

INTRODUCTION TO NON-DIMENSIONAL METHODS FOR THE MEASUREMENT OF PERFORMANCE OF TURBINE-ENGINED HELICOPTERS

1. NOTATION AND UNITS*

		SI	British
C_P	power coefficient, $\frac{P \times 550}{\rho \left(2\pi \frac{\Omega}{60} R \right)^3 \pi R^2}$ in British units, $\frac{P \times 10^3}{\rho \left(2\pi \frac{\Omega}{60} R \right)^3 \pi R^2}$ in SI units		
C_T	thrust coefficient, $\frac{F}{\rho \left(2\pi \frac{\Omega}{60} R \right)^2 \pi R^2}, \text{ or } \frac{W}{\rho \left(2\pi \frac{\Omega}{60} R \right)^2 \pi R^2}$ in hover		
D	representative engine dimension	m	ft
F	rotor thrust	N	lbf
f_1 to f_{17}	functions defined in Equations (3.1) to (3.12) and (4.1) to (4.5)		
G	fuel flow $\left\{ \begin{array}{l} \text{volume flow basis} \\ \text{weight flow basis} \\ \text{mass flow basis} \end{array} \right.$	l/h N/h kg/h	gal/h lbf/h slug/h
g	acceleration due to gravity	m/s ²	ft/s ²
K	constant used in Equations (A1.2), (A1.6) and (A1.7)		
m	helicopter mass	kg	slug
N_c	engine compressor speed	rev/min	rev/min
$N_{c_{ref}}$	reference value of engine compressor speed	rev/min	rev/min
n_c	per cent engine compressor speed, $(N_c/N_{c_{ref}})100$		
P	shaft power output from engine	kW	hp
P_R	shaft power required at rotor	kW	hp

* The units quoted here are those most commonly used in practice; they do not constitute coherent sets (hence, for normal analytical purposes, the use of conversion factors would be necessary). Such units are permissible here since the object of the Item is the presentation, in parametric form, of flight-measured data and it is only necessary that the same units be applied throughout any given piece of work.

p	atmospheric pressure*	N/m^2	lbf/ft^2
Q	torque	N m	lbf ft
Q_{ref}	reference value of torque	N m	lbf ft
q	per cent torque, $(Q/Q_{ref})100$		
R	rotor radius	m	ft
T	ambient temperature	K	$^{\circ}\text{C}$
T_{ref}	engine reference temperature	K	$^{\circ}\text{C}$
V	true airspeed*	km/h	kn
V_c	rate of climb*	m/min	ft/min
W	helicopter weight, mg	N	lbf
Δ	engine configuration parameter, <i>e.g.</i> inlet guide vane position		
δ	relative pressure, p/p_0		
Θ	collective pitch angle	deg	deg
θ	relative temperature, T/T_0		
ρ	density of air	kg/m^3	slug/ft^3
σ	relative air density, ρ/ρ_0		
Ω	rotor speed*	rev/min	rev/min
Ω_s	standard rotor speed*	rev/min	rev/min
ω	relative rotor speed, Ω/Ω_s		

Suffix

0	denotes value at sea level in International Standard Atmosphere (Item No. 68046 gives numerical values)
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* Units other than those quoted may also be in use.

2. INTRODUCTION

This Item introduces methods of measuring the steady-state performance of turbine-engined helicopters in terms of non-dimensional parameters (also known as compound, referred or reduced parameters). The *application* of these methods is described in a separate series of Items*.

The basis of the test methods is the arrangement of the relevant dimensional quantities into several non-dimensional parameters by means of dimensional analysis. The relationship between any given pair of non-dimensional parameters is established for a given flight condition by flying the helicopter to maintain constant values of each of the remaining non-dimensional parameters.

This approach to performance measurement removes the limitation placed by particular helicopter weights and atmospheric conditions of the test on the use of, for example, forward speed, rate of climb, power and fuel flow test data. A set of test measurements in non-dimensional form can be used to produce information relevant to atmospheric conditions other than those of the tests while neither requiring a lengthy reduction process nor having an adverse effect on the accuracy of the measured data. Further, by a suitable initial selection of test site(s) and choice of the ranges of values of the dimensional parameters, a relatively small number of tests, made in a limited range of atmospheric conditions, can produce information relevant to much of the helicopter's flight envelope.

3. HELICOPTER PERFORMANCE

Helicopter performance is measured in terms of the power, or in some cases the rotor collective pitch angle, required to maintain steady flight for various atmospheric conditions, over a range of weights and, if necessary, external configurations. In addition, engine fuel flow data are necessary to determine the helicopter's range performance (besides being one of the main parameters in the consideration of engine performance).

3.1 Application of Dimensional Analysis to Factors Affecting Helicopter Performance

Experience shows that the factors affecting the performance of a family of geometrically similar helicopters, in steady-state motion, may be assumed to be the engine power, helicopter weight or mass, rotor radius[†], forward speed, rate of climb, rotor speed and the ambient conditions. Dimensional analysis is used to show how these quantities can be related (see, for example, Appendix A) and different forms of non-dimensional relationships are obtained depending on the initial choice of dimensional parameters.

Apart from the alternative forms of non-dimensional relationships appropriate to measurements of power and collective pitch, different forms also arise depending on whether helicopter weight or mass is used and on the particular atmospheric parameters chosen.

Three methods of defining the atmospheric conditions are considered here:

- (i) density alone,
- (ii) density and temperature,
- (iii) pressure and temperature.

* See Item No. 73027 for hover performance and Item No. 74042 for level flight performance.

† Usually taken as the main rotor radius for a single rotor helicopter; in the case of a multi-rotor helicopter any radius may be used.

