

APPROXIMATE METHODS FOR ESTIMATION OF CRUISE RANGE AND ENDURANCE: AEROPLANES WITH TURBO-JET AND TURBO-FAN ENGINES

1. NOTATION AND UNITS*

		SI	British
A	temperature gradient (Item No. 77022 gives values for ISA conditions)	K/m	K/ft
a	speed of sound	km/h, kn	kn
C_D	drag coefficient, $D/\frac{\rho}{2}\left(\frac{rV}{3600}\right)^2 S$		
C_L	lift coefficient, $L/\frac{\rho}{2}\left(\frac{rV}{3600}\right)^2 S$		
c	specific fuel consumption, Q/F_N (see Appendix B)	weight flow basis mass flow basis	N/N h kg/N h lbf/lbf h slug/lbf h
c_1, c_2	constants defined in Section 2.3	kg/ N h	lbf/lbf h
D	drag	N	lbf
D_M	intake momentum drag (ram drag)	N	lbf
F_G	engine gross thrust	N	lbf
F_N	net thrust (see Item No. 69006 for thrust definitions)	N	lbf
g	acceleration due to gravity (see Appendix A of Item No. 73018 for values)	m/s ²	ft/s ²
H	geopotential height	m	ft
H_p	pressure height	m	ft
K_1, K_2	coefficients in drag equation (Equation (2.10))		
L	lift	N	lbf
M	Mach number, V/a		
m	aeroplane mass	kg	slug

* When using SI units it is usual to work in terms of aeroplane mass and fuel mass flow. This practice can of course be followed when using British units but the customary practice has been to work in terms of aeroplane weight and fuel weight flow. In Section 2 the basic equations are presented in terms of both weight and mass. Subsequent Sections, Tables, Figures and Appendices are, however, written in terms of aeroplane weight and fuel weight flow only, but the results given are equally applicable if aeroplane mass and fuel mass flow are used, provided the substitution $W = mg$ is made. The SI units have not necessarily received universal acceptance for the particular applications, i.e. other multiples and sub-multiples may be used. Note also that although the knot is given as an alternative unit for speed, some of the equations only apply if time units are not mixed.

N	engine speed	rev/s	rev/min
n	constant (see Appendix B, Section B3.2)		
p	pressure	N/m ²	lbf/ft ²
Q	total engine fuel consumption	$\left\{ \begin{array}{l} \text{weight flow basis} \\ \text{mass flow basis} \end{array} \right.$	$\left\{ \begin{array}{l} \text{N/h} \\ \text{kg/h} \end{array} \right.$ $\left\{ \begin{array}{l} \text{lbf/h} \\ \text{slug/h} \end{array} \right.$
R	gas constant for air, see page 3 for values	Nm/kg K	lbf ft/slug K
R_a	air range	km, n.mile	n.mile
R_{ref}	reference air range (see Equation (4.1))	km, n.mile	n.mile
r	conversion factor (values given at end of Notation)	m/km, m/n.mile	ft/n.mile
S	wing area	m ²	ft ²
T	ambient temperature	K	K
t	time, endurance	h	h
t_{ref}	reference endurance (see Equation (5.1))	h	h
V	true airspeed	km/h, kn	kn
V_e	equivalent airspeed, $V\sigma^{1/2}$	km/h, kn	kn
V_{mean}	mean cruise speed	km/h, kn	kn
W	aeroplane weight, mg	N	lbf
W_F	weight of fuel	N	lbf
x	quantity defined in Appendix D, Section D3		
y	ratio of airspeed to minimum drag speed, V/V_{md}		
α	angle of attack	degree	degree
γ	ratio of specific heat capacities of air		
γ_a	angle of climb in still air (air-path angle of climb)	degree	degree
Δ	denotes a change in the affixed parameter due to effects of flight path gradient and acceleration in cruise-climb (see Appendix D, Section D2.)		
δ	relative pressure, p/p_0		
θ	relative temperature, T/T_0		

ρ	density of air	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0		
τ	angle between projection of gross-thrust vector on to aeroplane plane of symmetry and longitudinal body axis; positive when F_G contributes to lift	degree	degree

Subscripts

b	denotes value at base of layer in International Standard Atmosphere
F	denotes fuel
f	denotes final condition
i	denotes initial condition
max	denotes maximum
md	denotes minimum drag condition in 1 g level flight
std	denotes value in International Standard Atmosphere, ISA
0	denotes value at sea level in ISA, numerical values given below

Constant and Standard Sea Level Values of Atmospheric Properties

a_0	1 225.06 km/h, 340.29 m/s	661.479 kn, 1 116.45 ft/s
g_0	9.806 65 m/s ²	32.174 0 ft/s ²
p_0	101 325.0 N/m ²	2 116.22 lbf/ft ²
R	287.052 87 N m/kg K	3 089.811 4 lbf ft/slug K
r	1 000 m/km, 1 852 m/n. mile	6 076.12 ft/n. mile
T_0	288.15 K	288.15 K
ρ_0	1.225 0 kg/m ³	$2.376\ 896 \times 10^{-3}$ slug/ft ³
γ	1.4	1.4

