

INTRODUCTION TO ESTIMATION OF RANGE AND ENDURANCE

1. NOTATION AND UNITS*

		<i>SI</i>	<i>British</i>
B	buoyancy force (aerostatic force)	N	lbf
D	drag	N	lbf
D_M	intake momentum drag (ram drag)	N	lbf
F_G	engine gross thrust	N	lbf
g	acceleration due to gravity (acceleration of free fall, see Appendix A)	m/s ²	ft/s ²
g_{app}	apparent acceleration due to gravity (see Appendix A)	m/s ²	ft/s ²
L	lift	N	lbf
m	aircraft mass	kg	slug
Q	total engine fuel consumption* $\left\{ \begin{array}{l} \text{mass flow basis} \\ \text{weight flow basis} \end{array} \right.$	kg/h N/h	slug/h lbf/h
R_a	air range	km, n.mile	n.mile
R_g	ground range	km, n.mile	n.mile
t	time, endurance	h	h
V	true airspeed (air-path velocity or resultant velocity with respect to air)	km/h, kn	kn
W	aircraft weight, mg	N	lbf
x_a	air-path-axis co-ordinate in direction of V	m	ft
z_a	air-path-axis co-ordinate at right angles to V and in aircraft plane of symmetry, positive in ventral sense	m	ft
α	angle of attack	deg	deg
β	angle of sideslip	deg	deg

* When using SI units it is usual to work in terms of aircraft mass and fuel mass flow. This practice can of course be followed when using British units but the customary practice has been to work in terms of aircraft weight and fuel weight flow. For the most detailed calculations the former of these approaches is the more satisfactory since the energy supplied to the engines depends on the fuel calorific value and mass flow whereas a weight flow, being dependent on the value of g , varies with latitude and height, see Appendix A. The SI units used here have not necessarily received universal acceptance for the particular applications, *i.e.* other multiples and submultiples may be used.

		<i>SI</i>	<i>British</i>
γ_a	air-path angle of climb (air-path inclination angle)	deg	deg
ε	angle between gross thrust vector and aircraft plane of symmetry, positive when F_G acts to port	deg	deg
λ	ratio of “non-propulsive” mass (weight) loss to mass (weight) loss due to engine fuel flow		
τ	angle between projection of gross-thrust vector on to aircraft plane of symmetry and longitudinal body axis; positive when F_G contributes to lift	deg	deg

Subscripts

f	final condition
i	initial condition

2. INTRODUCTION

This Item introduces the subject of range and endurance estimation and discusses the aircraft, engine, flight path and operational factors that need to be considered. While it is necessary to use numerical methods for the most exact calculations, close estimates of cruise range and endurance may be obtained using analytical methods. These are treated in Item No. 73019, which deals with turbo-jet and turbo-fan engined aeroplanes, and Item No. 75018 which considers propeller-driven aeroplanes. Equations given in the main text of the present Item are limited to those common to all classes of aircraft and those which provide a basis from which the simplified methods of Item Nos 73019 and 75018 are derived. Appendix A gives an approximate method for taking account of the effects of an aircraft’s latitude, height, speed and heading on the “apparent acceleration due to gravity”. Appendix B considers the effects of winds on optimum cruise speeds.

In any estimation of range or endurance the distance, fuel and time used in climb and descent should also be considered. An alternative to the detail calculation of these quantities is given in Item Nos 74018 and 77015 which offer simple, approximate methods of estimating “lost range”, “lost fuel” and “lost time” due to climb and descent suitable for use with the cruise performance methods of Item Nos 73019 and 75018 respectively. Item No. 74041 provides a worked example of a typical range-performance problem for a turbo-fan engined transport aeroplane in which the optimum cruise height to give minimum overall fuel consumed is sought for a number of stage lengths.

3. PRINCIPAL FACTORS AFFECTING AIRCRAFT RANGE AND ENDURANCE

Whatever method is used to estimate range or endurance, the principal factors to be considered are the total fuel available, the rate of fuel consumption and the flight profile chosen.

3.1 Fuel Available

The total fuel load that can be carried is limited either by the aircraft’s tank capacity or, in the case of flight with a heavy payload, by the maximum permitted weight. To estimate range or endurance it is first necessary to calculate the fuel available for the defined flight profile by deducting, from the total fuel carried, those quantities of fuel required to cover allowances, contingencies and reserves.

Allowance: This term covers fuel that is expected to be burnt during the flight but is not considered to contribute to range or endurance.

Contingency (allowance): This covers any fuel allocated for dealing with minor deviations from the defined flight profile.

Reserve: This is fuel allocated to provide for the safe completion of the flight in the event of some major unscheduled change in the flight profile, such as a diversion due to bad weather. It is not usual for the fuel reserve to be burnt and thus, besides limiting the quantity of fuel available for range, it has the effect of raising the aircraft mean weight above that required to complete the defined flight profile. Typically, a reserve is sufficient to permit a “hold” of given duration at a fairly low altitude and diversion to an alternative destination see, for example, Reference 7. In civil transport practice, the time spent and heights flown during holding, and the flight profile assumed for the diversion are usually determined by the operator in consultation with the appropriate authority.

