

A METHOD FOR ESTIMATING DRAG-RISE MACH NUMBER AT ZERO INCIDENCE OF SMOOTH OR BUMPY NON-DUCTED AXISYMMETRIC BODIES WITHOUT OR WITH FINS

SUMMARY

A method is given for estimating drag-rise Mach number at zero incidence of smooth or bumpy non-ducted, axisymmetric bodies without or with stabilizing fins. For finned bodies a feature of the sonic area rule is used as an artifice to derive an equivalent body to which the method is then applied.

The method is based on the observation in the limited number of experimental data available that shock-wave drag develops rapidly as shock waves form downstream of the axial location of the body maximum cross-sectional area. Analogy with the observed development of shock-wave drag on two-dimensional aerofoils suggests that the drag-rise Mach number is that at which the shock, either on first being formed or on moving from an upstream station, first appears to the rear of this location. The problem of predicting drag-rise Mach number is thus reduced to that of predicting the free-stream Mach number at which the shock first appears at or downstream of this location and is achieved by applying a simple criterion to the calculated equivalent shock-free pressure-distribution for the appropriate body.

The validity of the method has been demonstrated by comparison with experimental data for a number of bodies. On this basis it is concluded that the present method should estimate drag-rise Mach number to an accuracy in absolute terms of ± 0.015 for the majority of cases. The effect on M_D of changes in body shape from one configuration to another should be estimated to appreciably greater accuracy.

This Memorandum supersedes T.D. Memor. 71008 which was issued before the applicability of the criterion to bumpy and finned bodies had been established and was thus restricted to smooth bodies.

Issued August 1974

1

1. INTRODUCTION

A knowledge of the drag-rise Mach number of bodies is of importance in weapon design, design of aircraft external stores and in the context of trends towards wider fuselages and higher cruising speeds for civil aircraft.

For the specific class of smooth, non-ducted, axisymmetric bodies at zero incidence a method of estimating drag-rise Mach number was provided by T.D. Memor. 71008. The present Memorandum supersedes T.D. Memor. 71008 and extends the method to bumpy bodies and bodies with fins. The method should be sufficiently straightforward to use at any stage, either directly for individual bodies or indirectly through preparing generalised charts for families of related bodies. The detailed development of the method is described in References 1 and 2.

The drag-rise in question develops with increasing free-stream Mach number when a shock wave forms to terminate the region of local supersonic flow adjacent to the surface of the body and subsequently grows in strength and in height above the surface. The source of this wave drag is the loss of momentum at the shock, but the drag itself is transmitted to the body by readjustments in the pressure distribution over the surface. A full prediction of the wave drag development would thus require a detailed prediction of the shock growth or, alternatively, of the related development in the mixed subsonic/supersonic velocity distribution on the surface.

In the case of two-dimensional aerofoil sections it was found possible³ to define a stage in the development of the local supersonic flow up to which any shock-wave drag was generally trivial but beyond which the drag could be expected to rise rapidly. This stage was that at which the shock, either on first being formed or on moving from an upstream station, first appeared immediately to the rear of the aerofoil crest. Furthermore, this stage could be predicted directly by means of a simple criterion applied to a pressure distribution calculated for incompressible flow.

By analogy, on an axisymmetric body at zero incidence the stage at which shock-wave drag could be expected to rise rapidly would correspond to the shock appearing to the rear of the axial location of maximum cross-sectional area. Starting from this analogy it has now been demonstrated^{1, 2} that drag-rise Mach number of smooth or bumpy non-ducted, axisymmetric bodies without or with fins can be predicted directly by an equally simple criterion applied to the calculated equivalent shock-free axial velocity distribution.

2. NOTATION AND DEFINITIONS

2.1 Notation

A_F	cross-sectional-area of fins measured in plane normal to body axis
C_D	drag coefficient (based on maximum cross-sectional area of basic body)
C_p	pressure coefficient
d	maximum body diameter of basic body
l	body length

M_0	free-stream Mach number
M_D	drag-rise Mach number (see Section 2.2(vii))
M_L	local Mach number
R	maximum radius of basic body
r	local radius of basic body
r_b	radius of base excluding any equivalent area representing stabilizing fins
r_e	local radius of equivalent body
V	local flow velocity
V_0	free-stream velocity
v_r	perturbation velocity in radial direction
v_x	perturbation velocity in axial direction
x	axial distance (see Figure 1)
β	$\sqrt{1 - M_0^2}$
γ	ratio of specific heats of air
δx	length measured in x direction of bump added to basic body
δr	increment in local body radius due to adding bump to basic body
θ	body surface slope relative to longitudinal axis
ξ	axial distance non-dimensionalised by body length to closure (see Figure 1)

