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The stability and response of linear systems. Part III: methods of analysis based on frequency response

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THE STABILITY AND RESPONSE OF LINEAR SYSTEMS PART III: METHODS OF ANALYSIS BASED ON FREQUENCY RESPONSE

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THE STABILITY AND RESPONSE OF LINEAR SYSTEMS

PART III: METHODS OF ANALYSIS BASED ON FREQUENCY RESPONSE

1. INTRODUCTION

This Item is the third in a group dealing with methods of assessing the stability and response of linear systems. Part I, Item No. 74019^[3], introduces the mathematical description of linear systems, both with and without feedback, and introduces the concept of stability. Part II, Item No. 74020^[4], considers methods of graphical presentation that provide the means of displaying the effects on the characteristics of the system of varying some of its parameters.

The present Item treats the methods of stability analysis that are based on frequency response, including the Nyquist, Bode and Nichols presentations. Part IV, Item No. 80002^[5], considers the specification and measures of performance that may be used in conjunction with these methods of assessment. Part V, Item No. 80018^[6], deals with the function of the different system elements and with design techniques used to construct a system with the required characteristics. Part VI, Item No. 81008^[7], considers the factors that, in particular circumstances, affect the selection of a specific method of system assessment, and Part VII, Item No. 81040^[8], provides examples that illustrate use of the methods in Part I to VI.

The frequency response of a system is its steady-state response to a constant-amplitude sinusoidal input and it is shown in Section 5 that the mathematical function used in the techniques of this Item represents that response for a stable system. For that reason, the techniques are often said, as here, to be based on frequency response. However, they can also be applied to unstable systems when the mathematical function cannot represent the frequency response and the term has therefore been avoided in the following sections.

2. NYQUIST CRITERION

In Part I, Item No. 74019^[3], the application of the Routh-Hurwitz Criteria for the assessment of the stability of a system is outlined, and the effect of feedback on its stability and response is demonstrated.

Another method of assessing the stability of a system is a graphical one using the Nyquist plot by which, essentially, a check is made that no roots of the characteristic equation have positive real parts. As originally stated the Nyquist Criterion applies only to systems involving feedback but the principles on which it is based are applicable to any system.

To introduce the Nyquist Criterion consider first the function

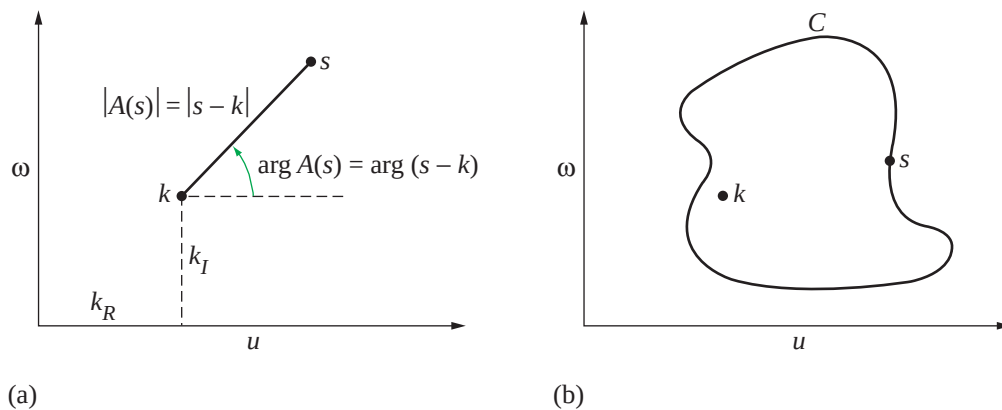
$$A(s) = s - k ,$$

where $s = u + i\omega$ and k is a complex constant ($k = k_R + ik_I$).

