

LOST RANGE, FUEL AND TIME DUE TO CLIMB AND DESCENT: AIRCRAFT WITH TURBO-JET AND TURBO-FAN ENGINES

1. NOTATION AND UNITS*

		British	SI
D	drag	lbf	N
F_N	total net thrust	lbf	N
g	acceleration due to gravity	ft/s ²	m/s ²
H	geopotential height	ft	m
h_e	energy height, $H + \left(\frac{rV}{3\ 600}\right)^2 / 2g_0$	ft	m
L	lift	lbf	N
m	aircraft mass	slug	kg
Q	total engine fuel consumption	$\left\{ \begin{array}{l} \text{weight flow basis} \\ \text{mass flow basis} \end{array} \right.$	$\left\{ \begin{array}{l} \text{N/h} \\ \text{kg/h} \end{array} \right.$
r	conversion factor (6 076.12 ft/n.mile, 1 000 m/km, 1 852 m/n.mile)	ft/n.mile	m/km, m/n.mile
s	distance along flight path	n.mile	km, n.mile
t	time	h	h
V	true airspeed	kn	km/h, kn
W	aircraft weight, mg	lbf	N
W_F	weight of fuel	lbf	N
Δh_e	change in energy height	ft	m
ΔR_a	lost range	n.mile	km, n.mile
δR_a	contribution to lost range due to change of energy height	n.mile	km, n.mile
Δt	lost time	h	h

* It is common British practice to work in terms of aircraft weight and fuel weight flow. However, in SI units, and occasionally with British units, aircraft mass and fuel mass flow are used. In Appendix A equations are derived in terms of aircraft weight and fuel weight flow only, but the results obtained and quoted in the Item are equally applicable if aircraft mass and fuel mass flow are used, provided the substitution $W = mg$ is made. The SI units have not necessarily received universal acceptance for the particular applications, *i.e.* other multiples and sub-multiples may be used.

δt	contribution to lost time due to change of energy height	h	h
ΔW_F	lost fuel	lbf	N
δW_F	contribution to lost fuel due to change of energy height	lbf	N

Suffixes

cl	climb
cr	cruise
d	descent
Δh_e	change in energy height
0	denotes value at sea level in International Standard Atmosphere (Item No. 68046 gives numerical values)

2. INTRODUCTION

This Item gives an approximate method for estimating “lost” range, “lost” fuel and “lost” time due to climb and descent. The use of these quantities permits estimates of still-air range, fuel and time based on cruise performance alone to be adjusted for the net loss of energy during climb (including acceleration) and descent (including deceleration). The method is best suited to preliminary calculations at a stage when the available airframe and engine data are minimal and for cases where the distances flown in climb and descent are not a substantial part of the total flight distance.

The subject of range estimation is introduced in Item No. 73018 and the full equations derived therein are used in Item No. 73019 to develop approximate methods for estimating cruise range. The present Item can be used in conjunction either with such data or with established cruise performance to estimate the *total* flight distance and time with a given fuel load or the *total* fuel required and time taken to cover a given *total* flight distance.

- (i) In the former case an estimate is first made of the distance that could be flown by burning all of the fuel under cruise conditions. The total flight distance is then obtained by subtracting the “lost” range from this cruise distance.
- (ii) In the latter case an estimate is first made of the fuel required to cover the total flight distance under cruise conditions. The total fuel is obtained by adding the “lost” fuel to this cruise fuel.

In both cases the total time is obtained by adding the “lost” time to the time taken to cover the total flight distance at the cruise speed.

A brief summary of the calculation procedure is given in Table 7.1.

3. THE LOST RANGE

The lost range is defined as

$$\Delta R_a = \Sigma \delta R_a = \Sigma W_{F, \Delta h_e} [(V/Q)_{cr} - (V/Q)_{\Delta h_e}] \quad (3.1)$$

where each contribution to lost range, δR_a , is the difference between the distance flown during a change of energy height*, Δh_e , in which a quantity of fuel $W_{F, \Delta h_e}$ is burnt and the distance that could be flown in cruise at the cruise height and speed on that quantity of fuel. In Appendix A a relationship is derived between δR_a and Δh_e and presented as Equation (A1.4).