3.1.1 Other factors affecting the fuel load carried

The fuel load carried on a particular flight may be affected by a wide range of additional factors, for example:

regulations relating to the ability safely to conclude the flight following an en-route engine failure,

fuel availability and cost at destination,

turn-round requirements and ability to complete further stages without refuelling.

Any factor that increases the aircraft weight has consequences both in terms of take-off performance (particularly in hot or high-altitude conditions) and in increased en-route fuel consumption due to the greater mean cruise weight.

3.2 Fuel Consumption

Estimates of range and endurance depend on the accuracy with which the fuel flow, Q , is predicted. For a known speed, weight and height the thrust or power required is calculated from the equations of motion (see below) and the corresponding fuel flow deduced from engine performance data. In cases where the engines are operated at a fixed “rating”, for example during climb or for maximum level speed, the equations are solved iteratively to give the equilibrium airspeed/thrust combination corresponding to the specified engine rating and aircraft weight and height.

Engine fuel flow data are usually available for different engine settings, off-takes and bleeds, ambient conditions and flight speeds or Mach numbers for a stated fuel calorific value. To obtain the most accurate estimates of range these data must be used and full account be taken of all “installation effects” which may affect the interrelationships of Q , and F_G and D_M for the engine(s) as installed and operated in the aeroplane.

When analytical methods are used, the engine fuel flow and thrust or power characteristics are represented by an algebraic expression, or “law” but no one such law can be expected to apply throughout the performance envelope. In general, the greater the range of thrust or power, height, temperature and speed or Mach number for which a law is required to be valid, the more complex it becomes. For the purposes of estimating approximate values of cruise range, the representation of engine fuel flow is simplified by the use of specific fuel consumption, i.e. fuel flow per unit thrust for turbo-jet and turbo-fan engines (see Item No. 73019, Appendix B) and fuel flow per unit shaft power or equivalent shaft power for piston and turbo-prop engines (see Item No. 75018, Appendix A).

3.2.1 The performance equation*

The engine thrust or power is calculated using air-path axes equations of motion. For example, Equations (4.3) and (4.4) of Item No. 79018 give, for turbo-jet or turbo-fan engined* aeroplanes,

$$F_G \cos \epsilon \cos \beta \cos (\alpha + \tau) - D_M - D - mg \sin \gamma_a + B \sin \gamma_a = m \ddot{x}_a, \quad (3.1)$$

$$mg \cos \gamma_a - L - F_G \cos \epsilon \sin (\alpha + \tau) - B \cos \gamma_a = m \ddot{z}_a. \quad (3.2)$$

Equations (3.1) and (3.2) can be much simplified and combined into a single expression as follows:

- (i) For aeroplanes the buoyancy, B , is neglected.
- (ii) For symmetric flight with all engines operating the effects of sideslip and of lateral thrust vector inclination are ignored so that $\beta = \epsilon = 0$. (See Item No. 79018, Section 4.1.1 for expressions for thrust components when τ and ϵ are different for each engine.)

When approximations (i) and (ii) are applied to Equations (3.1) and (3.2) and the equations combined, the following expression for thrust in terms of lift to drag ratio is obtained,

$$F_G \left[\cos (\alpha + \tau) + \frac{D}{L} \sin (\alpha + \tau) \right] - D_M - m \left[\frac{D}{L} (g \cos \gamma_a - \ddot{z}_a) + g \sin \gamma_a + \ddot{x}_a \right] = 0. \quad (3.3)$$

Further approximations, corresponding to typical cruise conditions are:

- (iii) For small angles of climb, $\cos \gamma_a \approx 1$, $\sin \gamma_a \approx 0$ (correction formulae for “cruise-climb” flight are developed in Appendix D of Item No. 73019).
- (iv) For steady level flight $\ddot{x}_a = 0$ and the only contribution to $\ddot{z}_a$ is the centripetal acceleration due to motion about the Earth. A simple method of including this in Equation (3.3) is to delete $\ddot{z}_a$ and to replace the acceleration due to gravity, g , by an “apparent” value, g_{app} (see Appendix A).

Applying approximations (iii) and (iv) to Equation (3.3) gives,

$$F_G \left[\cos (\alpha + \tau) + \frac{D}{L} \sin (\alpha + \tau) \right] - D_M - mg_{app} \frac{D}{L} = 0. \quad (3.4)$$

3.3 Flight Profile

The flight is conveniently separated into take-off, climb, cruise, descent, approach and landing and additional phases. It is usual to credit to range and endurance only the distance, time and fuel associated with each of the climb, cruise and descent phases although the fuel requirements must be evaluated for all phases.

3.3.1 Take-off and initial climb

The take-off and initial climb phase begins when the engines are started and includes taxiing and waiting; for the purposes of this Item it includes any part of the climb not credited to range. In many cases both “maximum ramp weight” and “maximum take-off weight” are specified; if an aeroplane is loaded to the

* Equations (3.1) to (3.4) are written for the case of aeroplanes with turbo-jet or turbo-fan engines although much of the material is also applicable to other types of powerplant; equations for propeller-driven aeroplanes are given in Item No. 75018.

former of these weights then the difference between them constitutes that part of the fuel load which must be used prior to take-off.

