower surfaces		
$\Delta_s(x_{tmax})$	additional velocity at maximum thickness position caused by sweepback, thickness taper, planform taper and aspect ratio	4.1(iii)	(4.9)
Δv	correction to spanwise velocity due to thickness	4.5	(3.2), (4.2), (4.51)
Δ_0	spanwise interpolation factor to allow for reduction in velocities due to thickness at maximum thickness position near tip of semi-infinite unswept wing	4.1(iii), Table	(4.9), (4.10), Fig. 3
Δ_1	spanwise interpolation factor to allow for effects of sweep on reduction of velocities due to thickness at maximum thickness position near tip of semi-infinite sheared wing	4.1(iii), Table	(4.9) to (4.11), Fig. 4
Δ_3	spanwise interpolation factor to allow for effects of surface crank on velocities at maximum thickness position	4.1(iii), Table	(4.9), Fig. 5
Δ_4	spanwise interpolation factor to allow for effects of planform taper on two-dimensional velocities due to thickness at maximum thickness position	4.1(iii), Table	(4.9), Fig. 6
Δ_5	spanwise interpolation factor to allow for effects of plan-taper crank on velocities due to thickness at maximum thickness position	4.1(iii), Table	(4.9), Fig. 7
θ	$\tan^{-1} \frac{d}{dy}(cz_{max})$: spanwise slope of wing maximum thickness	4.1(iii)	
Θ	exponent defined in Equation (4.31)	4.3(ii)	(4.31), (4.32)

Symbol	Description	Section	Equation
$\kappa \left(\frac{s_a - y_a}{c} \right)$	spanwise interpolation factor to allow for variation in velocities due to thickness near tip of wing of finite aspect ratio	4.1(iii)	
$\lambda \left(\frac{y_a}{c}, \frac{s_a - y_a}{c} \right)$	spanwise interpolation factor to allow for variations in shape of chordwise loading across span	4.3(i), 4.4(i)	(3.1), (4.1), (4.24) to (4.26), Fig. 2
Λ	wing sweep angle	4.2(ii)	
Λ_m	mean sweep; usually taken as sweep of mid-chord line	4.3(i)	(4.25)
Λ_t	sheared wing thickness sweep	4.1(ii), 8.1	(4.3), (4.4)
Λ_{tmax}	sweep of maximum thickness line	4.1(iii)	(4.8)
Λ_v	local sweep of vortex lines	4.3(i), 4.4	(4.21), (4.23), (4.40)
Λ_x	local geometric sweep	6.2	(6.6), (6.7)
Λ_x	centre effect thickness sweep	4.2(iii)	(4.14)
Λ_0	sweepback of leading edge	4.1(iii)	(4.9)
Λ_1	sweepback of trailing edge	4.1(iii)	(4.9)
Λ_t^*	$= \left\{ \begin{array}{l} \{ 1 - \tilde{K}_2 \} \Lambda_t \\ \{ 1 - \tilde{K}_2 \} \Lambda_x \end{array} \right\}$ interpolation between sweep at centre and tip stations	4.4(iv)	(4.50)
Λ_x^*		4.6	(4.52a)
σ	function used in Equation (4.28)	4.3(ii)	(4.28)
ω	downwash factor	4.7(i)	(4.55), (4.57)
<i>Suffix</i>			
<i>a</i>	pertaining to analogous wing	5.1	

This notation differs from that frequently found in the available literature (*e.g.* References 5 to 14) in several respects. For consistency with other Transonic Data Memoranda Λ has been used instead of ϕ , K_2 replaces μ , K_3 replaces τ and the factors Δ_0 and Δ_1 are introduced to replace κ and K_1 used in References 13 and 14.

2.1 Definition of Wing Geometry

In what follows x and y are orthogonal co-ordinates with the x -axis lying along the main stream direction and the y -axis spanwise. Formally, the origin of the system is taken at the wing apex, but it is convenient to use x to denote distances measured from a local origin at the wing leading edge, non-dimensionalised in terms of the local chord c . Values of y are measured from the centre-line of the wing; since there is no one quantity that can be used throughout to make y non-dimensional, it is convenient to use y to denote a dimensional spanwise distance in this document.

Values of z are measured perpendicular to the chord line joining the wing leading and trailing edges at a given spanwise station, and are non-dimensionalised with the local chord. If the upper surface and lower surface of the aerofoil are defined by

$$z = z_u(x) \text{ and } z = z_l(x), \text{ respectively,}$$

then the thickness and camber ordinates are defined by

$$2z_t = z_u - z_l$$

$$2z_s = z_u + z_l$$

2.2 Perturbation Velocity Components

Throughout the Memorandum all perturbation velocity components are non-dimensionalised with respect to the free-stream velocity, V_0 .

