



ESDU 73016

Issued July 1973

Supersedes Aero

W.02.04.08

Aero W.02.04.10

The mean skin friction coefficient for a rough flat plate with a turbulent two-dimensional boundary layer in compressible adiabatic flow, with application to wedges, cylinders and cones

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU Aero W.02.04.08 and Aero W.02.04.10, was monitored and guided by the Aerodynamics Committee. This Committee first met in 1942 and now has the following membership:

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THE MEAN SKIN FRICTION COEFFICIENT FOR A ROUGH FLAT PLATE WITH A TURBULENT TWO-DIMENSIONAL BOUNDARY LAYER IN COMPRESSIBLE ADIABATIC FLOW, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

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THE MEAN SKIN FRICTION COEFFICIENT FOR A ROUGH FLAT PLATE WITH A TURBULENT TWO-DIMENSIONAL BOUNDARY LAYER IN COMPRESSIBLE ADIABATIC FLOW, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

1. NOTATION AND UNITS

		SI	British
C_F	mean skin friction coefficient, $\frac{1}{l} \int_0^l C_f dx$		
C_f	local skin friction coefficient, $\tau_w / \frac{1}{2} \rho_1 u_1^2$		
k	equivalent sand roughness height (see Table 11.1)	m	ft
l	length of surface under consideration, measured along surface from virtual origin of turbulent boundary layer	m	ft
M	Mach number		
R_k	Reynolds number based on k , $u_1 k / \nu_1$		
R_l	Reynolds number based on l , $u_1 l / \nu_1$		
$(R_l)_{crit}$	maximum value of R_l for zero increase in C_F due to roughness		
u	velocity	m/s	ft/s
u_r	friction velocity, $(\tau_w / \rho_w)^{1/2}$	m/s	ft/s
x	distance measured along surface from virtual origin of turbulent boundary layer	m	ft
ν	kinematic viscosity	m ² /s	ft ² /s
ρ	density	kg/m ³	slug/ft ³
τ_w	wall shear stress	N/m ²	lbf/ft ²
Suffixes			
c	relating to cone flow		
w	relating to conditions at wall		
1	relating to condition at edge of boundary layer		

2. INTRODUCTION

This Item provides data for the mean skin friction coefficient of a perfectly insulated (adiabatic flow) flat plate with an aerodynamically smooth or, in the more general case, a uniformly rough surface at zero incidence in air at Mach numbers from 0 to 5. The data are derived with the assumption that fully-developed turbulent flow exists from the leading edge of the plate, which is therefore taken as the virtual origin.

The application of the Item is extended to cover the two-dimensional supersonic flow about sharp wedges, the axisymmetric flow over cylinders and the axisymmetric supersonic flow over pointed cones by means of factors applied to the flat plate data.

3. FLOW DESCRIPTION

The flow over a flat plate with a granular-type roughness distributed over the surface at the maximum density (*i.e.* with the grains as closely packed as possible) can be divided into three separate phases:

- (i) aerodynamically smooth, in which the height of the roughness particles is very much less than the thickness of the laminar sub-layer, so that the roughness has no effect on the skin friction coefficient, which has the smooth plate value,
- (ii) intermediate, in which the heights of the roughness particles are the same order as the thickness of the laminar sub-layer and in which the skin friction coefficient is dependent on R_l and l/k , and
- (iii) fully-rough flow, in which the heights of the roughness particles are very much larger than the thickness of the laminar sub-layer and in which the skin friction coefficient is independent of R_l and is a function of l/k only.

Near to the leading-edge of a flat plate with uniformly distributed roughness where the boundary layer is thin the roughness particles will be large compared with the thickness of the laminar sub-layer and the flow will be in the fully-rough phase. For given flow conditions R_k will be constant and as l , and thus R_l , increases the flow will pass into the intermediate phase and thence, if the plate is long enough, into the aerodynamically smooth phase. In practice, the complete change of phase from fully-rough to aerodynamically smooth extends over a very large R_l range; for example, never more than one change of phase occurs over the range $10^5 \leq R_l \leq 10^9$.

4. FLAT PLATE DATA*

Figure 1 presents data for $(R_l)_{crit}$ as a function of l/k and M_1 . These data were obtained from the expression

$$(R_l)_{crit} = \frac{l}{k} \left(\frac{u_r k}{\nu} \right)_w \left(\frac{C_f}{2} \right)^{-1/2} (1 + 0.178 M_1^2)^{1.26} \quad (4.1)$$

in which $(u_r k / \nu)_w$ was taken as 4.0 (see Derivation 1) and C_f is the smooth plate value of local skin friction coefficient taken from Item No. 68020.

Figures 2 to 9 each present, for a given Mach number in the range $0 \leq M_1 \leq 5$, data for the variation of

* Throughout this Item data are presented with reference to values of the flow parameters at the edge of the boundary layer. It will be realised that for the flat plate at zero incidence the flow properties at the edge of the boundary layer can be considered to be identical to those of the uniform flow upstream when the effects of induced pressure gradients are small, as assumed here.

mean skin friction coefficient with R_l and l/k . The data for $M_1 = 0$ were obtained via the method of Derivation 6 with the values at very low R_k modified by at the most two per cent to blend in to the smooth plate data derived from the Spalding and Chi method and given in Item No. 68020. The data for compressible flow were derived from those at $M_1 = 0$ by means of transformation functions described in Derivation 6 but related to the method of Spalding and Chi.