3.3.2 Climb and descent

No firm rule exists as to which parts of the climb and descent phases are credited to range and endurance. The important point is to be clear what the distinction is in any particular case. In some instances, particularly in flights of relatively short duration, the climb and descent phases constitute the greater part of the flight. However, even if the distance covered or time spent in climb and descent is but a small part of the flight, the fuel used must still be estimated. A further consideration is that much of the acceleration to, and deceleration from, cruise occurs during these phases and the fuel consumed is affected accordingly. During climb, the path followed may be dictated by any of a number of considerations, for example:

the requirements of air traffic control to hold a given forward speed and to pass through points of specified height and position,

an operational requirement to reach cruise altitude, for example, at minimum cost, with minimum fuel consumption, in the shortest possible time or while covering the greatest possible distance (see Item Nos Perf. EG 3/3 to EG 3/5).

During descent, the requirements of air traffic control again frequently determine the flight path. For civil aircraft, rates of descent are usually limited so as to avoid excessive rates of change of cabin pressure.

3.3.3 Cruise

For cruising flight at least one of the following quantities is usually held constant: Mach number, airspeed, pressure height, engine setting, lift/drag ratio. For aeroplanes the maximum range is usually obtained if a “cruise-climb” technique is used in which the height is allowed to increase as the weight decreases and the lift/drag ratio remains approximately constant. However, due to the requirements of air traffic control, such cruise techniques are only permitted in areas or at heights where there is very little traffic. Accordingly, most cruising flight is at constant pressure height although, where economically justified, a “stepped cruise” of several stages at different heights is used. The cruise height can also be influenced by a desire to take advantage of prevailing winds or to minimise their adverse effects. References 12 to 14 provide information on the winds likely to be encountered during flight.

Cruising flight at a nominally constant Mach number or airspeed requires a particular engine thrust or power setting for each combination of height and mass (or weight); thus, as fuel is consumed the engine thrust or power has to be reduced to maintain constant speed or Mach number. With the exception of cases where the autopilot can be coupled to the engine control system, it is customary to avoid frequent adjustment of the engine controls by flying at a fixed engine setting and to accept, within certain limits, the variations in speed and Mach number that occur as the fuel load decreases and/or ambient temperature changes.

When long endurance is required, the fuel flow must be minimised. Most practical cases, for example “holding” operations and military patrol duties, again involve flight at constant height and, in some cases, at constant speed (see Item No. 73019, Section 5).

3.3.4 Approach and landing

For present purposes, this phase includes the initial and final approach, the landing and subsequent taxiing.

3.3.5 Additional phases

If the flight includes any phase not covered in Sections 3.3.1 to 3.3.4, then the fuel required must be calculated. The main events that have to be considered here apply only to military aircraft, for example:

phases of flight where speed and/or height are of prime importance, for example the dash to a target or during combat,

changes in aircraft mass (or weight) due to release of weapons or cargo, expenditure of ammunition and inflight refuelling,

the use of an auxiliary power unit when the aircraft is on stand-by.

4. SPECIFIC RANGE, RANGE AND ENDURANCE

4.1 Specific Range, Specific Air Range

The most general method of considering an aircraft's range performance is in terms of specific range, *i.e.* the distance travelled per unit of fuel used. For estimation purposes it is usual only to consider the air-distance travelled per unit of fuel used, *i.e.* the specific air range (or specific range in still air). Specific air range is an instantaneous property of the aircraft-engine combination and is usually presented as a function of aircraft mass, true air speed or Mach number, pressure height and ambient temperature (see Item No. 70022, Sections 3 of Item Nos 73019 and 75018 and Item No. 74041 for examples of such data).

Specific air range can also be used,

- (i) to give speed(s) and height(s) necessary to obtain the absolute maximum cruise air range for the aircraft,
- (ii) when at least one of the cruise parameters (for example, height) has been fixed, to determine the values of the other cruise parameters necessary to obtain maximum air range.

For most operational purposes, the various cruise techniques which constitute case (ii) are of greater interest than case (i) since the latter often corresponds to a cruise-climb. Case (i) does, however, provide a simple means of comparing the range performance of different aircraft and is frequently used for this purpose. In all cases the necessary condition for maximum range is that throughout the flight the specific air range be the maximum value that is consistent with the chosen cruise technique and Sections 3 of Item Nos 73019 and 75018 consider the various cases that arise for turbo-jet and turbo-fan engined aeroplanes and propeller-driven aeroplanes respectively. Appendix B of the present Item gives formulae and graphical data to show how the true air speed for maximum specific range, *i.e.* relative to the ground, varies with wind speed for jet, propeller-driven and unpowered aeroplanes.

Expressions for specific air range are developed below on the bases of fuel mass and weight flows

$$\left. \begin{aligned} Q &= - \frac{1}{(1 + \lambda)} \frac{dm}{dt} \text{ kg/h, slug/h} \\ Q &= - \frac{1}{(1 + \lambda)} \frac{dW}{dt} \text{ N/h, lbf/h} \end{aligned} \right\} \quad (4.1)$$