3. THE BASIC FORMULA

The general expression for the velocity at any point on the surface of a swept wing is

$$\begin{aligned}
 \left(\frac{V}{V_0} \right)^2 = & \left\{ \cos \alpha_e \left[1 + \frac{\beta_n}{B_n} \left(\frac{K_{3a} \cos \Lambda_t S^{(1)}(x)}{(1 - M_0^2 \cos^2 \Lambda_t)^{1/2}} \right. \right. \right. \\
 & \left. \left. - \frac{K_{2a} \cos \tilde{\Lambda}_x f(\tilde{\Lambda}_{xa}) S^{(2)}(x)}{\left\{ 1 + (S^{(2)}(x)/B_n)^2 \right\}^{1/2} (1 - M_0^2 \cos^2 \tilde{\Lambda}_x)^{1/2}} \right] \left[D \left(\frac{u}{l} \right) \right]^{-1/2} \right. \\
 & \left. \pm \frac{\beta_n}{B_n} \left[\frac{\cos \alpha_e}{\{ (1 - M_0^2) (1 - M_0^2 \cos^2 \Lambda_m) \}^{1/2}} \frac{\cos \Lambda_m}{\cos (\lambda_a \Lambda_{ma})} \frac{\gamma_a}{2 V_0} \right. \right. \\
 & \left. \left. + H_a \sin \alpha_e \frac{\cos \Lambda_m}{\{ 1 - M_0^2 \cos^2 \Lambda_m \}^{1/2}} \left\{ 1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \left(\frac{1-x}{x} \right)^{n_a} \right\} \left[D \left(\frac{u}{l} \right) \right]^{-n_a^*} \right\}^2 \right. \\
 & \left. + \frac{\beta_n^2}{B_n^2} \left[\frac{\cos \alpha_e (1 - |\tilde{K}_{2a}|) \sin \Lambda_t S^{(1)}(x)}{(1 - M_0^2 \cos^2 \Lambda_t)^{1/2}} \left[D \left(\frac{u}{l} \right) \right]^{-1/2} \right. \right. \\
 & \left. \pm \cos \alpha_e \left\{ \frac{(1 - M_0^2 \cos^2 \Lambda_m) \cos^2 (\lambda_a \Lambda_{ma}) - (1 - M_0^2) \cos^2 \Lambda_m}{(1 - M_0^2) (1 - M_0^2 \cos^2 \Lambda_m) \cos^2 (\lambda_a \Lambda_{ma})} \right\}^{1/2} \frac{\gamma_a}{2 V_0} \right. \\
 & \left. \left. + H_a \sin \alpha_e \left\{ \frac{(1 - M_0^2 \cos^2 \Lambda_m) \cos^2 (\lambda_a \Lambda_{ma}) - (1 - M_0^2) \cos^2 \Lambda_m}{(1 - M_0^2 \cos^2 \Lambda_m)} \right\}^{1/2} \right. \right. \\
 & \left. \left. \left\{ 1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \left(\frac{1-x}{x} \right)^{n_a} \right\} \left[D \left(\frac{u}{l} \right) \right]^{-n_a^*} \right\}^2 \right. \\
 & \left. + (1 - \tilde{K}_{2a}^2) \sin^2 \tilde{\Lambda}_x \cos^2 \alpha_e \left\{ 1 - \left[D \left(\frac{u}{l} \right) \right]^{-1} \right\} \right\} \quad (3.1)
 \end{aligned}$$

where

$$\left[D \left(\frac{u}{l} \right) \right] = 1 + \left(\frac{S^{(2)}(x) \pm S^{(5)}(x)}{B_n \cos \Lambda_x^*} \right)^2$$

The symbol $\pm$ refers to the upper and lower surfaces, the positive sign being taken for the upper surface and the negative sign for the lower surface.

All the symbols are defined in Sections 4 to 6. Many of the symbols (for example K_2 , K_3) represent terms that are functions of spanwise station, y . To reduce the complexity of the mathematical text this dependence is not explicitly shown in Equation (3.1).