2. INTRODUCTION

Approximate methods are given for estimating the cruise range and endurance performance of aeroplanes powered by turbo-jet and turbo-fan engines. The methods are particularly suited to project-type calculations and provide results applicable in still air. A separate Item, No. 73018, introduces the subject of range and endurance estimation, discusses the factors that govern these aspects of performance and derives the basic equations. The present Item, by making several simplifying assumptions, develops approximate versions of those equations that have analytical solutions for cases of cruising flight in which at least one of the parameters; airspeed, Mach number, pressure height and engine control setting is held constant. The conditions in which specific air range and specific endurance may be maximised are examined and expressions for air range and endurance presented for each cruise case. Curves are presented which permit range and endurance in several cruise cases to be derived once the range or endurance in one reference cruise case has been calculated. Appendix A presents relationships derived from a parabolic drag law that are used elsewhere in the Item. Appendix B deals with the representation of fuel consumption data with particular reference to range calculations. Appendix C discusses the implications for engine thrust at “constant control setting” – one of the cruise cases considered in the Item. Appendix D gives additional information on cruise-climb performance including simple expressions for the flight path followed and the correction to range due to flight path gradient and acceleration. An Example is included in Appendix D to demonstrate the use of these expressions for several types of aeroplane and in particular, to illustrate the order of magnitude of the range correction. The effects of wind on the airspeed for maximum specific range are considered in Appendix B of Item No. 73018.

An approximate method for adjusting estimations of cruise range, fuel and time to allow for climb and descent is given in Item No. 74018 and an example of the combined use of that Item and the present Item is given in Item No. 74041. The range performance of propeller-driven aeroplanes is considered in Item Nos 75018 and 77015.

2.1 Simplifying Assumptions

Except where otherwise stated, the following assumptions are used in this Item to simplify the performance equations and, in particular, to derive the equations given in Section 2.2.

- (i) The aeroplane is a rigid body, has a plane of symmetry and has its mass located at its centre of gravity.
- (ii) Changes in aeroplane mass or weight, other than those due to fuel consumption, are ignored, *i.e.* in Equations (4.1) to (4.5) of Item No. 73018 the quantity $\lambda = 0$.
- (iii) The effect of buoyancy is neglected.
- (iv) The aeroplane is in symmetric flight, *i.e.* no sideslip, and the inclination of the thrust vector to the flight path is small and can be neglected (some effects of thrust vector inclination are considered in Appendix B).
- (v) The climb angle is small* so that, to a satisfactory approximation, $\cos \gamma_a = 1$, $\sin \gamma_a = 0$.
- (vi) There is no acceleration along the flight path*.

* In Appendix D, Section D2., correction terms are developed which allow for finite values of both climb rate and acceleration along the flight path for the case of cruise-climb at constant L/D .

- (vii) Variations in the value of g with latitude and height are ignored*.
- (viii) The centripetal acceleration due to the aeroplane's motion about the Earth is ignored*.

2.2 Basic Equations

The aeroplane performance equations, such as those in Section 3.2.1 of Item No. 73018, are simplified by applying the above assumptions and combined with the definition of engine specific fuel consumption in Equation (B2.1) of Appendix B to give an expression for fuel flow, see Equations (B2.2) and (B2.3). Equation (B2.3) is combined with Equations (4.1), (4.3), (4.4) and (4.9) of Item No. 73018 to give the following two sets of expressions for specific air range, air range, specific endurance and endurance.

If aeroplane weight and fuel weight flow are used:

$$-\frac{dR_a}{dW} = \frac{V}{Q} = \frac{V}{cF_N} = \frac{1}{W} \frac{V}{c} \frac{L}{D}, \quad (2.1)$$

$$R_a = \int_{W_f}^{W_i} \frac{V}{Q} dW = \int_{W_f}^{W_i} \frac{1}{W} \frac{V}{c} \frac{L}{D} dW, \quad (2.2)$$

$$-\frac{dt}{dW} = \frac{1}{Q}, \quad (2.3)$$

$$t = \int_{W_f}^{W_i} \frac{dW}{Q} = \int_{W_f}^{W_i} \frac{1}{cW} \frac{L}{D} dW. \quad (2.4)$$

If aeroplane mass and fuel mass flow are used:

$$-\frac{dR_a}{dm} = \frac{V}{Q} = \frac{V}{cF_N} = \frac{1}{W} \frac{V}{c} \frac{L}{D} = \frac{1}{mg} \frac{V}{c} \frac{L}{D}, \quad (2.5)$$

$$R_a = \int_{m_f}^{m_i} \frac{V}{Q} dm = \int_{m_f}^{m_i} \frac{1}{mg} \frac{V}{c} \frac{L}{D} dm, \quad (2.6)$$

$$-\frac{dt}{dm} = \frac{1}{Q}, \quad (2.7)$$

$$t = \int_{m_f}^{m_i} \frac{dm}{Q} = \int_{m_f}^{m_i} \frac{1}{c} \frac{L}{mg} \frac{1}{D} dm. \quad (2.8)$$

In this Item analytical methods[†] are used to derive from Equations (2.1) to (2.4) (*i.e.* those based on aircraft weight and fuel weight flow) the conditions necessary to obtain maximum specific air range and endurance and expressions for cruise air range and endurance. To do this, substitutions are required for the variables c , L/D and V and, consequently, further assumptions are required in addition to those made in Section 2.1.

* If required, the combined effects of the variation in acceleration due to gravity with latitude and height and of the centripetal acceleration due to motion about the Earth can be taken into account if g is replaced by an "apparent acceleration due to gravity, g_{app} ". Values of g_{app} are given in Appendix A of Item No. 73018.

[†] With one exception, see Section 4.2.2.

2.3 Additional Characteristics of the Aeroplane, Engine and Flight Path

Additional assumptions made and used to derive the results presented in this Item are as follows.

