

ESTIMATION OF THE LOCAL BUCKLING STRESS UNDER BIAXIAL COMPRESSION OF AN ISOTROPIC SKIN WITH FIBRE REINFORCED INTEGRAL UNFLANGED STIFFENERS

1. INTRODUCTION

This Item illustrates by reference to a specific example the use of Item No. 73007 for estimating the local buckling* stress under biaxial loading of an isotropic skin stiffened by orthotropic unflanged stringers.

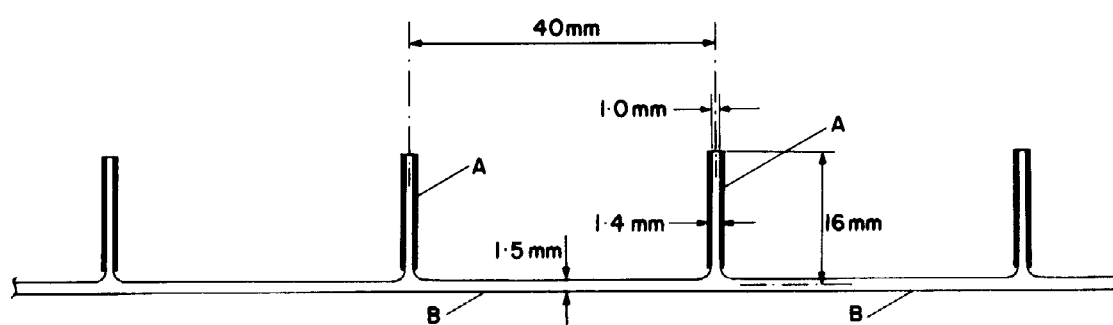
2. NOTATION

The notation used is the same as that of Item Nos. Struct 02.01.28, 73007 and 94003. The suffixes listed below are added to facilitate description of the calculations.

px	refers to panel and to x -direction
skx, sky	refers to skin and to x - and y -directions respectively
stx	refers to stiffener and to x -direction

It should be noted that the sign convention of Item Nos. Struct 02.01.28 and 73007 is compressive stresses positive whereas the sign convention of Item No. 94003 is tensile stresses positive.

3. EXAMPLE



The panel shown above is subjected to a transverse compressive stress of 20 N/mm^2 and to a longitudinal loading such that the stringers remain unbent in their own plane. It is required to find the longitudinal load at which local buckling will occur. The material of the plate and stiffeners is aluminium alloy which has a Young's modulus of $73\,000 \text{ N/mm}^2$, a shear modulus of $28\,100 \text{ N/mm}^2$ and a Poisson's ratio of 0.3. The stiffeners are reinforced by layers (5 each side) of 0.04 mm thick carbon fibre reinforced plastic. The properties of an individual layer of reinforcement under direct in-plane stresses are: Young's modulus of

* Local buckling is described and discussed in Item No. 72012 "Information on the use of Data Items on the buckling of plates and compression panels manufactured from isotropic materials".

207 000 N/mm² along the fibres and 7600 N/mm² across the fibres, shear modulus parallel and normal to the fibres of 5000 N/mm² and Poisson's ratio under load along the fibres of 0.3. The fibres are arranged, starting with the innermost layer and going outwards, at angles of 0°, -15°, 0°, +15°, 0° to the longitudinal axis of the stiffeners.

4. DESCRIPTION OF THE CALCULATION PROCEDURE

4.1 General

The basic method of determining the local buckling stress of panels with unflanged integral stiffeners is described in Section 4.1 of Item No. Struct 02.01.28 and is shown to be dependent upon the relationship

$$T_{\theta}/\theta = A_1 + 2B_2 \quad (4.1)$$

between the unbalanced couple T_{θ} at the junctions of component flats and the angular rotations θ at the junctions. The stiffness A_1 denotes a value of μ_1 for A and B_2 a value of μ_2 for B . The values of μ_1 and μ_2 depend upon the buckle half wavelength, the longitudinal and transverse stresses and the plate stiffnesses.

Under the action of the longitudinal compressive stress local buckling occurs when the value of T_{θ}/θ in Equation (4.1) is zero. The calculation procedure consists of a search for the wavelength that yields the minimum stress at $T_{\theta}/\theta = 0$. This minimum stress is then the local buckling stress. Notes on the initial choice of the stress and wavelength appear in Item No. Struct 02.01.28 and in Section 5.6 of this Item.