Now when the points k and s are plotted in the complex plane, $A(s)$ is obtained as indicated in Sketch 2.1a from

$$|A(s)| = |s - k| = \left((u - k_R)^2 + (\omega - k_I)^2 \right)^{1/2} \quad (2.1)$$

$$\text{and} \quad \arg A(s) = \arg(s - k) = \tan^{-1} \left[\frac{\omega - k_I}{u - k_R} \right]. \quad (2.2)$$



Sketch 2.1 Complex plane plot of $A(s)$

Suppose that s traces, in a clockwise direction, a closed curve C that encloses the point k as shown in Sketch 2.1b. Then for one complete circuit around C , $\arg A(s)$ changes by -2π , i.e. if the change in $\arg A(s)$ is written as $\Delta[\arg A(s)]$ then

$$\Delta[\arg A(s)] = \Delta[\arg(s - k)] = -2\pi. \quad (2.3)$$

If the curve C does not enclose the point k then

$$\Delta[\arg A(s)] = 0. \quad (2.4)$$

Next, consider the function

$$B(s) = \frac{1}{s-k}.$$

Then $|B(s)| = \left| \frac{1}{s-k} \right| = \left((u-k_R)^2 + (\omega-k_I)^2 \right)^{-1/2}$ (2.5)

and $\arg B(s) = \arg \left[\frac{1}{s-k} \right] = \tan^{-1} \left[-\frac{\omega-k_I}{u-k_R} \right] = -\arg(s-k).$ (2.6)

Thus when s traces, in a clockwise direction, a closed curve C that encloses k

$$\Delta \left[\arg \frac{1}{s-k} \right] = +2\pi. \quad (2.7)$$

If the curve C does not enclose the point k then

$$\Delta \left[\arg \frac{1}{s-k} \right] = 0. \quad (2.8)$$

These arguments can be extended to apply to a function $T(s)$ that consists of a number of factors in the numerator and the denominator, *i.e.*

$$T(s) = \frac{N(s)}{D(s)} = \lambda \frac{\prod_{i=1}^n (s-k_i)}{\prod_{i=1}^m (s-l_i)},$$

where k_i ($i = 1, 2, \dots, n$) and l_i ($i = 1, 2, \dots, m$) are in general complex constants. Then

$$|T(s)| = \lambda \frac{\prod_{i=1}^n |s-k_i|}{\prod_{i=1}^m |s-l_i|} \quad (2.9)$$

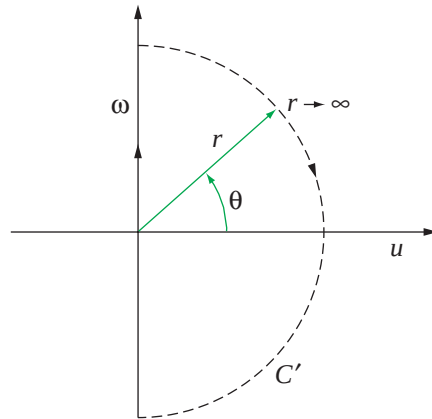
and $\arg T(s) = \sum_{i=1}^n \arg(s-k_i) - \sum_{i=1}^m \arg(s-l_i).$ (2.10)

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The points $s = k_i$ are known as the zeros of $T(s)$ and the points $s = l_i$ are known as the poles of $T(s)$. Then as s traces a closed curve C that encloses the n zeros and the m poles of $T(s)$

$$\Delta[\arg T(s)] = 2\pi(m-n). \quad (2.11)$$

As indicated earlier, the Nyquist Criterion determines whether any roots of the characteristic equation of a system have positive real parts. Therefore the positive half of the complex plane is the domain of interest. Thus a closed curve C' that is suitable because it encloses that domain consists of the imaginary axis and a semi-circle of infinite radius as shown in Sketch 2.2.



Sketch 2.2 Closed curve C'

Then as s traces around C' clockwise there will be contributions to $\Delta[\arg T(s)]$ from s tracing along the imaginary axis and around the infinite semi-circle thus

$$\Delta[\arg T(s)] = \left[\arg T(i\omega) \right]_{\omega = -\infty}^{\omega = +\infty} + \left[\arg T(re^{i\theta}) \right]_{\substack{\theta = -\pi/2 \\ \theta = +\pi/2 \\ r \rightarrow \infty}} \quad (2.12)$$

$$= 2\pi(P - Z), \quad (2.13)$$

where P = number of poles of $T(s)$ in the right half-plane (*i.e.* enclosed by C')
and Z = number of zeros of $T(s)$ in the right half-plane.