- (i) The relationship between lift and drag during cruise is expressed either by

$$L/D = \text{constant} \quad (2.9)$$

or by the parabolic drag law

$$C_D = K_1 + K_2 C_L^2. \quad (2.10)$$

- (ii) The engine fuel flow characteristics are expressed in terms of specific fuel consumption, c (see Appendix B). Two cases are considered:

$$c = c_1, \quad (2.11)$$

which is the simplest fuel flow law of practical use, or

$$c = c_2 \sqrt{\theta} M^n \quad (2.12)$$

which is fairly easy to apply and, within the limited range of engine speed and flight Mach number appropriate to cruise, gives a reasonable approximation to the specific fuel consumption of current turbo-jet and turbo-fan engines. Appendix B presents several types of fuel flow law; those defined by Equations (2.11) and (2.12) are considered in Sections B3.1 and B3.2.

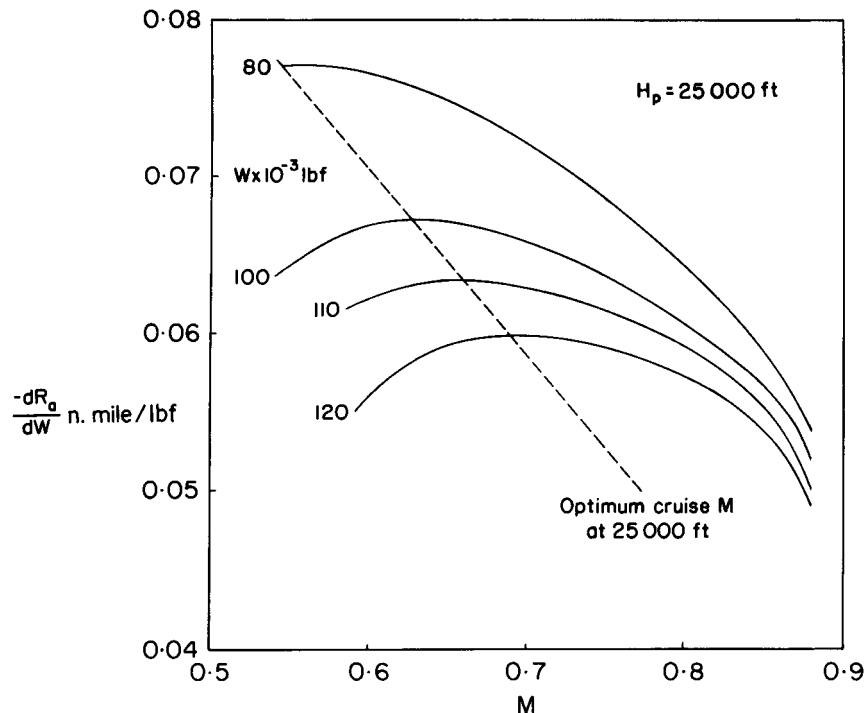
- (iii) The cruise is flown with one or more of the following parameters held constant: true airspeed, Mach number, pressure height, engine setting (see Appendix C).
- (iv) Ambient conditions are as defined for the International Standard Atmosphere, see Item No. 77022. Still-air conditions are assumed throughout this Item*.

3. SPECIFIC AIR RANGE

This section is principally concerned with determining the flight conditions necessary to maximise cruise specific air range* and thereby to maximise the range itself.

Sketch 3.1 shows a typical example of specific air range performance data for a current subsonic transport aeroplane plotted as curves of $-dR_a/dW$ against M for fixed values of W at a given value of H_p . Item No. 70022 describes alternative, more compact methods of presenting such data in “non-dimensional” form and Figure 2 of that Item presents the complete specific air range performance of another transport aeroplane as curves of WV/Q (i.e. $-W dR_a/dW$) plotted against M for fixed values of W/δ (WV/Q is sometimes referred to as the “range factor”).

* The effect of winds on the airspeed for maximum specific range, i.e. relative to the ground, is considered in Appendix B of Item No. 73018.



Sketch 3.1

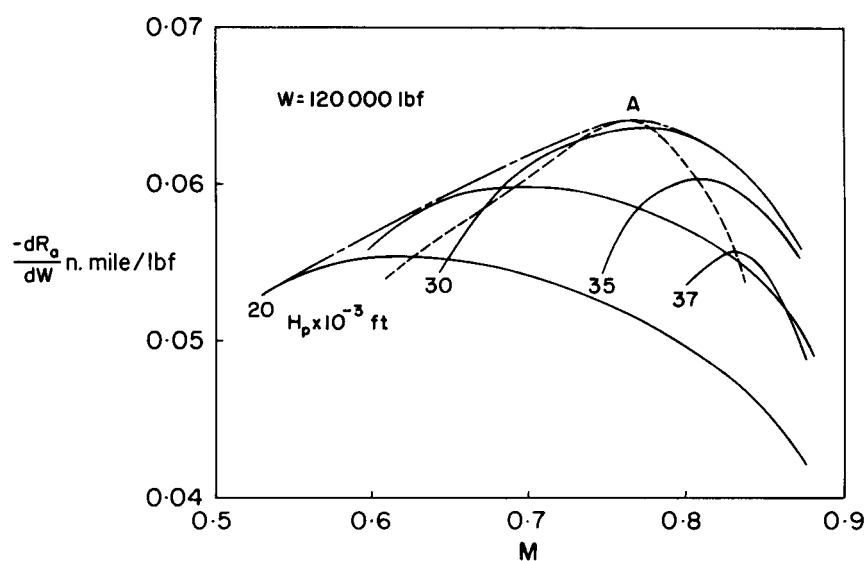
Since, for any aeroplane or flight, the value of W is only controllable to the extent that take-off and landing weights may be selected, most interest is centred in finding the conditions necessary for maximum specific air range to be obtained at any value of W .