It is convenient to write Equation (3.1) in the form

$$\begin{aligned} \left(\frac{V}{V_0}\right)^2 = & \left\{ \cos \alpha_e \left[1 + \frac{\beta_n}{B_n} \left(u_{t1} + \frac{u_{t2}}{\left\{ 1 + (S^{(2)}(x)/B_n)^2 \right\}^{1/2}} \right) \right] \left[D\left(\frac{u}{l}\right) \right]^{-1/2} \right. \\ & \left. \pm \frac{\beta_n}{B_n} \left(u_c \cos \alpha_e + u_\alpha \left[1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \right] \right) \left[D\left(\frac{u}{l}\right) \right]^{-n_a^*} \right\}^2 \\ & + \left(\frac{\beta_n}{B_n} \right)^2 \left\{ v_t \cos \alpha_e \left[D\left(\frac{u}{l}\right) \right]^{-1/2} \pm \left[v_c \cos \alpha_e + v_\alpha \left[1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \right] \right] \left[D\left(\frac{u}{l}\right) \right]^{-n_a^*} \right\}^2 \\ & + (\Delta v)^2 \cos^2 \alpha_e \left\{ 1 - \left[D\left(\frac{u}{l}\right) \right]^{-1} \right\}. \end{aligned} \quad (3.2)$$

The first term in curly brackets is approximately the velocity component in the chordwise free-stream direction. The second term in curly brackets is approximately the velocity component in the spanwise direction. The last term in Equation (3.2) is a correction to the approximate spanwise velocity component.

The linear terms represented by u_{t1} , u_c and u_α are the contributions due to thickness, camber and incidence in the free-stream direction; v_t , v_c and v_α are the corresponding contributions in the spanwise direction. The term u_{t2} is also a thickness term, resulting from the kinked nature of the sources representing the thickness distribution at the centre.

The sum of these linear terms in Equation (3.1) or (3.2) then represents an approximation to the result given by calculation according to full linearised theory for a given wing. The various interpolation functions have been derived by comparison with known linearised theory results for particular wings, and with experimental results.

To improve on the linearised theory the factors

$$\left[D\left(\frac{u}{l}\right) \right]^{-1/2}, \left[D\left(\frac{u}{l}\right) \right]^{-n_a^*} \text{ and } \left\{ 1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \right\}$$

have been introduced into Equation (3.2). These factors approximate to the effect of calculating the velocities at the aerofoil surface rather than on the chord line, and serve to eliminate the infinite values

predicted by linearised theory at the leading edge. They are discussed in Section 4.6 and 4.4(iv).

The derivation of Equation (3.1) will be discussed in two parts – first in Section 4 the various terms are discussed for the case of incompressible flow, then in Sections 5 and 6 the modifications that are needed to deal with subsonic compressible flow are considered.

However, before proceeding to Section 4, it is appropriate to observe that since the form of Equation (3.1) was first devised there have been important developments in numerical methods for solving full linearised theory (References 51 to 55, 65 to 69) that enable values of the components u and v to be determined that are more accurate than the approximations developed in Sections 4, 5 and 6. These more accurate values may be used in an alternative form of Equation (3.2) namely

$$\begin{aligned} \left(\frac{V}{V_0}\right)^2 = & \left\{ \cos \alpha \left[1 + \frac{\beta_n}{B_n} (u_t \pm u_c) \right] \pm \frac{\beta_n}{B_n} u_\alpha \left(1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \right) \right\}^2 \left[D \left(\frac{u}{l} \right) \right]^{-1} \\ & + \left(\frac{\beta_n}{B_n} \right)^2 \left\{ \cos \alpha (v_t \pm v_c) \pm v_\alpha \left(1 + \frac{S^{(3)}(x)/B_n}{\cos \Lambda_t^*} \right) \right\}^2 \left[D \left(\frac{u}{l} \right) \right]^{-1} \\ & + (1 - \tilde{K}_{2a}^2) \sin^2 \Lambda_x \cos^2 \alpha \left\{ 1 - \left[D \left(\frac{u}{l} \right) \right]^{-1} \right\}, \end{aligned} \quad (3.3)$$

where (u_t, v_t) , (u_c, v_c) and (u_α, v_α) are the perturbation velocity components (in the plane $z = 0$) due to, respectively, thickness, camber and incidence (plus twist) calculated by *compressible* linearised theory.

