



ESDU 73017

Issued August 1973
With Amendments A to C
January 2013
Supersedes ESDU Aero
00.02.01, 02, 03, 04

Reynolds number, speed of sound, dynamic viscosity, kinetic pressure and total pressure coefficient in air



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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU Aero 00.02.01, Aero 00.02.02, Aero S.00.02.03 and Aero 00.03.04, was monitored and guided by the Aerodynamics Committee. At the time of publication this Committee, which first met in 1942, had the following membership:

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REYNOLDS NUMBER, SPEED OF SOUND, DYNAMIC VISCOSITY, KINETIC PRESSURE AND TOTAL PRESSURE COEFFICIENT IN AIR

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REYNOLDS NUMBER, SPEED OF SOUND, DYNAMIC VISCOSITY, KINETIC PRESSURE AND TOTAL PRESSURE COEFFICIENT IN AIR

1. NOTATION AND UNITS

		SI	British
A_0 to A_4	coefficients in Equation (3.1)		
a	speed of sound, see Section 3 and Figure 3	m/s	ft/s
C_{pH}	total (or stagnation) pressure coefficient, Equations (5.2) and (5.3), see Figure 8		
H	geopotential height*, see ESDU 77021	m	ft
l	characteristic length in Reynolds number R	m	ft
M	Mach number, V/a		
p	static pressure	N/m ²	lbf/ft ²
p_H	total (reservoir or stagnation) pressure in Equation (5.2)	N/m ²	lbf/ft ²
p_0	standard (sea level) pressure, 101.325×10^3 N/m ² or 2.11622×10^3 lbf/ft ² , 1 atmosphere (1 atm)	N/m ²	lbf/ft ²
q	kinetic pressure, Equation (5.1), see Section 5 and Figures 6 and 7	N/m ²	lbf/ft ²
R	Reynolds number, Equation (2.1), see Sections 2 and 4 and Figures 1, 2 and 5		
S	constant in Sutherland's equation, Equation (3.2), taken as 110.4 K	K	K
s	parameter in Equation (3.1)		
T	static temperature	K	K
T_H	total (reservoir or stagnation) temperature	K	K
T_r	reference value for T in Equation (3.2)	K	K
V	velocity	m/s	ft/s
γ	ratio of specific heat capacities, taken as 1.4 for air		

* The geopotential height is the height, in a uniform gravitational field, which would give the same potential energy as the point under consideration in the actual (variable) gravity field.

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μ	dynamic viscosity of dry air, see Section 3 and Figure 4	kg/m s	slug/ft s
μ_r	value of μ corresponding to T_r , see Equation (3.2)	kg/m s	slug/ft s
ρ	density of air	kg/m ³	slug/ft ³

2. REYNOLDS NUMBER IN THE INTERNATIONAL STANDARD ATMOSPHERE

Reynolds number is defined as

$$R = \frac{\rho V l}{\mu}, \quad (2.1)$$

and so Reynolds number per unit length, commonly called “unit Reynolds number”, is given by

$$\frac{R}{l} = \frac{\rho V}{\mu}, \quad (2.2)$$

which, with the substitution of $V = Ma$, becomes

$$\frac{R}{l} = \frac{\rho Ma}{\mu}. \quad (2.3)$$

Figures 1 and 2 present R/l for Mach numbers, M , up to 10 and geopotential heights, H , up to 32000 m in Figure 1 and 105000 ft in Figure 2.

The properties of the atmosphere from which the curves in Figures 1 and 2 were derived were obtained from ESDU 77021 (Derivation 9), based on the US Standard Atmosphere, 1976 (Derivation 8) which, over the range of height considered here, is identical to the International Standard Atmosphere (Derivation 7). It is evident from Equation (2.3) that Reynolds number is a linear function of Mach number, so that Reynolds numbers for Mach numbers outside the range presented may be obtained as simple multiples of the data given; for example, the Reynolds number at $M = 20$ is twice the Reynolds number at $M = 10$. The tropopause, at 11000 m (36089 ft) in the standard atmosphere, is indicated by the broken lines in Figures 1 and 2.

3. SPEED OF SOUND AND DYNAMIC VISCOSITY

Speed of sound and dynamic viscosity data for the International Standard Atmosphere are tabulated in Derivations 7 to 9.

Data given by Hilsenrath *et al.* (Derivation 2) show that the speed of sound not only has a strong dependency on temperature, it is also weakly dependent on pressure over quite a wide range, as shown in Figure 3. It is evident that, except at pressures well above 10^5 N/m² (approximately 1 atm) and at the higher temperatures where real gas effects are more significant, the dependence on pressure is small.

Figure 4 presents the variation of the viscosity of dry air with temperature. Over the temperature range from 300 K to 2200 K the curve is a plot of tabulated data due to Watson (Derivation 5), a treatment that indicates a systematic error in many of the experimental data (particularly for high temperatures) used to derive the previously established values such as those tabulated by Hilsenrath *et al.* in Derivation 2. This

part of the curve is given by the equation

$$\mu = \frac{T^{1/2} \times 10^{-6}}{A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4} \text{ kg/m s}, \quad (3.1)$$

where

$$s = 100/T \text{ and}$$

$$A_0 = 0.552795$$

$$A_1 = 2.81089$$

$$A_2 = -13.5083$$

$$A_3 = 39.3531$$

$$A_4 = -41.4194.$$

For the temperature range from 300 K down to 70 K, Sutherland's equation has been used, *i.e.*

$$\mu = \mu_r \left(\frac{T_r + S}{T + S} \right) \left(\frac{T}{T_r} \right)^{3/2}, \quad (3.2)$$

in which Sutherland's constant, S , has been taken as 110.4 K, as used in Table A2.3 of ESDU 77021 and as applied in the unit Reynolds number data presented here in Figures 1 and 2. For Figure 4, the reference temperature, T_r , was taken as 300 K, for which Equation (3.1) gives $\mu_r = 18.5244 \times 10^{-6}$ kg/m s. Note that Equation (3.2) then gives a value of $\mu = 17.956 \times 10^{-6}$ kg/m s at the standard sea level temperature of 288.15 K, which is about 0.35% greater than the value of $\mu = 17.894 \times 10^{-6}$ kg/m s given in ESDU 77021. While the latter value is considered to be the more reliable, the difference is insignificant in the graphical presentation of Figure 4.