For example, in Sketch 3.2 curves of $-\frac{dR_a}{dW}$ are plotted against M for fixed values of H_p at one value of W . Two additional curves are shown in Sketch 3.2 and each of these has a maximum value at point "A". The broken line joins the points of maximum specific air range for each value of H_p while the chain dotted line is an envelope curve which shows the maximum specific air range at any value of M . Sketch 3.2 thus demonstrates that, for a turbo-jet or turbo-fan powered aircraft, the range performance generally increases with increase in cruise height and Mach number and the limit to such increase in performance is reached in this case at point "A". Such a limit may, for example, be due to

- (i) the rise in engine specific fuel consumption associated with increase in M (see Appendix B) or
- (ii) the increase in aircraft drag associated with operation at higher values of M .

In some cases it may not be possible to attain the flight conditions associated with point "A", for example due to

- (i) there being insufficient cruise thrust available,
- (ii) the point being outside the permitted flight envelope.



Sketch 3.2

From Equation (2.1) the necessary condition for obtaining the maximum specific air range at any given value of W is that $(V/c)(L/D)$ (i.e. WV/Q) be a maximum. Sections 3.1 and 3.2 consider the conditions for maximum specific air range for particular cruise cases.

3.1 General Lift and Drag Conditions for Maximum Specific Air Range

Table 7.1 presents expressions for $-dR_a/dW$ and the conditions in terms of lift and drag that are necessary if, at any instant, $|dR_a/dW|_{max}$ is to be obtained while flying at some given value of V or M or H_p or engine setting. The expressions are presented for each of the fuel flow laws given in Section 2.3 and it should be noted that they are *independent of the way in which drag varies with lift*.

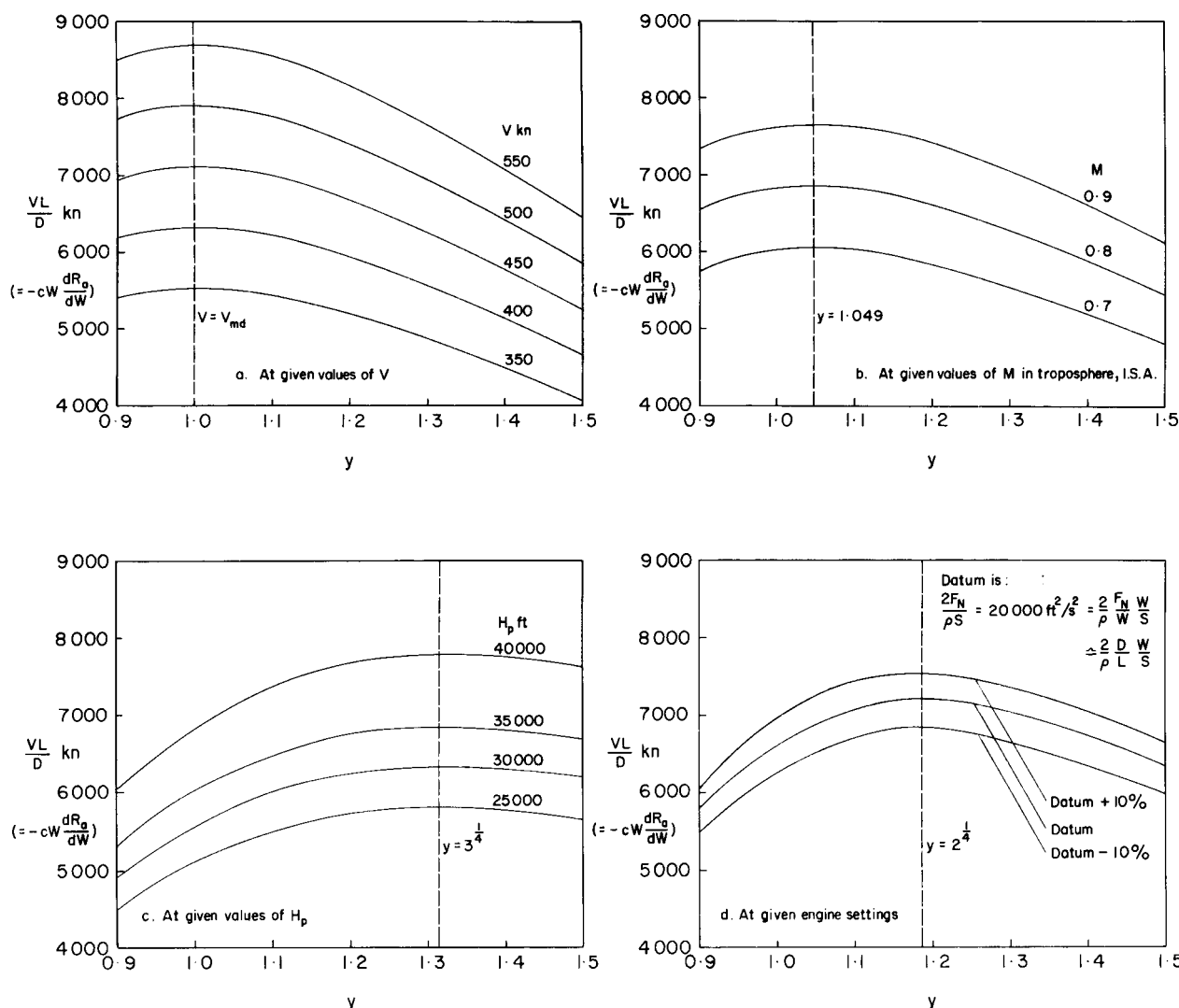
3.2 Specific Air Range Using a Parabolic Drag Law

Further deductions may be made regarding the flight conditions necessary to achieve the maximum specific air range if the relationship between the cruise lift and drag coefficients is assumed to be that given in Equation (2.10). Appendix A summarises a number of standard results that may be derived from Equation (2.10) and are required here. In particular, Equation (A1.8) defines the relationship between the L/D ratio and speed, see Figure 1. Table 7.2 presents the conditions for maximum specific air range at any given W when cruise lift and drag are related as in Equation (2.10).