The first term on the right-hand side of Equation (2.12) can be written

$$\left[\arg T(i\omega) \right]_{\omega = -\infty}^{\omega = +\infty} = \Delta[\arg T(i\omega)] \quad (2.14)$$

and the second term for large r becomes, from Equation (2.10)

$$\begin{aligned} \left[\arg T(re^{i\theta}) \right]_{\substack{\theta = -\pi/2 \\ \theta = +\pi/2 \\ r \rightarrow \infty}} &= \left[\sum_{i=1}^n \arg(re^{i\theta} - k_i) - \sum_{i=1}^m \arg(re^{i\theta} - l_i) \right]_{\substack{\theta = -\pi/2 \\ \theta = +\pi/2 \\ r \rightarrow \infty}} \\ &= (m - n)\pi. \end{aligned} \quad (2.15)$$

Hence, from Equations (2.12) to (2.15),

$$\Delta[\arg T(i\omega)] = 2\pi(P - Z) - (m - n)\pi. \quad (2.16)$$

The number of poles, P , in the right half-plane can be found from this equation and a knowledge of $\Delta[\arg T(i\omega)]$ and $(m - n)\pi$ only if the number of zeros, Z , in the right half-plane is already known.

However, the application of Equation (2.16) to systems with feedback leads to a more useful result. Consider a system whose open-loop transfer function is $G(s)H(s)$ (see Section 3.2 of Item No. 74019^[3]). The closed-loop system will be stable if there are no zeros of $[1 + G(s)H(s)]$ in the right half-plane. From Equation (2.16)

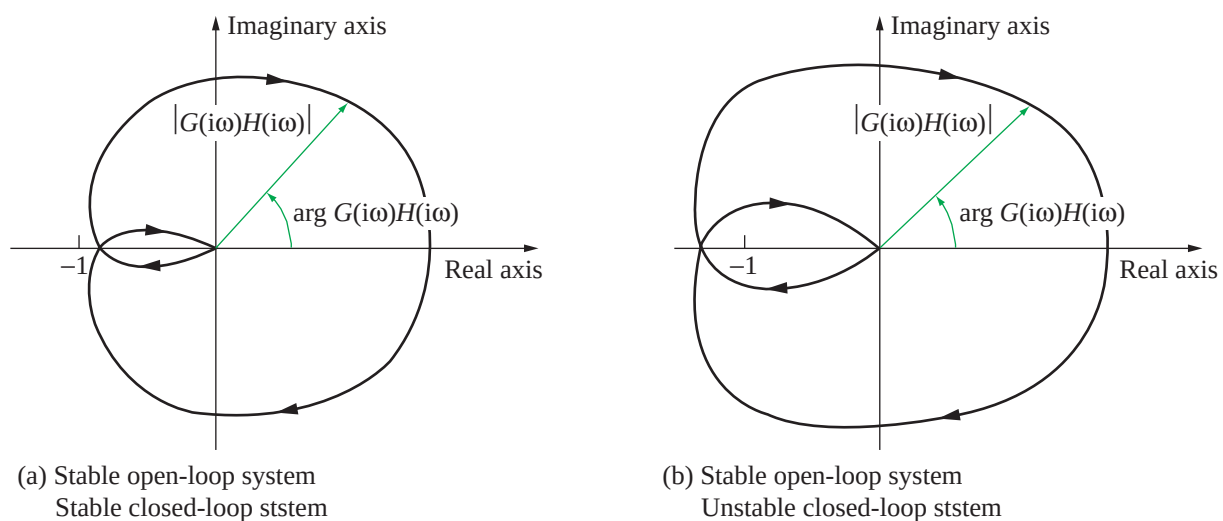
$$\Delta\left[\arg\left(1 + \frac{G(i\omega)}{H(i\omega)}\right)\right] = 2\pi(P - Z) - (m - n)\pi, \quad (2.17)$$

where P and Z are the numbers of poles and zeros respectively of $[1 + G(s)H(s)]$ in the right half-plane, and m and n are the orders of the denominator and numerator respectively of $[1 + G(s)H(s)]$.