A preference for this alternative form, where facilities permit its use, is *strongly recommended*, although (if desired) because the spanwise terms involving v_t , v_c and v_α are small (of second order), they might still be estimated using the approximations (Section 5.1)

$$v_t = -(1 - |\tilde{K}_{2a}|) \frac{S^{(1)}(x) \sin \Lambda_t}{(1 - M_0^2 \cos^2 \Lambda_t)^{1/2}} \quad (5.14)$$

$$v_c = -(1 - M_0^2)^{1/2} u_c \tan \Lambda_{va} \quad \text{cf Equations (5.16), (5.17)}$$

$$v_\alpha = -(1 - M_0^2)^{1/2} u_\alpha \tan \Lambda_{va} \quad \text{cf Equations (5.20),}$$

where $\cos \Lambda_{va} = \frac{\cos \Lambda_{ma}}{\cos (\lambda_a \Lambda_{ma})} \quad \text{cf Equation (4.40)}$

Alternatively, v_c and v_α may be derived from known values of u_c and u_α using Equation (A2.8), namely:

$$v_{c,\alpha}(x, y) = -u_{c,\alpha}(x, y) \tan \Lambda_x + \int_0^x \frac{\partial}{\partial y} [c(y) u_{c,\alpha}(x, y)] dx.$$

4. INTERPRETATION OF THE BASIC FORMULA FOR INCOMPRESSIBLE FLOW

When $M_0 = 0$, the basic equation, Equation (3.1), reduces to

$$\begin{aligned} \left(\frac{V}{V_0}\right)^2 = & \left\{ \cos \alpha_e \left[1 + K_3 \cos \Lambda_t S^{(1)}(x) - \frac{K_2 \cos(\tilde{\Lambda}_x) f(\tilde{\Lambda}_x) S^{(2)}(x)}{\left\{ 1 + [S^{(2)}(x)]^2 \right\}^{1/2}} \right] \left[D\left(\frac{u}{l}\right) \right]^{-1/2} \right. \\ & \left. \pm \left[\frac{\cos \alpha_e \cos \Lambda_m}{\cos(\lambda \Lambda_m)} \frac{\gamma}{2V_0} + H \sin \alpha_e \cos \Lambda_m \left[1 + \frac{S^{(3)}(x)}{\cos \Lambda_t^*} \right] \left(\frac{1-x}{x} \right)^n \left[D\left(\frac{u}{l}\right) \right]^{-n^*} \right]^2 \right. \\ & + \left\{ \cos \alpha_e (1 - |\tilde{K}_2|) \sin \Lambda_t S^{(1)}(x) \left[D\left(\frac{u}{l}\right) \right]^{-1/2} \pm \left[\cos \alpha_e \frac{\{ \cos^2(\lambda \Lambda_m) - \cos^2 \Lambda_m \}^{1/2}}{\cos(\lambda \Lambda_m)} \frac{\gamma}{2V_0} \right. \right. \\ & \left. \left. + H \sin \alpha_e \{ \cos^2(\lambda \Lambda_m) - \cos^2 \Lambda_m \}^{1/2} \left[1 + \frac{S^{(3)}(x)}{\cos \Lambda_t^*} \right] \left(\frac{1-x}{x} \right)^n \left[D\left(\frac{u}{l}\right) \right]^{-n^*} \right]^2 \right. \\ & \left. \left. + (1 - \tilde{K}_2^2) \sin^2 \tilde{\Lambda}_x \cos^2 \alpha_e \left\{ 1 - \left[D\left(\frac{u}{l}\right) \right]^{-1} \right\} \right. \right. \end{aligned} \quad (4.1)$$

The various terms in Equation (4.1) will now be discussed separately, assuming the form of Equation (3.2) *i.e.*

$$\begin{aligned} \left(\frac{V}{V_0}\right)^2 = & \left\{ \cos \alpha_e \left[1 + u_{t1} + \frac{u_{t2}}{\left\{ 1 + [S^{(2)}(x)]^2 \right\}^{1/2}} \right] \left[D\left(\frac{u}{l}\right) \right]^{-1/2} \right. \\ & \left. \pm \left[u_c \cos \alpha_e + u_\alpha \left(1 + \frac{S^{(3)}(x)}{\cos \Lambda_t^*} \right) \right] \left[D\left(\frac{u}{l}\right) \right]^{-n^*} \right]^2 \\ & + \left\{ v_t \cos \alpha_e \left[D\left(\frac{u}{l}\right) \right]^{-1/2} \pm \left[v_c \cos \alpha_e + v_\alpha \left(1 + \frac{S^{(3)}(x)}{\cos \Lambda_t^*} \right) \right] \left[D\left(\frac{u}{l}\right) \right]^{-n^*} \right]^2 \\ & \left. + (\Delta v)^2 \cos^2 \alpha_e \left\{ 1 - \left[D\left(\frac{u}{l}\right) \right]^{-1} \right\} \right. \end{aligned} \quad (4.2)$$

4.1 Sheared Wing Thickness Terms

These are:–

$$u_{t1} = K_3 S^{(1)}(x) \cos \Lambda_t, \quad (4.3)$$

$$v_t = -(1 - |\tilde{K}_2|) S^{(1)}(x) \sin \Lambda_t. \quad (4.4)$$

The significance of the various terms in these expressions is explained below.