If Equations (2.1) and (A1.8) are combined then the specific air range is given by

$$-\frac{dR_a}{dW} = \frac{2}{cW} V_{md} \left(\frac{L}{D} \right)_{max} \left(\frac{y}{y^2 + \frac{1}{y^2}} \right) \quad (3.1)$$

Sketch 3.3 presents specific air range data derived from Equation (3.1) as curves of $VL/D (= -cW dR_a/dW)$ for an aeroplane for which it is assumed that $K_1 = 0.02$, $K_2 = 0.05$, $W/S = 100$ lbf/ft² and $c = \text{constant}$ (the numerical values chosen are broadly representative of subsonic transport aeroplanes). In addition to the specific air range information the Sketch illustrates the conclusions derived in Table 7.2 regarding the speeds necessary for $|dR_a/dW|_{max}$. A similar set of data could be constructed for any other fuel flow law and drag constants.



Sketch 3.3 Typical variations of $\frac{VL}{D}$ for subsonic transport aeroplane in cruise

c is constant,

$$C_D = 0.02 + 0.05 C_L^2,$$

$$\frac{W}{S} = 100 \text{ lbf/ft}^2$$

4. AIR RANGE

The methods of estimating range presented in this Item are, with one exception (see Section 4.2.2), based on the analytical integration of Equation (2.2). For each of the cruise cases considered the appropriate assumptions regarding the aircraft, engine and flight path characteristics are selected from those given in Section 2.3. The cruise cases may be subdivided into two groups depending upon the relationship assumed between cruise lift and drag (see Section 2.3, item (i)) and are considered in this way in Sections 4.1 and 4.2.

Expressions for R_a are presented for each cruise case in Tables 7.3 and 7.4 for the two fuel flow laws given in Section 2.3, item (ii). These Tables also present expressions for the range ratio, R_a/R_{ref} , where R_{ref} is a reference range obtained by integrating Equation (2.2) with c , L/D and V held constant, i.e.*

$$R_{ref} = \left(\frac{VL}{cD} \right)_i \log_e \left[\frac{1}{1 - \frac{W_F}{W_i}} \right], \quad (4.1)$$

and having initial cruise conditions identical with those assumed for R_a . To assist in calculating values of R_{ref} the log function in Equation (4.1) is plotted against W_F/W_i in Figure 2. In general, $R_a/R_{ref} \neq 1.0$ and values are plotted against W_F/W_i in Figures 3 to 7; the appropriate Figure for each cruise case may be selected by reference to Tables 7.3 and 7.4. By using these Figures, values of R_a may be deduced as follows for any given set of initial conditions (i.e. y_i , $[(V/c)(L/D)]_i$) once the corresponding value of R_{ref} has been calculated.

Calculation sequence for R_a

- (i) By reference to Tables 7.3 and 7.4, locate the appropriate Figure for the cruise case under consideration.
- (ii) Calculate $[(V/c)(L/D)]_i$ and y_i (if the latter is required as a parameter to enter the Figure located in (i) above).
- (iii) Calculate R_{ref} using $[(V/c)(L/D)]_i$ from (ii) and $\log_e [1/(1 - W_F/W_i)]$ from Figure 2 for the required value of W_F/W_i (see Item No. 73018, Section 3.1, for guidance regarding the fuel available for cruise).
- (iv) Using the Figure located at (i), read R_a/R_{ref} for the required value of W_F/W_i and, if appropriate, y_i and n .
- (v) Cruise range is the product of R_a/R_{ref} and R_{ref} .

4.1 Cruise Air Range in Constant L/D Flight

Cruise at constant L/D provides a relatively simple case for analysis since the aircraft is flown at a fixed angle of attack and hence constant C_L and C_D . From the expressions

$$W = mg = L = \frac{C_L}{2} \rho \left(\frac{rV}{3600} \right)^2 S = \frac{C_L}{2} \rho_0 S \left(\frac{rV}{3600} \right)^2 \sigma = \frac{C_L}{2} \rho_0 S \left(\frac{ra_0 M}{3600} \right)^2 \delta, \quad (4.2)$$

* Equation (4.1) is the form taken by the classical Breguet range equation when applied to turbo-jet and turbo-fan engined aircraft cruising at constant L/D (i.e. constant angle of attack) and speed and assuming constant specific fuel consumption.

it can be seen that cruise at constant L/D can be attained in several ways of which the following are amenable to analysis (and are considered further in Sections 4.1.1 and 4.1.2).

- (i) Cruise at constant V with height varying such that W/σ is constant (“cruise-climb at constant V ”).
- (ii) Cruise at constant M with height varying such that W/δ is constant (“cruise-climb at constant M ”).
- (iii) Cruise at constant height with W/V^2 or W/M^2 constant.

4.1.1 Cruise-climb at constant V or M

Although now of limited operational use (see Item No. 73018, Section 3.3.3), these cruise techniques provide methods of operating at or near the maximum range capability of the aeroplane. In addition, the cruise range so obtained is often used as a standard of comparison between aeroplanes. Values of R_a/R_{ref} are plotted against W_F/W_i in Figures 3 and 4 (for those cases when $R_a/R_{ref} \neq 1.0$).