(The effects of other atmospheric parameters, such as viscosity and humidity, are usually small enough to be neglected.) A test method in which only density is used affords a more compact presentation of results but does not permit the effects of changes in rotor Mach number to be investigated. The effect of disregarding rotor Mach number varies and depends on the range of temperature in which tests, including hover, are carried out as well as the flight condition considered. Consequently, significant errors can arise if such test results are used to predict performance in temperatures markedly different from those in which the tests were conducted.

3.2 Practical Forms of the Non-dimensional Relationships

In practice the basic non-dimensional relationships (for example, see Appendix A, Equation (A1.7)) are rarely used since more convenient, modified, forms can be derived if:

- (i) linear dimensions are omitted since they are constant for the helicopter under test,
- (ii) atmospheric properties are expressed as ratios to sea level standard conditions,
- (iii) rotor speed and torque, and hence power, are each expressed as fractions or percentages of reference values.

In consequence of (i) to (iii) above, the parameters often used are not non-dimensional; however, it is usual to refer to them as “non-dimensional” and the practice is retained here. In addition, the rules of dimensional analysis permit the non-dimensional parameters to be regrouped by multiplication or division, provided the number of such parameters is unchanged (see Appendix A, Equations (A1.8) and (A1.9)).

In this Item, non-dimensional relationships are derived and quoted in terms of engine power and helicopter weight for each of the three methods of defining the atmospheric conditions listed in Section 3.1. Additional notes are included to describe the changes necessary if collective pitch and/or helicopter mass is used. In general the relationships given in the following Sections follow current British practice, but where alternative forms exist that are in widespread use they are also quoted.

3.2.1 Non-dimensional relationships based on atmospheric density

In this case the engine power* required to maintain flight is

$$P = f_1(W, V, V_c, \Omega, R, \rho) \quad (3.1)$$

from which the following non-dimensional relationship* can be derived

$$\frac{P}{\rho \Omega^3 R^5} = f_2\left[\left(\frac{W}{\rho \Omega^2 R^4}\right), \left(\frac{V}{\Omega R}\right), \left(\frac{V_c}{\Omega R}\right)\right]. \quad (3.2)$$

Equation (3.2) can be modified as described in Section 3.2 to give*

$$\frac{P}{\sigma \omega^3} = f_3\left[\left(\frac{W}{\sigma \omega^2}\right), \left(\frac{V}{\omega}\right), \left(\frac{V_c}{\omega}\right)\right]. \quad (3.3)$$

* If rotor collective pitch angle, Θ , is used in place of engine power then, in Equations (3.1) to (3.3) the power parameter is simply replaced by Θ and new functions (f'_1 , f'_2 , f'_3 , say) inserted on the right hand sides of the equations.

If, for the helicopter under test, it is not possible to vary the rotor speed, then Equation (3.3) may be simplified to

$$\frac{P}{\sigma} = f_4 \left[\left(\frac{W}{\sigma} \right), (V), (V_c) \right]. \quad (3.4)$$

If helicopter mass is used instead of weight, then the weight parameter in Equation (3.2) is replaced by $mg/\rho\Omega^2 R^4$ and that in Equations (3.3) and (3.4) by $m/\sigma\omega^2$.

Note that the test methods based on Equations (3.2) to (3.4) are not capable of taking account of rotor Mach number effects (see Section 3.1, and Sections 3.2.2 and 3.2.3).

3.2.2 Non-dimensional relationships based on atmospheric density and temperature

Rotor Mach number effects may be taken into account if ambient temperature is included with the parameters in Equation (3.1). When this is done, one form of the non-dimensional relationship for engine power required (see also footnote to Section 3.2.1) is:

$$\frac{P}{\rho\Omega^3 R^5} = f_5 \left[\left(\frac{W}{\rho\Omega^2 R^4} \right), \left(\frac{V}{\Omega R} \right), \left(\frac{V_c}{\Omega R} \right), \left(\frac{T}{\Omega^2 R^2} \right) \right]. \quad (3.5)$$

It is usual for the term $T/\Omega^2 R^2$ to be replaced by $\Omega R/T^{1/2}$ (which is effectively the rotor tip Mach number at zero forward speed) and in British practice the most commonly used form of Equation (3.5) is

$$\frac{P}{\sigma\omega^3} = f_6 \left[\left(\frac{W}{\sigma\omega^2} \right), \left(\frac{V}{\omega} \right), \left(\frac{V_c}{\omega} \right), \left(\frac{\omega}{\theta^{1/2}} \right) \right]. \quad (3.6)$$

Another form of Equation (3.6) is obtained by multiplying the forward speed and rate of climb parameters in Equation (3.6) by the rotor speed parameter to give

$$\frac{P}{\sigma\omega^3} = f_7 \left[\left(\frac{W}{\sigma\omega^2} \right), \left(\frac{V}{\theta^{1/2}} \right), \left(\frac{V_c}{\theta^{1/2}} \right), \left(\frac{\omega}{\theta^{1/2}} \right) \right]. \quad (3.7)$$

Thus in Equation (3.7) the forward speed and rate of climb are effectively expressed as Mach numbers while in Equation (3.6) they are expressed as advance ratios.

In the hover case it can be seen from Equation (3.5) that Equations (3.6) and (3.7) can be written

$$C_P = f_8 \left[(C_T), \left(\frac{\omega}{\theta^{1/2}} \right) \right]. \quad (3.8)$$

This form is used mainly in American practice for the presentation of hover performance data. The coefficients C_P and C_T can also be introduced into Equations (3.6) and (3.7) for cases other than hover; however, since in forward or climbing flight $W \neq F$, it is then more correct to speak of a weight coefficient unless, of course, F is itself measured.