- (i) $S^{(1)}(x)$ is the velocity increment according to linearised theory on a two-dimensional aerofoil with the specified thickness distribution, $z_t(x)$. It is given by the expression:

$$S^{(1)}(x) = \frac{1}{\pi} \int_0^1 \left(\frac{dz_t}{dx} \right)_{x=x'} \frac{dx'}{x-x'}. \quad (4.5)$$

This can conveniently be approximated by the formula given by Weber⁶:

$$S^{(1)}(x_v) = \sum_{\mu=1}^N s_{\mu v}^{(1)} z_t(x_\mu), \quad (4.6)$$

where the formula for the coefficients $s_{\mu v}^{(1)}$ and their values for $N = 16$ are given in Table A4.7. The specified stations x_μ , x_v are given by:

$$x_\mu = \frac{1}{2} \left(1 + \cos \frac{\mu\pi}{N} \right), \quad x_v = \frac{1}{2} \left(1 + \cos \frac{v\pi}{N} \right), \quad (4.7)$$

where N is usually taken equal to 16. Values of x_μ are tabulated in Table A4.1. Coefficients have been tabulated in Reference 6 for $N = 32$, and may be needed when the surface slope changes rapidly over part of the profile. To use these successfully very accurate and smooth values of z_t are required. Other methods of evaluating the integral in Equation (4.5) may be preferable in some circumstances (see for example Reference 18).

- (ii) Λ_t is the sheared wing thickness sweep. On an untapered wing of constant section this is simply the geometric wing sweep. On a tapered wing with constant section shape (with either constant or varying thickness-chord ratio), it was originally proposed that Λ_t should be a function of x and taken as the sweep of a line joining points of equal x . More recently comparisons⁵⁰ with results calculated from full linear theory⁵² have suggested that greater accuracy near the leading edge will result from modifying this proposal by taking Λ_t equal to $\Lambda_{t\max}$, the angle of sweep of the maximum thickness line*.

On a wing with rapidly varying section such that near the root $\Lambda_{t\max} > \Lambda_0$, acceptable results at the root and inboard stations (particularly near $x_{t\max}$) have been obtained by taking Λ_t varying linearly between the leading-edge sweep and the sweep of the maximum thickness line for the portion of wing ahead of the maximum thickness position.

* For that part of the wing free from root or tip effects (*i.e.* near $y/s = 1/2$) rather better accuracy can be obtained⁵⁰ by taking $\Lambda_t = \Lambda_m$ (*i.e.* sweep of the mid-chord line). However, this definition was found to be unsatisfactory near the root when seeking a compromise definition suitable for any spanwise station on any wing, including those with the thickness form varying across the span.

Therefore the following definition of Λ_t , which also complies with a requirement that Λ_t should be continuous across the span, is now recommended.

$$\Lambda_t = \Lambda_{t_{\max}}, \text{ except that if } \Lambda_{t_{\max}} > \Lambda_0 \text{ then}$$

$$\tan \Lambda_t = \tan \Lambda_0 + \frac{x}{x_{t_{\max}}} \tan (\Lambda_{t_{\max}} - \tan \Lambda_0)$$

for $0 \leq x \leq x_{t_{\max}}.$

However, it must be stressed that for wings having a complicated variation of spanwise section this recommendation is based on only very sparse information and attention is drawn to the remarks in Section 4.2(iii) on the choice of Λ_t and Λ_x .

(iii) The function

$$K_3 = 1 + \frac{\Delta_s(x_{t_{\max}})}{S^{(1)}(x_{t_{\max}}) \cos \Lambda_{t_{\max}}} \quad (4.8)$$

is the ratio of the supersonic velocity at the maximum thickness position of any spanwise station of a swept wing to the supersonic velocity at the maximum thickness of the corresponding infinite sheared wing. It is a function of y and can be applied with reasonable accuracy to the sheared-wing supersonic velocities calculated for any chordwise position, as shown in Reference 14, even though it is derived only for the maximum thickness position.