Appendix D considers the flight paths and engine settings that correspond to cruise-climb. In addition, since the level-flight assumptions, $F_N = D$, $L = W$ (see Section 2.1) are not strictly correct in cruise-climb, Appendix D also presents correction terms for thrust, fuel flow and air range due to flight path gradient and aeroplane acceleration.

4.1.2 Cruise at constant L/D , constant H_p

Case (iii) of Section 4.1 permits L/D to be held constant while flying at constant H_p . During this type of cruise, V or M must be reduced to maintain W/V^2 or W/M^2 constant. Values of R_a/R_{ref} for this cruise case are plotted against W_F/W_i in Figure 5 for each of the two fuel flow laws.

4.2 Cruise Air Range at Constant H_p Assuming a Parabolic Drag Law

In this section, in contrast to Section 4.1, cruise C_L is assumed to vary in such a manner as to keep cruise height constant as fuel is used and C_D is assumed to vary in the manner defined by Equation (2.10)

4.2.1 Cruise at constant H_p and constant M or V

For this cruise technique, Equation (4.2) shows that the angle of attack must be steadily reduced during the flight so that W/C_L remains constant. The corresponding reduction in C_D means that the engine thrust must be reduced in order to maintain constant V or M . Cruise at constant H_p and M is, nominally at least, the commonest method of cruising for turbo-jet and turbo-fan powered aeroplanes (see also Section 4.2.2 and Item No. 73018, Section 3.3.3). Values of R_a/R_{ref} for this cruise case are plotted against W_F/W_i and y_i in Figure 6; note that in this case the one set of curves applies to each of the fuel flow laws.

4.2.2 Cruise at constant H_p and constant F_N

In the cases considered so far it is assumed that either engine thrust or cruise height is adjusted throughout the flight so as to maintain the specified cruise condition. To avoid frequent adjustments to the engine controls, it is more customary to fly with the engines at a fixed control setting and to accept, within certain limits, the variations in M and V that occur as W , C_L and C_D decrease (see Item No. 73018, Section 3.3.3). The term “constant engine setting” refers here to the controls operated by the pilot; the consequences, in terms of engine performance, depend on the type of control system used and are discussed in Appendix C. Equation (C2.2) suggests that for cruise at constant H_p , provided M does not vary to any great extent, an adequate approximation to constant engine setting is to assume constant thrust. Values of R_a/R_{ref} for this

cruise case are plotted against W_F/W_i and y_i in Figure 7a for $c = c_1$ (or, if $n = 0$, for $c = c_2\sqrt{\theta}M^n$) and in Figures 7b to 7d for $n = 0.2, 0.4$ and 0.6 in the second of these fuel flow laws (see also Figure B1). Figures 8a and 8b present curves of V_{mean}/V_i plotted against W_F/W_i and y_i for $c = c_1$ (or, if $n = 0$, for $c = c_2\sqrt{\theta}M^n$ and for $n = 0.6$ in $c = c_2\sqrt{\theta}M^n$, respectively. Only two values of n are considered because of the very small effect on mean speed. Note that the curves in Figures 7b to 7d and in 8b are obtained from a numerical integration of the form of Equation (2.2) presented for the constant thrust case in Table 7.4.

In Appendix C it is shown that, for flight in the stratosphere, a constant engine setting corresponds almost exactly to the engine setting required for a cruise-climb at constant L/D and constant V . Thus, Section 4.1.1 may be considered to deal with a constant engine setting case for flight at constant L/D in the stratosphere.

5. ENDURANCE

5.1 Specific Endurance

The purpose of considering specific endurance here is to determine the conditions necessary for maximum endurance. From Equation (2.3), the main requirement is that the fuel flow be minimised and thus, at any instant, the maximum specific endurance, i.e. $|dt/dW|_{max}$ is obtained by operating at the minimum thrust necessary to maintain flight. The general lift and drag conditions necessary to obtain $|dt/dW|_{max}$ are given in Table 7.1 and in Table 7.2 the conditions are further examined for the case when a parabolic drag law is assumed (see Equation (2.10) and Appendix A).

5.2 Endurance

Tables 7.3 and 7.4 present expressions for endurance and for the endurance ratio, t/t_{ref} , for each of the cruise cases considered previously in Section 4. The reference endurance, t_{ref} , is obtained by integrating Equation (2.4) with c and L/D held constant, i.e.

$$t_{ref} = \left(\frac{1}{c} \frac{L}{D}\right)_i \log_e \left[\frac{1}{1 - \frac{W_F}{W_i}} \right], \quad (5.1)$$

and has initial conditions identical with those assumed in calculating the endurance, t . The log term in Equation (5.1) is the same as that in Equation (4.1) and values may be obtained from Figure 2 to assist in calculating t_{ref} . By using the Figures of this Item as directed in Tables 7.3 and 7.4 and following a calculation sequence similar to that described for range in Section 4, values of t may be deduced for *any given set of initial conditions* (i.e. y_i , $[(1/c)(L/D)]_i$) once the corresponding value of t_{ref} has been calculated. The principal endurance cases of practical significance are described briefly in Sections 5.2.1 to 5.2.3.

5.2.1 Military patrol, etc.

Such operations are usually flown at a nominal $(V_{md} + \Delta V)$ while maintaining a constant cruise height (the ΔV is included as a safety margin and its magnitude would depend, in part, on the handling qualities of the aeroplane at the minimum drag condition). The relevant cruise case is flight at constant L/D and H_p .

During this type of flight, since $V_{md} \propto W^{1/2}$ (see Equation (A1.7)), engine thrust must be reduced not only to prevent the usual build up of speed but also to follow the gradual reduction in V_{md} as fuel is used.