Suffixes

a	denotes analogous body
A	denotes afterbody
max	denotes maximum value
N	denotes nose

2.2 Definitions

(i) *Smooth Body*

For the purposes of this Memorandum a smooth body is defined as one without discontinuities in surface slope for which the local radius defining the cross-section increases with axial distance (x) without inflection points to a maximum and stays constant or decreases without inflection points to a minimum. The requirement for continuous surface slope in the longitudinal direction does not require the radius of curvature of the body generator to be continuous. The base area need not be zero.

(ii) *Bumpy Body*

Similarly, for the purposes of this Memorandum a bumpy body is the counterpart of a smooth body in that it *does have* one or more inflection points in the longitudinal surface slope. For the strict application of the method, however, discontinuities in surface slope are still excluded.

(iii) *Equivalent Body*

A hypothetical body derived from a finned body by wrapping the actual cross-sectional-area distribution of the fins, measured in a plane normal to the body axis, concentrically around the actual (or basic) body to form an “equivalent” axisymmetric body. Unless some form of area ruling has been undertaken in the original design, this equivalent body will be bumpy.

(iv) *Geometric Terms*

The terms nose, cylindrical centrebody and afterbody are used in a conventional sense but for clarity are illustrated in Figure 1.

(v) *The Crest*

The crest is defined as the axial location for which the body surface is tangential to the undisturbed free-stream direction. For a body without a cylindrical centrebody and at zero incidence the crest corresponds to the position of the maximum cross-sectional area. For a body with a cylindrical centrebody and at zero incidence the crest is at the nose/cylinder junction.

An ambiguity can arise in this definition in the special case of a body in which the basic configuration has a cylindrical centrebody and the generation of the equivalent body adds to this centrebody a localised bump covering less than the total centrebody length. In this case the crest should still be interpreted as the nose/cylinder junction but unless the nose is appreciably blunted or the bump is trivial the drag-rise Mach number will be determined by the minimum pressure coefficient generated by the bump or boattailing and the definition of the crest is of no practical significance.

(vi) *Fineness Ratio*

The overall fineness ratio of the basic body is defined as $l/d = l/2R$.

The fineness ratio of a bump superimposed onto a basic body is defined as $\delta x/2(\delta r)_{max} = (\delta x/l)/2(\delta r/l)_{max}$.

(vii) *Drag-Rise Mach Number*

(a) Observed value.

The definition of observed drag-rise Mach number is open to some debate but for the analysis here and in References 1 and 2 has been taken as that free-stream Mach number for which the variation of C_D with free-stream Mach number first noticeably departs from an extrapolation of the subcritical curve.

(b) Estimated value.

The estimated drag-rise Mach number is given by the criterion in Section 3.

3. THE CRITERION FOR ESTIMATING DRAG-RISE MACH NUMBER

M_D is the lower of the free-stream Mach numbers for which the local Mach number at the crest (see Section 2.2(v)) or at the local-velocity peak downstream of the crest (should such a peak exist[†]), equals 1.02 in the equivalent shock-free flow calculated using incompressible potential-flow theory and the Goethert compressibility rule as described in Section 4.2.

4. THE METHOD

The method, which requires a computer program (see footnotes to Section 4.1) to calculate the nominally-exact incompressible-flow velocity distribution for an axisymmetric body from a knowledge of body ordinates, consists of the following steps.

- (i) For a finned body calculate the x -wise distribution of the total cross-sectional-area of the fins, $A_F(x)$, measured in a plane normal to the body axis.
- (ii) Calculate the radius of the equivalent body

$$r_e(x) = \left[r(x)^2 + \frac{A_F(x)}{\pi} \right]^{1/2}.$$

For a body without fins the two above steps are, of course, omitted.

- (iii) Derive analogous bodies from the given[‡] body by shrinking the body radii by a factor $\beta = \sqrt{1 - M_0^2}$ (see Section 4.2) for selected values of free-stream Mach number likely to embrace^{††} M_D for the given body.

[†] Typically, a local-velocity peak can occur downstream of the crest on smooth configurations with afterbodies

(a) when the afterbody is less slender than the nose,

(b) when a cylindrical centrebody is present,

and, generally, on any body

(c) that has a bump downstream of the crest,

(d) that has a sustained region of sufficiently-low curvature approaching, embracing, or downstream of the crest followed by a region of curvature sufficiently high to allow the flow to expand.