The factor K_3 takes into account the effect of finite aspect ratio, planform taper and thickness taper on the sheared wing supersonic velocities due to thickness in Equation (4.1), and is determined from the geometric characteristics of the wing by using Equation (4.8) in conjunction with the following expression for $\Delta_s(x_{t_{\max}})$.

$$\begin{aligned} \Delta_s(x_{t_{\max}}) = & \left[\Delta_0 \left(\frac{s-y}{c_T} \right) + \Delta_0 \left(\frac{s+y}{c_T} \right) \right] \frac{t_T}{c_T} \cos \Lambda_{t_{\max}} \\ & + \left[0.375 + 1.25(1 - X_{t_{\max}}) \right] \left[\Delta_1 \left(\frac{s-y}{c_T} \right) + \Delta_1 \left(\frac{s+y}{c_T} \right) \right] \frac{t_T}{c_T} \\ & + \left[0.375 + 1.25 X_{t_{\max}} \right] \Delta_1 \left(\frac{y}{c_R} \right) \frac{t_R}{c_R} \\ & + 2 \Delta_3 \left(\frac{y}{c_R} \right) \tan \delta_R + \left[\Delta_3 \left(\frac{s-y}{c_T} \right) + \Delta_3 \left(\frac{s+y}{c_T} \right) \right] \tan \delta_T + \sum_{\text{Thickness cranks}} \Delta_3 \left(\frac{y-y_c}{c_c} \right) \tan \delta \\ & + \Delta_4 \left(\frac{y}{s'} \right) \left| \tan \Lambda_0 - \tan \Lambda_1 \right| \frac{t_R}{c_R} \\ & + 2 \Delta_5 \left(\frac{y}{c_R} \right) \left[\tan \Lambda_0 - \tan \Lambda_1 \right] \frac{t_R}{c_R} + \left[\Delta_5 \left(\frac{s-y}{c_T} \right) + \Delta_5 \left(\frac{s+y}{c_T} \right) \right] \left[\tan \Lambda_0 - \tan \Lambda_1 \right] \frac{t_T}{c_T}, \quad (4.9) \end{aligned}$$

where $X_{t_{\max}} = x_{t_{\max}}$ for sweptback wings

and $= 1 - x_{t_{\max}}$ for sweptforward wings.

This expression is derived for an isolated wing; it can be applied to a wing-fuselage combination if full reflection of the wing in the body side is assumed.

The functions Δ_0 to Δ_5 in Equation (4.9) are defined as follows.

(a) Δ_0 is plotted in Figure 3 as a function of the spanwise distances of a point from the tips of a wing in terms of the tip chord. The product $[\Delta_0(t_T/c_T)]$ is the reduction in the velocity near the tip of a semi-infinite unswept wing and replaces the velocity ratio factor κ used in a number of earlier reports. The relationship between κ and Δ_0 is:-

$$S^{(1)}(x_{t\max}) + \Delta_0 \frac{t_T}{c_T} = \kappa S^{(1)}(x_{t\max}).$$

Δ_0 has the advantage of being theoretically independent of section shape except in a region very close to the tip whereas κ varies with section shape (according to linearised theory) for all spanwise stations other than the tip itself.

Experimental support for the applicability of Δ_0 to rectangular wings is provided, amongst others, by Figure 5 of Reference 19 and Figure 6 of Reference 49.

For a wing with large end plates it can be presumed that $\Delta_0 = 0$ at the tips; other bodies, such as tip tanks, which would act as partial end plates would also be expected to reduce the numerical value of Δ_0 near the tips.

(b) Δ_1 is plotted in Figure 4 as a function of the spanwise distances of a point from the root and tips of a wing in terms of the root and tip chords. The term $\{[0.375 + 1.25X_{t\max}]\Delta_1(t/c)\}$ is the increase in supervelocity at the maximum thickness position near the tip of a semi-infinite sheared wing, relative to that obtained by applying a factor $\cos \Lambda_{t\max}$ to the supervelocities at the maximum thickness of the corresponding unswept semi-infinite wing, *i.e.* Δ_1 allows for the effect of sweep on Δ_0 . The expression $[0.375 + 1.25X_{t\max}]$ is an empirical factor allowing for the effects of change of section shape.

The total supervelocity at the maximum thickness of a semi-infinite sheared wing can be expressed as:-

$$u_t = S^{(1)}(X_{t\max}) \cos \Lambda + \Delta_0 \left(\frac{t}{c}\right) \cos \Lambda + [0.375 + 1.25X_{t\max}]\Delta_1 \left(\frac{t}{c}\right), \quad (4.10)$$

$X_{t\max}$ representing $x_{t\max}$ for an upstream tip and $1 - x_{t\max}$ for a downstream tip.