5.2.2 Holding operations

These are generally flown at constant height and nominally constant speed or Mach number.

5.2.3 Flight at constant engine setting

In practice, the pilot technique used in the methods of Sections 5.2.1 and 5.2.2 is to adjust the engine setting at intervals to maintain flight reasonably near the nominal speed or Mach number. Consequently, the constant engine setting case is usually applicable over segments of the flight. Note, however, that for some military missions frequent throttle adjustments may well be applied in order to obtain the best performance from the aeroplane.

6. REFERENCES

1. BORE, C.L. A range formula for jet aircraft having variable specific fuel consumption. Hawker Siddeley Aviation Ltd, Kingston. Project Office Note No. 1276, June 1968.
2. PECKHAM, D.H. Range performance in cruising flight. RAE tech. Rep. 73164, 1973.



7. TABLES

TABLE 7.1
General Lift and Drag Conditions for Maximum Specific Air Range and Endurance

Cruise at a Given Value of	Specific Air Range $-\frac{dR_a}{dW}$	Specific Endurance $-\frac{dt}{dW}$	Flight Conditions for			
			Maximum Specific Air Range with		Maximum Specific Endurance with	
			$c = c_1$	$c = c_2 \theta^{1/2} M^n$	$c = c_1$	$c = c_2 \theta^{1/2} M^n$
V	$\frac{1}{W} \frac{V C_L}{c C_D}$, i.e. $\frac{1}{W} \frac{VL}{c D}$	$\frac{1}{W} \frac{C_L}{c C_D}$, i.e. $\frac{1}{W} \frac{L}{c D}$	Maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$	In troposphere, maximum $\frac{C_L}{C_D} \theta^{(n-1)/2}$, i.e. maximum $\frac{L}{D} \theta^{(n-1)/2}$; In stratosphere, maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$	Maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$	In troposphere, maximum $\frac{C_L}{C_D} \theta^{(n-1)/2}$, i.e. maximum $\frac{L}{D} \theta^{(n-1)/2}$; In stratosphere, maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$
M	$\frac{1}{W} \frac{Ma_0 \sqrt{\theta} C_L}{c C_D}$ i.e. $\frac{1}{W} \frac{Ma_0 \sqrt{\theta} L}{c D}$	as above	In troposphere, maximum $\frac{C_L}{C_D} \theta^{1/2}$, i.e. maximum $\frac{L}{D} \theta^{1/2}$; In stratosphere, maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$	Maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$	as above	In troposphere, maximum $\frac{C_L}{C_D} \theta^{-1/2}$, i.e. maximum $\frac{L}{D} \theta^{-1/2}$; In stratosphere, maximum $\frac{C_L}{C_D}$, i.e. $\left(\frac{L}{D}\right)_{max}$
H_p	$\frac{1}{c} \left(\frac{2}{\rho W S}\right)^{1/2} \frac{C_L^{1/2}}{C_D}$	as above	Maximum $\frac{C_L^{1/2}}{C_D}$	Maximum $\frac{C_L^{(1+n)/2}}{C_D}$	as above	Maximum $\frac{C_L}{C_D} V^{-n}$
Engine setting: i) fixed H_p and F_N , or ii) in stratosphere, fixed F_N/ρ . (See Appendix C).	$\frac{1}{cW} \left(\frac{2F_N}{\rho S}\right)^{1/2} \frac{C_L}{C_D^{3/2}}$	as above	Maximum $\frac{C_L}{C_D^{3/2}}$	Maximum $\frac{C_L}{C_D^{(3-n)/2}}$	as above	Maximum $\frac{C_L}{C_D^{1-n/2}}$

TABLE 7.2
Conditions for Maximum Specific Air Range Assuming a Parabolic Drag Law

Cruise at a Given Value of:	Conditions for Maximum Specific Air Range	
	$c = c_1$	$c = c_2 \theta^{1/2} M^n$
V	$C_L = C_{L_{md}}, \frac{L}{D} = \left(\frac{L}{D}\right)_{max}, y = \frac{V}{V_{md}} = 1$ see Appendix A. From Equation (A1.7), for any W , $V_{md} \sigma^{1/2} = \text{constant} = V_{e_{md}}$. If V is required speed, $\left \frac{dR_a}{dW}\right _{max}$ is obtained at H_p where $\sigma^{1/2} = V_{e_{md}}/V$.	In troposphere, $C_L = C_{L_{md}} \left(\frac{1+n+2g_0/AR}{3-n+2g_0/AR}\right)^{1/2} = \left(\frac{9.512-n}{7.512+n}\right)^{1/2} C_{L_{md}}$, $\frac{L}{D} = \left(\frac{L}{D}\right)_{max} \frac{[(1+n+2g_0/AR)(3-n+2g_0/AR)]^{1/2}}{2+2g_0/AR}$ $= \frac{(9.512-n)^{1/2}(7.512+n)^{1/2}}{8.512} \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = \left(\frac{3-n+2g_0/AR}{1+n+2g_0/AR}\right)^{1/4} = \left(\frac{7.512+n}{9.512-n}\right)^{1/4}$; In stratosphere, as for case $c = c_1$.
M	In troposphere, $C_L = C_{L_{md}} \left(\frac{2g_0/AR+1}{2g_0/AR-1}\right)^{1/2} = 0.909 C_{L_{md}}$, $\frac{L}{D} = \frac{2\left(\frac{L}{D}\right)_{max}}{1+\left(\frac{2g_0/AR-1}{2g_0/AR+1}\right)^{1/2}} = 0.952 \left(\frac{L}{D}\right)_{max}$, $y = \frac{M}{M_{md}} = \left(\frac{2g_0/AR-1}{2g_0/AR+1}\right)^{1/4} = 1.049$; In stratosphere, as for case with $c = c_2 \theta^{1/2} M^n$	$C_L = C_{L_{md}}$, $\frac{L}{D} = \left(\frac{L}{D}\right)_{max}$, $y = \frac{M}{M_{md}} = 1$.
H_p	$C_L = C_{L_{md}}/3^{1/2} = 0.577 C_{L_{md}}$, $\frac{L}{D} = \frac{3^{1/2}}{2} \left(\frac{L}{D}\right)_{max} = 0.866 \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = 3^{1/4} = 1.316$.	$C_L = \frac{(1+n)^{1/2}}{(3-n)^{1/2}} C_{L_{md}}$, $\frac{L}{D} = \frac{[(1+n)(3-n)]^{1/2}}{2} \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = \left(\frac{3-n}{1+n}\right)^{1/4}$.
Engine setting: i) fixed H_p and F_N , or ii) in stratosphere, fixed F_N/ρ . (See Appendix C)	$C_L = \frac{C_{L_{md}}}{2^{1/2}} = 0.707 C_{L_{md}}$, $\frac{L}{D} = \frac{2(2)^{1/2}}{3} \left(\frac{L}{D}\right)_{max} = 0.943 \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = 2^{1/4} = 1.189$.	$C_L = \frac{C_{L_{md}}}{(2-n)^{1/2}}$, $\frac{L}{D} = 2 \left(\frac{L}{D}\right)_{max} \frac{(2-n)^{1/2}}{(3-n)}$, $y = \frac{V}{V_{md}} = (2-n)^{1/4}$.