The order of the numerator of $G(s)H(s)$ is less than, or at most equal to, that of its denominator, so that for $[1 + G(s)H(s)]$ the value of m must be equal to n . Also, when the open-loop system is stable, P is zero. Thus, with those restrictions, the closed-loop system is stable when

$$\Delta\left[\arg\left(1 + \frac{G(i\omega)}{H(i\omega)}\right)\right] = 0. \quad (2.18)$$

Thus for stability, the curve traced out by $[1 + G(i\omega)H(i\omega)]$ as ω goes from $-\infty$ to $+\infty$ must not enclose the origin. Sketch 2.3 shows Nyquist diagrams for two open-loop stable systems. The system shown in Sketch 2.3a is closed-loop stable because the closed-loop origin (the point $(-1, 0)$) is outside the locus. However, the system shown in Sketch 2.3b must be closed-loop unstable since there are two clockwise encirclements of the closed-loop origin; in terms of Equation (2.18) this means that $\Delta[\arg(1 + G(i\omega)H(i\omega))] = 4\pi$.



Sketch 2.3

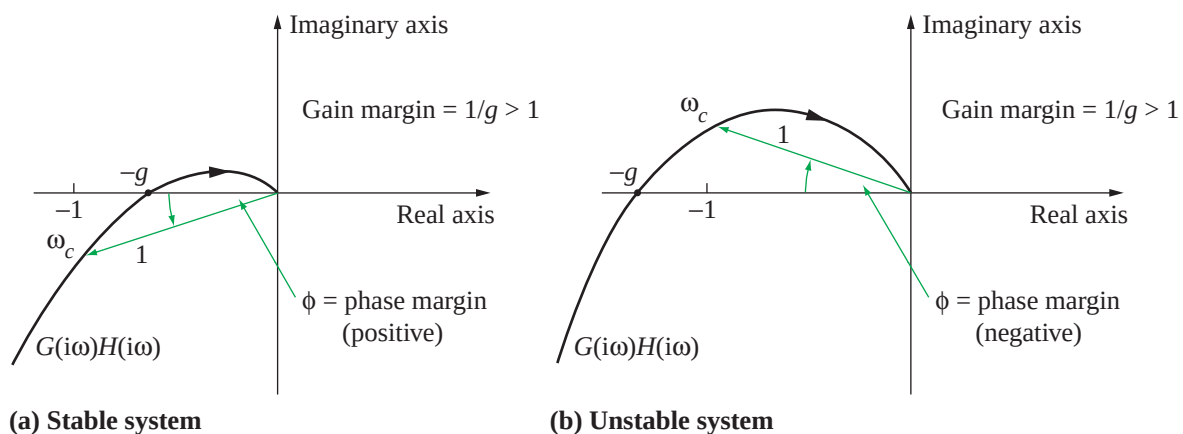
Equation (2.17) is the formal statement of the general Nyquist stability criterion. Any closed-loop system must satisfy this condition to be stable. A simpler statement of the general Nyquist stability criterion is obtained by considering the manner in which the locus passes the closed-loop origin (the point $(-1, 0)$). The locus of $G(s)H(s)$ as s follows the Nyquist contour is a closed path which may cross itself a number of times. Each intersection of the locus with itself may form an additional closed path upon the Nyquist diagram, and any closed path which encircles the point $(-1, 0)$ is called an encirclement at the point $(-1, 0)$.

Expressed in terms of encirclements, the Nyquist stability criterion is as follows.

- (i) No encirclement by $G(s)H(s)$ of the point $(-1, 0)$ implies that the system will be
 - (a) closed-loop stable if it is open-loop stable, or
 - (b) closed-loop unstable if it is open-loop unstable.
- (ii) Any clockwise encirclements by $G(s)H(s)$ of the point $(-1, 0)$ imply that the system is closed-loop unstable, irrespective of the open-loop stability.
- (iii) Where there are counter-clockwise encirclements by $G(s)H(s)$ of the point $(-1, 0)$ then, for the system to be closed-loop stable, there must be the same number of encirclements as there are open-loop poles with positive real parts. Otherwise, the system will be closed-loop unstable.