An infinite sweptback wing is formed by combining two semi-infinite wings having upstream tips, the influence of one wing on the velocities at the maximum thickness of the other being approximately $(-\Delta_0(t/c) \cos \Lambda)$. Hence the supervelocity at the maximum thickness of an infinite swept wing can be expressed as:-

$$u_t = S^{(1)}(x_{t\max}) \cos \Lambda_{t\max} + [0.375 + 1.25x_{t\max}]\Delta_1 \frac{t}{c}. \quad (4.11)$$

Substituting $\Delta_1 = \frac{(K_1 - 1)S^{(1)}(x_{t\max}) \cos \Lambda_{t\max}}{(0.375 + 1.25x_{t\max})t/c}$ in the above equation gives:-

$$u_t = K_1 S^{(1)}(x_{t\max}) \cos \Lambda_{t\max}, \quad (4.12)$$

K_1 being the factor first suggested by Haines¹³ to relate the supervelocities on an infinite swept wing to those on the corresponding infinite sheared wing.

The main advantage of using Δ_1 rather than K_1 lies in the fact that Δ_1 is independent of section shape (at

least for conventional sections) and hence a single set of values, obtained for the simple biconvex parabolic arc aerofoil section, is adequate. The value of Δ_1 is also independent of the sign of the angle of sweep, whereas different values of K_1 are required for sweptback and sweptforward wings.

Values of K_1 (and hence Δ_1) may be computed for wings with unconventional aerofoil shapes using the equation

$$\pi(K_1 - 1)S^{(1)}(x) = \int_0^1 \left(\frac{dz}{dx} \right)_{x'} \left\{ \frac{\frac{y^2 \tan \Lambda_t}{\cos \Lambda_t} - y \tan \Lambda_t \left[\frac{y^2}{\cos^2 \Lambda_t} + 2y \tan \Lambda_t (x - x') + (x - x')^2 \right]^{1/2}}{\left[2y \tan \Lambda_t (x - x') + (x - x')^2 \right] \left[\frac{y^2}{\cos^2 \Lambda_t} + 2y \tan \Lambda_t (x - x') + (x - x')^2 \right]^{1/2}} \right\} dx. \quad (4.13)$$

(c) Δ_3 is plotted in Figure 5 a function of the spanwise distances of a point from the root, the tips or any chordwise surface crank, in terms of the chord at the surface discontinuity. The effects of such a crank spread equally on either side of the discontinuity, so Δ_3 is a symmetric function.

$\Delta_3 \tan \delta$ is the velocity increment at the maximum thickness position associated with a change, $\tan \delta$, in the spanwise slope of the wing surface. If

$$\tan \theta = \frac{d}{dy}(cz_{\max})$$

is defined to be the spanwise slope at the wing maximum thickness, then δ is the change in θ across any discontinuity in slope, reckoned positive when the crank produces a concavity in the frontal elevation of the wing. For simplicity, the contributions due to the wing root and tip discontinuities have been given separately in Equation (4.9). At the wing tip, $\delta_T = -\theta$; at the root of a wing without fuselage, $\delta_R = \theta$.

This term was derived from calculations by linearised theory on uncambered wings with spanwise variations in thickness, and the velocity increments are associated with the source distributions representing the thickness variations. However, the term is physically associated with a discrete change in the spanwise slope of the wing surface, and it is convenient (and physically plausible) to assume that the same velocity increment arises from a similar discrete change of slope due to dihedral or wing twist for example. (This assumption is likely to be valid only for small values of θ .) Consequently, the definition of the surface slope, $\tan \theta$, is given above in terms of the ordinate $z_{\max}$ rather than the thickness ordinate $z_{t\max}$: in other words θ is a measure of the slope of the wing in frontal elevation. At the junction of a wing and fuselage, where ψ is the included angle between the wing surface and the fuselage side (at the maximum thickness position of the wing), it is suggested that $\delta_R = 90^\circ - \psi$ is used as a crude approximation, provided this value is not large.

On a wing that tapers smoothly in thickness, it is suggested that the maximum thickness line should be approximated by a series of tangents, ensuring that any physical discontinuities in spanwise slope are correctly represented. This may introduce some additional, fictitious, slope discontinuities, and velocities due to these should not be calculated close to the discontinuities.

(d) Δ_4 is plotted in Figure 6 as a function of the spanwise distance of a point from the wing root, in

terms of the distance from the root of the point of intersection of the projected leading and trailing edges.

The term $\left\{ \Delta_4 \frac{t_R}{c_R} \left| \tan(\Lambda_0 - \Lambda_1) \right| \right\}$

is the increase in velocity on a tapered wing of constant thickness-chord ratio relative to that on the corresponding infinite sheared wing, after allowing for the effects of discontinuities in planform and thickness at the centre and tips, *i.e.*,

$$S^{(1)}(x_{tmax}) \cos \Lambda_{tmax} + \Delta_4 \frac{t_R}{c_R} \left| \tan \Lambda_0 - \tan \Lambda_1 \right|$$

corresponds, for a tapered panel, to the sheared-wing value for an untapered wing. The parameter Δ_4 is independent of section shape.