[‡] i.e. the actual body for a body without fins or the equivalent body for a body with fins.

^{††} Guidance on the likely value of M_D for smooth bodies having the maximum local velocity at, or forward of the crest and for smooth closed bodies having the maximum local velocity downstream of the crest is provided by Figures 10 and 11 respectively. These figures are based on simple correlations in Reference 1. No simple guidance can be given for other classes of bodies.

- (iv) Compute the incompressible-flow velocity distribution for the analogous bodies selected in step (iii) using nominally-exact potential-flow theory (Section 4.1).
- (v) Derive the axial and radial velocity increments for the analogous bodies from the expressions

$$\frac{v_{xa}}{V_0} = \left(\frac{V_a}{V_0} \cos \theta_a \right) - 1 ,$$

$$\frac{v_{ra}}{V_0} = \frac{V_a}{V_0} \sin \theta_a .$$

- (vi) Derive the axial and radial velocity increments and the total local velocity for the given body at the selected free-stream Mach numbers from the expressions

$$\frac{v_x}{V_0} = \left(\frac{v_{xa}}{V_0} \right) \frac{1}{1 - M_0^2} ,$$

$$\frac{v_r}{V_0} = \left(\frac{v_{ra}}{V_0} \right) \frac{1}{\sqrt{1 - M_0^2}} ,$$

$$(V/V_0) = [\{1 + (v_x/V_0)\}^2 + (v_r/V_0)^2]^{1/2} .$$

- (vii) Interpolate between calculated values over the selected range of M_0 to find the lower of the free-stream Mach numbers ($\equiv M_D$) for which the local Mach number at the crest, or at the local-velocity peak down-stream of the crest (should such a peak exist), equals 1.02.

Local velocity is related to local Mach number through the two following well-known expressions for pressure coefficient.

$$1 + \frac{1}{2} \gamma M_0^2 C_p = \left(1 + \frac{\gamma - 1}{2} M_0^2 \left[1 - \left(\frac{V}{V_0} \right)^2 \right] \right)^{\gamma/(\gamma - 1)}$$

$$= \left(\frac{2 + (\gamma - 1) M_0^2}{2 + (\gamma - 1) M_L^2} \right)^{\gamma/(\gamma - 1)}$$

which when the ratio of the specific heats of air is taken as 1.40 yield

$$\left(\frac{V}{V_0} \right)^2 = \frac{M_L^2 (1 + 0.2 M_0^2)}{M_0^2 (1 + 0.2 M_L^2)} .$$

The relationship between (V/V_0) and M_0 when M_L has the specific value of 1.02 is plotted in Figure 2. The equivalent relationship between C_p and M_0 is also shown.

4.1 Nominally-Exact Incompressible Potential-Flow Theory

Numerical solutions of exact equations yielding the axial velocity distribution at the surface of an axisymmetric body at zero incidence in incompressible potential flow are presented in References 4 and 5[†].

In using these methods some care is needed to ensure that the specified body ordinates are smooth otherwise a smooth velocity distribution will not be obtained. Difficulties in obtaining smooth velocity distributions will also be experienced if the step-length between the ordinates specified is changed discontinuously. (See also Section 4.3, item (iv).

Experience has shown that the methods may be used quite successfully to obtain velocity distributions on truncated bodies having non-zero base area, provided results are not required in the immediate vicinity (say, within one base diameter) of the truncation.

4.2 Equivalent Shock-Free Flow and the Goethert Compressibility Rule

The term equivalent shock-free flow is used to identify a velocity (or pressure) distribution for just-supercritical free-stream Mach numbers calculated from low-speed data by accounting for compressibility through a simple rule; at such Mach numbers the calculated pressure distributions would be expected to differ from the real flow because of the presence of shock waves in the latter. Nevertheless the equivalent shock-free flow can be used in an empirical way to predict the appearance of these shock waves and hence the onset of shock-wave drag.

The basis on which equivalent shock-free flow is calculated in the procedure of Section 4 is the existence of a relationship (in linearised theory) between the compressible flow past a given body and the incompressible flow about an analogous body affinely related to the given body.