4.2 Particular Calculations for the Example

The reinforced stiffener may be represented as a single orthotropic plate. The Example of Item No. 94003 calculates the in-plane and flexural stiffness matrices **A** and **D** respectively.

Owing to the different in-plane stiffnesses of the skin and the stringer and the transverse load in the skin the stresses acting on these members will not, in general, be equal for a given panel strain. For this reason it is simpler in this particular example to investigate a range of panel loads in terms of the stresses acting in the component flats.

5. THE CALCULATION

5.1 Data Calculated in Item No. 94003

From the example of Item No. 94003 for the stiffener

$$A = \begin{bmatrix} 159.2 & 26.92 & 0 \\ 26.92 & 83.48 & 0 \\ 0 & 0 & 32 \end{bmatrix} \times 10^3 \text{ N/mm}, A^{-1} = \begin{bmatrix} 0.00664 & -0.00214 & 0 \\ -0.00214 & 0.0127 & 0 \\ 0 & 0 & 0.0312 \end{bmatrix} \times 10^{-3} \text{ mm/N}$$

$$\text{and } D = \begin{bmatrix} 35.4 & 3.03 & 0.351 \\ 3.03 & 7.87 & 0.034 \\ 0.351 & 0.034 & 3.76 \end{bmatrix} \times 10^3 \text{ N mm.}$$

The terms D_{13} , D_{23} , D_{31} and D_{32} of the matrix $\mathbf{D}$ defined in Item No. 73007 are zero. In the above $\mathbf{D}$ matrix these terms are sufficiently small in relation to the other terms of $\mathbf{D}$ for Item No. 73007 to be applicable.

5.2 Distribution of Applied Loads Between Skin and Stiffeners

For a given panel load intensity, N_{px} , the stiffener and skin loads must sum to N_{px} and their strains must be equal. The stiffener strain is obtained from the relationship $\mathbf{e} = \mathbf{A}^{-1}\mathbf{N}$ (see Equation (4.6) and Section 6.3.1 of Item No. 94003). Since the stiffener is loaded only in the x -direction by N_{stx} the only relevant term of $\mathbf{A}^{-1}$ is the leading term of 0.00664×10^{-3} mm/N so that $e_{stx} = 0.00664 \times 10^{-3} \times N_{stx}$. The skin strain is given by

$$e_{skx} = (N_{skx} - \sigma N_{sky})/Et.$$

Equating the skin and stiffener strains and putting $N_{stx}h + N_{skx}b = N_{px}b$ yields the skin and stiffener loads for a given N_{px} . Hence, equating the strains gives

$$\begin{aligned} N_{stx} &= (N_{skx} - 0.3(-20) \times 1.5)/(0.00664 \times 10^{-3} \times 73000 \times 1.5) \\ &= (N_{skx} + 9)/0.727 \text{ N/mm}. \end{aligned}$$

Then, summing the load contributions gives

$$(N_{skx} + 9) \times 16/0.727 + N_{skx} \times 40 = N_{px} \times 40$$

from which $N_{skx} = (N_{px} - 4.95)/1.55 \text{ N/mm}$

giving $f_{skx} = (N_{px} - 4.95)/2.32 \text{ N/mm}^2$.

Similarly, $f_{stx} = (0.887N_{px} + 7.98)/1.4 \text{ N/mm}^2$.

5.3 Calculation of Expressions for α and β for the Skin

The skin is isotropic so that, using Section 1.2 of Item No. 73007,

$$f_I = \frac{\pi^2 \times 73000}{3(1 - 0.3^2)} \times \left(\frac{1.5}{40}\right)^2 = 371 \text{ N/mm}^2$$

and, noting that compressive stresses are positive when using Item No. 73007,

$$\beta_I = 1 - 2(+20/371)/(b/\lambda)^2 = 1 - 0.108/(b/\lambda)^2.$$

The stress parameter α_I is given by

$$\alpha_I = \frac{f_{skx}}{371\beta_I} + \frac{1}{4}\left(\frac{b}{\lambda}\right)^2 \left\{ \beta_I - \frac{1}{\beta_I} \right\}.$$