The degree of stability of a closed-loop system with feedback can be inferred from the Nyquist diagram by considering the gain margin and phase margin. The gain margin is the reciprocal of the value of $|G(i\omega)H(i\omega)|$ on the negative real axis as shown in Sketch 2.4a; the corresponding value of ω is known as the gain margin frequency. The gain margin is a direct measure of by how much the gain of the open-loop system, (i.e. $|G(i\omega)H(i\omega)|$), can be increased before the closed-loop system becomes unstable. The phase margin is given by the angle made with the negative real axis by the unit vector drawn from the origin to the $G(i\omega)H(i\omega)$ curve. For particularly complicated systems there may be several gain and phase margins (see Part IV, Item No. 80002^[5]).

Sketch 2.4 shows the phase and gain margins for two open-loop stable systems; (a) is closed-loop stable and (b) is closed-loop unstable. It will be seen that in this case the stable system is associated with a positive phase margin and a gain margin greater than one, and that instability is associated with a negative phase margin and a gain margin which is less than one. The frequencies on the $G(i\omega)H(i\omega)$ loci at the intercept of the unit vector are known as the phase margin or crossover frequencies, ω_c .



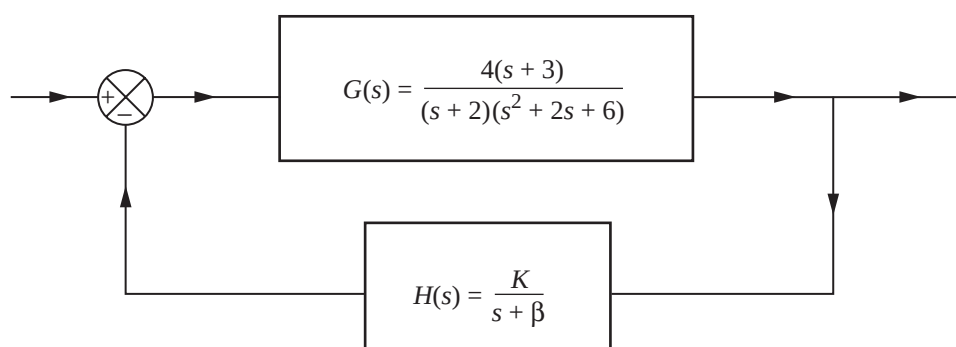
Sketch 2.4 Phase and gain margins of stable and unstable systems

For a unity feedback system, the Nyquist diagram can be used to show, in a particularly simple way, the vector relationship between the output from and the input to the system, and in the case of the inverse Nyquist diagram this leads to a useful result (see Section 4.3).

For the general system with a frequency-dependent feedback function, the simplicity of the unity feedback vector diagram is lost but again the simplicity of the Nyquist contour leads to a constructional method of estimating from the diagram the pair of closed-loop complex conjugate poles with the least damping. Appendix D shows how these closed-loop poles may be estimated from the Nyquist plot of the open-loop system. This technique is general and is often the most expedient means of examining the closed-loop system response.

2.1 Example on the Application of the Nyquist Criterion

For the system[†] defined by the block diagram in Sketch 2.5 construct the Nyquist diagram for $\beta = 5$ and $K = 5$ and 15. Determine how stable the system is by estimating the gain and phase margins and if it is unstable determine the number of unstable roots.



Sketch 2.5

The open-loop transfer function for this system with $\beta = K = 5$ is

$$G(s)H(s) = \frac{20(s+3)}{(s+2)(s^2+2s+6)(s+5)}.$$

Although, as will be seen in Section 3, it is often simpler to construct the Nyquist diagram from the Bode diagram, the procedure for the direct preparation of the Nyquist diagram is illustrated here. The first step is to evaluate $|G(i\omega)H(i\omega)|$ and $\arg G(i\omega)H(i\omega)$ from[‡]

$$|G(i\omega)H(i\omega)| = 20(\omega^2 + 9)^{1/2}(\omega^2 + 4)^{-1/2}[(6 - \omega^2)^2 + 4\omega^2]^{-1/2}(\omega^2 + 25)^{-1/2} \quad (2.19)$$

$$\text{and} \quad \arg G(i\omega)H(i\omega) = \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{2\omega}{6 - \omega^2} - \tan^{-1}\frac{\omega}{5}, \quad (2.20)$$

for a range of values of ω as shown in Table 2.1.