The analogous body is derived from the given body by shrinking the body radii by a factor $\beta = \sqrt{1 - M_0^2}$. The velocity increments in compressible flow on the surface of the given body are then related to the incompressible increments on the analogous body by

$$\frac{v_x}{V_0} = \left(\frac{v_{xa}}{V_0} \right) \frac{1}{1 - M_0^2} ; \quad \frac{v_r}{V_0} = \left(\frac{v_{ra}}{V_0} \right) \frac{1}{\sqrt{1 - M_0^2}} .$$

The above relationship was established by Goethert⁸ and is often referred to as the Goethert compressibility rule.

The adequacy of the rule depends on the velocity perturbation ratios v_x/V_0 and v_r/V_0 being small compared with unity and so the rule is clearly wrong in the vicinity of a stagnation point. However, in the region of interest in this Memorandum, namely the crest or at the location of a local-velocity peak downstream of the crest (where such a peak exists) the rule is reliable (except possibly for bodies of very low fineness ratio[‡]) and is known^{1, 2} to give good agreement with experiment up to free-stream Mach numbers closely approaching that for which local sonic conditions first appear on the body.

[†] The method of Reference 4, extended to include the Goethert compressibility rule, has been programmed in ALGOL at Hawker Siddeley Aviation Ltd, Hatfield and Aircraft Research Association Ltd, Bedford for use in the ICL 1900 series and 4100 computers respectively, and also in Fortran IV for IBM computers at British Aircraft Corporation (Military Aircraft Division), Warton.

The programs in Reference 5, also extended to include the Goethert compressibility rule, have been re-written in ALGOL 60 for use at the Royal Aircraft Establishment, Bedford on the ICL 4130 computer. For bodies with very localised changes in area distribution (as may occur in bumpy bodies or bodies equivalent to finned bodies) these programs may be the less suitable because the numerical integration procedure embodied in the programs uses a prescribed distribution (Gaussian quadrature) of 40 points on the body which may or may not adequately represent the geometry of the bump. In principle, of course, the programs could be modified to increase the number of points.

[‡] i.e. small values of l/d .

4.3 Applicability and Accuracy

An assessment^{1, 2} of experimental data has shown that the method has a wide range of applicability. This stems from the general validity of the concept that the first occurrence of shock waves anywhere downstream of the crest corresponds to the onset of shock-wave drag, and hence by definition to drag-rise Mach number, and from the accuracy with which this occurrence can be predicted.

The method appears equally applicable to smooth and bumpy bodies without and with fins provided in the latter case the artifice is adopted whereby the local cross-sectional area of the fins is treated as an equivalent body cross-sectional-area distribution and provided the bumps and fins are not too severe in size and shape. Furthermore, there is some evidence² that the criterion may also be applicable to simple wing-body configurations when similarly treated. In the case of finned bumpy bodies, however, there are insufficient experimental data satisfactorily to evaluate the criterion and a complete absence of systematic data that might be used to establish the range of bump fineness ratios and shapes for which the criterion would be applicable.

In principle the applicability is conditional upon the flow over the body being everywhere attached but in practice good agreement between observed and predicted drag-rise Mach number has been obtained when some degree of flow separation on the afterbody has almost certainly been present. However, in such cases the separation that is permissible for good agreement still to exist must be sufficiently small for the calculated velocity distribution to represent the velocity distribution in real viscous flow at least up to the crest or local-velocity peak downstream of the crest (when such a peak exists).

During the assessment of the method against experimental data some limitations have become apparent that can conveniently be expressed in terms of body geometry.

(i) Overall Fineness Ratio

Lack of experimental data and concern over possible limitations of a compressibility rule founded in linearized theory has restricted the evaluation of the method, except for one configuration without an afterbody, to overall fineness ratios between 5 and 13.

(ii) Nose Fineness Ratio

The method¹ has been found to give *slightly optimistic predictions* when the nose is between[†] $1/12$ and $1/4$ of the body length l . However, even in these cases the estimated values of M_D corresponded to values of free-stream Mach number where the measured drag coefficients had risen by less than $7\frac{1}{2}$ per cent above their subcritical values and thus the drag-rise criterion remained, for most practical purposes, an acceptable method of estimation.

(iii) Bodies Without Boat-tailing

The method has *failed*¹ on smooth configurations without fins when no afterbody was present ($r_b/R = 1.0$) and this is expected to be a general result. Thus for such configurations its use is *not recommended*. But the criterion does appear to be applicable² to fin-stabilized bodies without boat-tailing if an equivalent area distribution is used to represent the fins. However only 2 cases have been evaluated.

[†] Configurations with noses shorter than $1/12l$ have not been evaluated.