The chain-dotted line in each of Figures 2 to 9 indicates the boundary between the fully-rough and intermediate flow phases and was obtained by means of an expression identical to that of Equation (4.1) in which $(u_r k / \nu)_w$ was taken as 100 (see Derivation 1). In this context C_f is the fully-rough flow value of local skin friction coefficient derived by a process similar to that previously described for mean skin friction coefficient.

The lowest curve in each of Figures 2 to 9 corresponds to aerodynamically smooth flow and, as such, agrees with the data given in Item No. 68020.

Although the data have been presented for Reynolds numbers down to $R_l = 10^5$ it should be pointed out that there is some evidence (Derivation 3) to suggest that a turbulent boundary layer cannot exist at Reynolds numbers based on momentum thickness less than a value of about 320, for incompressible flow over a smooth flat plate at least. The hatched boundaries on Figures 4 to 9 were drawn on the assumption that this value holds for a flat plate with uniformly distributed roughness and they were derived by the application of the previously mentioned compressibility parameters to the boundary obtained for $M_1 = 0$.

In order to interpolate in any of Figures 2 to 9 for values of l/k intermediate to those given, it is useful to plot on the Figure a curve corresponding to an R_k value calculated from the required values of R_l and l/k via the relation

$$R_k = \frac{R_l}{l/k}. \quad (4.2)$$

The curve corresponding to this fixed value of R_k is drawn for a range of l/k values to straddle that required and the value of C_F can then be read off from this curve at the required value of R_l .

5. EQUIVALENT SAND ROUGHNESS HEIGHT

The values of mean skin friction coefficient given in this Item are based on the sand-type roughness used by Nikuradse in his pipe-flow experiments (Derivation 1). In those experiments the sand was distributed at maximum density, *i.e.* the individual grains were attached to the surface as close together as possible. In most practical applications the roughness density is less than this and the roughness itself is not granular. It is convenient to classify a particular type of uniformly distributed surface roughness in terms of its equivalent sand roughness height, which is defined as that value of k required to produce the same mean skin friction coefficient as the actual roughness when used in conjunction with the data of this Item. Table 11.1, taken from Derivation 5, provides equivalent sand roughness heights for a number of common surfaces.

It is pointed out in Derivation 5 that experiments have shown that when the surface roughness is non-uniform, as it is likely to be in practice, the dip in the curve of C_F against R_l (when l/k is fixed) in the intermediate phase does not occur, and the fully-rough flow value of C_F is applicable as soon as the flow departs from aerodynamically smooth.

6. SUPERSONIC WEDGE FLOW

Since the supersonic flow over a sharp wedge is two-dimensional and since the flow properties between the shock and the boundary layer are constant, the data given in the Figures may be applied directly to this case, provided that the flow properties downstream of the shock are used in the calculation.

7. AXISYMMETRIC FLOW

7.1 Parallel Circular Cylinder

Provided that the displacement thickness is very much less than the radius (say one per cent) then flat plate data are expected to apply directly. For checking purposes the displacement thickness may be assumed to be independent of roughness height, so that the data of Item No. 68020 may be used.

7.2 Right-circular Cone in Supersonic Flow

As an approximation it will be assumed that the relationship for aerodynamically smooth cones given in Item No. 68020 applies to cones with evenly distributed granular roughness, *i.e.*

$$(C_F)_c = 1.05 C_F . \quad (7.1)$$

Equation (7.1) applies when the flow conditions at the edge of the boundary layer on the flat plate and the cone are identical (*i.e.* $u_1 = (u_1)_c$, $v_1 = (v_1)_c$, $M_1 = (M_1)_c$) and when the values of l and k are identical.

Equation (7.1) is only valid for displacement thicknesses of the order of one per cent or less of the local radius. For checking purposes the displacement thickness may be assumed to be independent of roughness height, so that the smooth flat plate data of Item No. 68020 may be used together with the displacement thickness relationship given in that Item.

8. COMPARISON WITH EXPERIMENTAL DATA

The available experimental data for the mean skin friction of the aerodynamically smooth flat plate turbulent boundary layer are correlated by the method of this Item with an r.m.s. error of 2 to 3 per cent for incompressible flow and 9 per cent for compressible adiabatic flow (see Item No. 68020).

Experimental data for surfaces with evenly distributed granular-type roughness are very few, there being only two sets, the one obtained from flat plates (Derivation 2) and the other from tests on ogive-cylinders (Derivation 4). The roughness used in these tests was such that the flow was either in the fully-rough phase or bordering on it, in the majority of cases. The method of this Item generally underestimates the data of Derivation 2 (covering the Mach number range $1.62 \leq M_1 \leq 2.23$) by up to about 10 per cent and overestimates the data of Derivation 4 (covering the Mach number range $0.7 \leq M_1 \leq 4.5$) by up to about 20 per cent. The data of Derivation 2 are, it is felt, likely to be the more reliable of the two sets owing to the problems inherent in the test technique of Derivation 4.