[†] This is the system whose characteristic equation for the case of $\beta = 5$ is considered in Section 2.3 of Item No. 74019^[3].

[‡] It is easier to treat a quadratic factor with imaginary roots without factorising it. In this case substituting $s = i\omega$ yields $(6 - \omega^2) + 2i\omega$ whence the corresponding terms in Equations (2.19) and (2.20) follow.

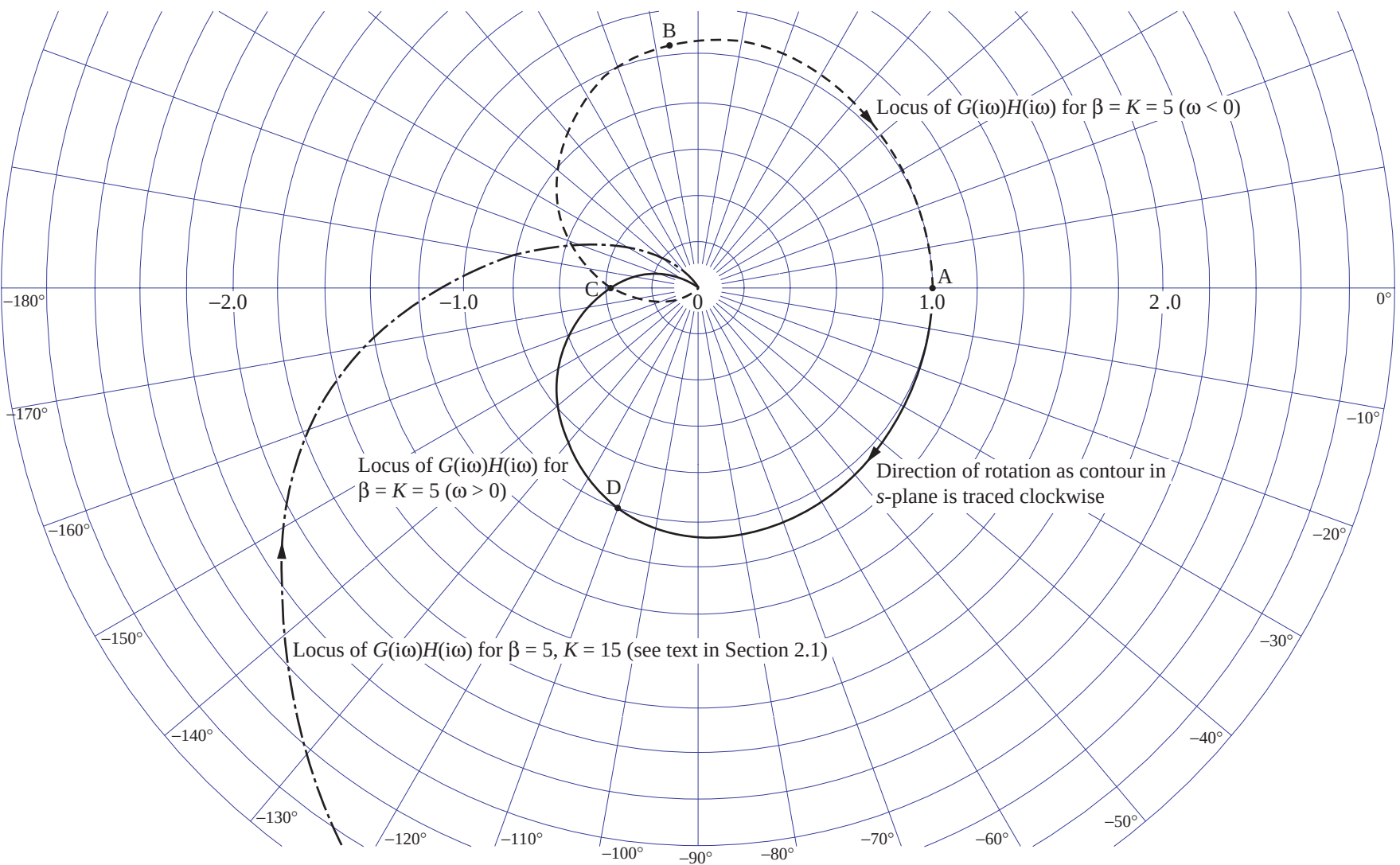
TABLE 2.1

ω	$ G(i\omega)H(i\omega) $	$\arg G(i\omega)H(i\omega)$
0	1	0°
0.5	1.0061	$-20^\circ 9'$
1.0	1.0300	$-41^\circ 15'$
1.5	1.0703	$-65^\circ 40'$
2.0	1.0587	$-96^\circ 33'$
2.5	0.8717	$-130^\circ 58'$
3.0	0.6016	$-158^\circ 51'$
3.5	0.3993	$-177^\circ 37'$
4.0	0.2727	$-190^\circ 18'$
5.0	0.1426	$-206^\circ 25'$
6.0	0.0841	$-216^\circ 32'$

The curve of $G(i\omega)H(i\omega)$ for $\omega > 0$ from Table 2.1 is shown plotted as the solid curve in the polar diagram of Sketch 2.6. The dotted portion for $\omega < 0$ is obtained by symmetry. The point $(-1, 0)$ lies outside the $G(i\omega)H(i\omega)$ contour so that from Equation (2.18) the system is stable since the open-loop system is known to be, a priori, stable.

The gain margin of the system is given by $1/OC = 1/0.375 = 2.67$ (i.e. the value of the gain, K , that just makes the system unstable is $2.67 \times 5 = 13.35$) and the phase margin, given by the angle COD between the unit vector to the contour and the negative real axis, is $(180^\circ - 112^\circ) = 68^\circ$.

For the system with $\beta = 5$ and $K = 15$, consideration of Equations (2.19) and (2.20) shows that $\arg G(i\omega)H(i\omega)$ will be unchanged from the values in Table 2.1 while $|G(i\omega)H(i\omega)|$ will be increased by a factor of 3. Part of the contour of $G(i\omega)H(i\omega)$ for $\omega > 3$ obtained from the values in Table 2.1 is shown plotted as the chain dotted