In Figure 4, the curve has been terminated at $T = 70$ K to indicate the possible onset of liquefaction.

Equation (3.1) has also been used to extend the curve in Figure 4 from 2200 K to 3000 K. However, that part of the curve is shown broken due to the possible onset of dissociation.

For a comparison between experimental data and Equation (3.1) reference can be made to Watson (Derivation 6), who quotes uncertainties in μ values from the use of the equation as within 0.25% from 300 K to 600 K, 1% from 610 K to 1600 K and 2% from 1610 K to 2200 K. Although the data in Derivation 5 were based on measurements at or below atmospheric pressure, it is stated in Derivation 5 that the data may be used with little loss of accuracy at higher pressures (to within 1% at pressures up to 1.2×10^6 N/m² (*i.e.* about 12 atm) at 300 K and 2.2×10^6 N/m² (about 22 atm) at 500 K; at higher temperatures the influence of pressure is further reduced).

A very well researched study, produced subsequent to that of Watson at the NEL, is given in a paper by Kadoya *et al.* (Derivation 10). That work, in 1985, took into account the effects of both high temperature and high pressure on the dynamic viscosity and thermal conductivity of dry air. Four reliable sources of test data up to $T = 1600$ K published after Watson's work (1972) were included in the assessment. Kadoya *et al.* provide a correlation equation, together with tabulated data over their recommended range

of applicability from 85 K to 2000 K, covering a wide range of pressures from 10^4 N/m² (about 0.1 atm) to 10^8 N/m² (about 1000 atm). The discrepancies for μ at a pressure of 10^5 N/m² (about 1 atm) when compared with Figure 4 are generally small. Over the range 160 K to 600 K they are within about $\pm 0.2\%$, rising steadily to about $+0.8\%$ to remain constant between 1000 K and 1800 K, rising again to just under $+1\%$ at the recommended limit of 2200 K. The discrepancies for temperatures outside that overall range are larger, approaching $+2.6\%$ at 85 K and $+1.7\%$ at 3000 K. The uncertainties quoted earlier for Equation (3.1) are all within those quoted for the method in Derivation 10. So, in view of the quite small differences between the two methods and the claimed uncertainties for each, it is considered that no further changes to Figure 4 are needed at this time (2012).

3.1 Applicability of Figure 4

Figure 4 is applicable to any situation requiring the dynamic viscosity of dry air with a fixed molecular species content*, such as in the determination of Reynolds number for aircraft performance calculations or wind-tunnel testing. For higher pressures Derivation 10 provides tabulated data. More specialised applications, such as turbo jet or rocket performance calculations, will require the effects of fuel/air ratio and combinations of high pressure and temperature, assuming either fixed species content or – currently more likely – chemical equilibrium, including the products of combustion where applicable. For fixed species, References 11 to 13 provide useful data, and for chemical equilibrium, References 14 to 16 are similarly helpful. However, even in these extreme circumstances, while the effects on other thermodynamic properties such as the specific heat capacities and conductivity are seriously affected, the effect on dynamic viscosity is much less significant, see Reference 12, for example.

4. REYNOLDS NUMBER IN A WIND-TUNNEL WORKING SECTION

Figures 5a and 5b may be used to give the unit Reynolds number, R/l , in the working section of a wind tunnel in terms of the working-section Mach number, M , reservoir pressure, p_H , and reservoir temperature, T_H . The curves present $R/l(p_H/p_0)$ as functions of M and T_H . Figure 5a covers the Mach number range $0 \leq M \leq 3$ and Figure 5b covers the overlapping range $2 \leq M \leq 10$. Note that p_0 , the standard (sea level) pressure, is used here as a reference value to normalise p_H .

The data for these curves were calculated by the method described by Wilson and Regan in Derivation 3. Essentially, the method provides approximate corrections for the real gas effects in the expansion from the reservoir to the working section conditions. The corrections are strongly dependent on T_H but only weakly dependent on p_H , so that normalisation of the unit Reynolds number by p_H/p_0 is acceptable. Briefly, with the assumption of conservation of entropy and total enthalpy, an equivalent perfect gas is determined as that perfect gas which has the same entropy and enthalpy as the real gas in the reservoir. In the process, correction factors to be applied to the reservoir pressure and temperature are derived. For simplicity in the calculations, values in the working section for the speed of sound at one atmosphere (taken as 10^5 N/m²) from Figure 3 were used, together with values of dynamic viscosity from Figure 4. The hatched line in Figure 5b corresponds to $T = 70$ K and is included to indicate the possible onset of liquefaction. An analysis of liquefaction temperatures was carried out by Daum and Gyarmathy in Derivation 4.

* The standard content of dry air (by volume fraction) is usually given (Derivation 8) as N₂ 0.780840, O₂ 0.209476, Ar 0.009340 and CO₂ 0.000314. The remaining volume of 0.000030 is made up of other, more minor, constituents.