NOTE: Using the value of lapse rate appropriate to the troposphere in the International Standard Atmosphere,

the parameter $\frac{g_0}{AR} = -5.2559$.

TABLE 7.3
Conditions for Maximum Specific Endurance Assuming a Parabolic Drag Law

Cruise at a Given Value of:	Conditions for Maximum Specific Endurance	
	$c = c_1$	$c = c_2 \theta^{1/2} M^n$
V	$C_L = C_{L_{md}}, \frac{L}{D} = \left(\frac{L}{D}\right)_{max}, y = \frac{V}{V_{md}} = 1$ see Appendix A. From Equation (A1.7), for any W , $V_{md} \sigma^{1/2} = \text{constant} = V_{e_{md}}$. If V is required speed, $\left \frac{dt}{dW}\right _{max}$ is obtained at H_p where $\sigma^{1/2} = V_{e_{md}}/V$.	In troposphere, $C_L = C_{L_{md}} \left(\frac{1+n+2g_0/AR}{3-n+2g_0/AR}\right)^{1/2} = \left(\frac{9.512-n}{7.512+n}\right)^{1/2} C_{L_{md}}$, $\frac{L}{D} = \left(\frac{L}{D}\right)_{max} \frac{[(1+n+2g_0/AR)(3-n+2g_0/AR)]^{1/2}}{2+2g_0/AR}$ $= \frac{(9.512-n)^{1/2}(7.512+n)^{1/2}}{8.512} \left(\frac{L}{D}\right)_{max}$ $y = \frac{V}{V_{md}} = \left(\frac{3-n+2g_0/AR}{1+n+2g_0/AR}\right)^{1/4} = \left(\frac{7.512+n}{9.512-n}\right)^{1/4}$; In stratosphere, as for case $c = c_1$.
M	$C_L = C_{L_{md}}, \frac{L}{D} = \left(\frac{L}{D}\right)_{max}, y = \frac{M}{M_{md}} = 1$. If M is the required Mach number, height to fly is given by Equations (D1.2) and (D1.3)	$C_L = C_{L_{md}} \left(\frac{1-AR/2g_0}{1+AR/2g_0}\right)^{1/2} = 1.100 C_{L_{md}}$, $\frac{L}{D} = \left(\frac{L}{D}\right)_{max} (1-AR/2g_0)^{1/2} (1+AR/2g_0)^{1/2} = 0.995 \left(\frac{L}{D}\right)_{max}$, $y = \frac{M}{M_{md}} = \left(\frac{1+AR/2g_0}{1-AR/2g_0}\right)^{1/4} = 0.954$.
H_p	$C_L = C_{L_{md}}$, $\frac{L}{D} = \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = 1$.	$C_L = C_{L_{md}} \left(\frac{2+n}{2-n}\right)^{1/2}$, $\frac{L}{D} = \frac{[(2-n)(2+n)]^{1/2}}{2} \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = \left(\frac{2-n}{2+n}\right)^{1/4}$.
Engine setting: i) fixed H_p and F_N , or ii) in stratosphere, fixed F_N/ρ (See Appendix C)	$C_L = C_{L_{md}}$, $\frac{L}{D} = \left(\frac{L}{D}\right)_{max}$, $y = \frac{V}{V_{md}} = 1$; $V_{md} = \left(\frac{2F_N}{\rho S C_{D_{md}}}\right)^{1/2}$.	$C_L = \frac{C_{L_{md}}}{(n+1)^{1/2}}$, $\frac{L}{D} = 2 \left(\frac{L}{D}\right)_{max} \frac{(n+1)^{1/2}}{(n+2)}$, $y = \frac{V}{V_{md}} = (n+1)^{1/4}$.

NOTE: Using the value of lapse rate appropriate to the troposphere in the International Standard Atmosphere,

the parameter $\frac{g_0}{AR} = -5.2559$.