3.1 Lost Range Due to Climb

From Equation (A1.4) the contribution to lost range due to a climb is

$$\delta R_{a, cl} = \frac{\Delta h_{e, cl}}{r} \left(\frac{W}{F_N - D} \right)_{cl} \left[\frac{(V/Q)_{cr}}{(V/Q)_{cl}} - 1 \right]. \quad (3.2)$$

To obtain a more convenient form of Equation (3.2) a satisfactory approximation for present purposes is obtained if it is assumed that:

- (i) the angle of climb is small, *i.e.*

$$\left(\frac{W}{D} \right)_{cl} = \left(\frac{L}{D} \right)_{cl},$$

- (ii) the equivalent airspeeds in climb and cruise are similar, *i.e.*

$$\left(\frac{L}{D} \right)_{cl} \approx \left(\frac{L}{D} \right)_{cr}.$$

When the assumptions (i) and (ii) are incorporated in Equation (3.2) the expression for $\delta R_{a, cl}$ can be written as

$$\delta R_{a, cl} = \frac{\Delta h_{e, cl}}{r} \left(\frac{L}{D} \right)_{cr} \left[\frac{\frac{(V/Q)_{cr}}{(V/Q)_{cl}} - 1}{\left(\frac{F_N}{W} \right)_{cl} \left(\frac{L}{D} \right)_{cr} - 1} \right] \quad (3.3)$$

and may be used in this form provided that satisfactory estimates can be obtained for $(F_N/W)_{cl}$ and $(V/Q)_{cr}/(V/Q)_{cl}$. Derivation 2 presents the results of an examination of the climb, cruise and descent performance in ISA conditions of a variety of aircraft types and shows that for the purposes of this Item the relevant climb and cruise data for turbo-jet and turbo-fan aircraft are sufficiently well represented by the relationship

* The concept of energy height, in relation to climb performance, is introduced in Item No. Perf. EG3/3.

$$\frac{(V/Q)_{cr}}{(V/Q)_{cl}} = 1.4 \left(\frac{F_N}{W} \right)_{cl} \left(\frac{L}{D} \right)_{cr} - 0.4 . \quad (3.4)$$

When Equations (3.3) and (3.4) are combined the resulting expression for the contribution to lost range due to a climb is

$$\delta R_{a, cl} = 1.4 \frac{\Delta h_{e, cl}}{r} \left(\frac{L}{D} \right)_{cr} . \quad (3.5)$$

3.2 Lost Range Due to Descent

From Equation (A1.4) the contribution to lost range due to a descent is

$$\delta R_{a, d} = \frac{\Delta h_{e, d}}{r} \left(\frac{W}{F_N - D} \right)_d \left[\frac{(V/Q)_{cr}}{(V/Q)_d} - 1 \right] . \quad (3.6)$$

Note that for descent $\Delta h_{e, d}$ is negative.

A more convenient form of Equation (3.6) can be obtained if similar assumptions are made to those used for climb in Section 3.1 items (i) and (ii). The expression for $\delta R_{a, d}$ can then be written

$$\delta R_{a, d} = \frac{\Delta h_{e, d}}{r} \left(\frac{L}{D} \right)_{cr} \left[\frac{\frac{(V/Q)_{cr}}{(V/Q)_d} - 1}{\left(\frac{F_N}{W} \right)_d \left(\frac{L}{D} \right)_{cr} - 1} \right] \quad (3.7)$$

and may be used in this form, provided that satisfactory estimates can be obtained for $(F_N/W)_d$ and $(V/Q)_{cr}/(V/Q)_d$. For the purposes of this Item Derivation 2 suggests that the relevant descent and cruise data for the majority* of turbo-jet and turbo-fan aircraft may be sufficiently well represented by

$$\frac{(V/Q)_{cr}}{(V/Q)_d} = 0.6 \left(\frac{F_N}{W} \right)_d \left(\frac{L}{D} \right)_{cr} + 0.4 . \quad (3.8)$$

When Equations (3.7) and (3.8) are combined, the resulting expression for the contribution to lost range due to a descent is

$$\delta R_{a, d} = 0.6 \frac{\Delta h_{e, d}}{r} \left(\frac{L}{D} \right)_{cr} . \quad (3.9)$$

* Note, however, that for various reasons some aircraft have descent fuel flows that are exceptionally high, see Derivation 2.