(e) Δ_5 is plotted in Figure 7 as a function of the spanwise distance of a point from the root and tips of a wing in terms of the root and tip chords.

The term $\left\{ 2\Delta_5 \frac{t_R}{c_R} [\tan \Lambda_0 - \tan \Lambda_1] \right\}$

is the increase in velocity at x_{tmax} for a tapered wing, caused by the discontinuity in planform taper at the root.

The term $\left\{ \Delta_5 \frac{t_T}{c_T} [\tan \Lambda_1 - \tan \Lambda_0] \right\}$

is the corresponding value caused by the discontinuity in planform taper at the tip and Δ_5 is again independent of section shape.

(f) It should be noted that for untapered wings, only the terms including the factors Δ_0 and Δ_1 are required in Equation (4.9) while for large angles of sweep ($\Lambda_t > 50^\circ$) the values of Δ_3 and Δ_5 may in general be neglected.

(g) In a digital computer program it may not be convenient to express the parameters Δ_0 to Δ_5 in tabular form.

Formulas for Δ_0 and Δ_1 are available; the empirically derived formula for Δ_1 immediately precedes Equation (4.12). A disadvantage of this formula lies in the fact that Δ_1 is expressed as a function of the factor K_1 which is strongly dependent on both section shape and sweep angle. The parameter Δ_1 , however, is independent of section shape (at least for conventional sections) and of the sign of the sweep angle. Thus an alternative approximate formula for Δ_1 has been developed and may be found to be more convenient in practice. Also, approximate formulas for Δ_3 to Δ_5 have been developed by Wooller at the Royal Aircraft Establishment, Farnborough. The formulas are given below.

1. Formula for Δ_0 .

$$\Delta_0 = -\frac{2}{\pi} \left\{ 1 - \frac{y}{c} \log_e \left(\frac{\sqrt{1 + 4\left(\frac{y}{c}\right)^2} + 1}{\sqrt{1 + 4\left(\frac{y}{c}\right)^2} - 1} \right) \right\},$$

where $\frac{y}{c}$ is to be interpreted as $\frac{s_a - y_a}{c_T}$ or $\frac{s_a + y_a}{c_T}$.

2. Formula for Δ_1 .

$$\Delta_1 = k \sin^q \left[180 e^{-p \frac{y}{c}} \right] \text{ for } 14.4^\circ \leq |\Lambda_{t\max}| \leq 70^\circ$$

$$\Delta_1 = 0 \quad \text{for } |\Lambda_{t\max}| \leq 14.4^\circ,$$

where

$$k = -0.0541 + 0.00375 |\Lambda_{t\max}|,$$

$$q = -0.2075 + 0.02415 |\Lambda_{t\max}| \text{ for } |\Lambda_{t\max}| \geq 50^\circ$$

$$q = 1 \quad \text{for } |\Lambda_{t\max}| < 50^\circ,$$

$$p = 15.95 - 1.385 |\Lambda_{t\max}| + 5.30 \times 10^{-2} \Lambda_{t\max}^2 - 0.854 \times 10^{-3} |\Lambda_{t\max}|^3 + 0.0508 \times 10^{-4} \Lambda_{t\max}^4,$$

$\frac{y}{c}$ is to be interpreted as $\frac{y_a}{c_R}$, $\frac{s_a - y_a}{c_T}$ or $\frac{s_a + y_a}{c_T}$, and $180 e^{-p \frac{y}{c}}$ is measured in degrees.

3. Formula for Δ_3 .

$$\Delta_3 = \frac{0.088 - 0.04 \left(\frac{y}{c} + 0.011 |\Lambda_{t\max}| \right)}{0.291 + \left(\frac{y}{c} + 0.011 |\Lambda_{t\max}| \right)} - \frac{0.015 |\Lambda_{t\max}|}{20}$$

$$\text{for } \frac{y}{c} < \frac{88 - 0.658 |\Lambda_{t\max}| - 0.00825 \Lambda_{t\max}^2}{40 + 0.75 |\Lambda_{t\max}|}$$

$$\Delta_3 = 0$$

$$\text{for } \frac{y}{c} \geq \frac{88 - 0.658 |\Lambda_{t\max}| - 0.00825 \Lambda_{t\max}^2}{40 + 0.75 |\Lambda_{t\max}|},$$