curve on Sketch 2.6. However, since the effect of factoring K is simply to factor $|G(i\omega)H(i\omega)|$ by the same amount, it is not necessary in fact to draw a new contour if it is required to consider a different value of K ; the existing contour can be used and the radius scale considered to have been reduced by the reciprocal of that factor. In the particular case when K is increased from 5 to 15, the scale is considered to be reduced by a third and the contour for $K = 5$ then applies. Since the contour encircles the point that corresponds to $(-1, 0)$ on the scaled figure, from Equation (2.17) for the closed-loop system $P - Z \neq 0$. The closed-loop system characteristic equation has no poles with positive real parts, because the open-loop system is stable, so there must be zeros with positive real parts and the closed-loop system must be unstable. However, to illustrate the method more clearly it is worthwhile to check that in fact the contour of $G(i\omega)H(i\omega)$ is traced clockwise as s traces the contour of Sketch 2.2 clockwise. Consider a vector drawn from the point that is $(-1, 0)$ to $G(i\omega)H(i\omega)$ in Sketch 2.6. Then $s = -\infty i$ corresponds to point 0, $s = -2i$ to point B and $s = 0$ to point A so that as the imaginary axis of Sketch 2.2 is traced from $-i\omega$ to 0 (i.e. clockwise) the vector rotates once clockwise about the point $(-1, 0)$. Hence in tracing the complete contour there will be two rotations of the vector about $(-1, 0)$ and, P being equal to zero, there must be two zeros of $1 + G(i\omega)H(i\omega)$ in the right half-plane.



Sketch 2.6

3. BODE DIAGRAM

The Bode diagram is essentially a method of plotting the function $T(i\omega)$ where $T(s)$ is the system transfer function and takes the form

$$T(s) = \frac{N(s)}{D(s)} = \lambda \frac{\prod_{i=1}^n (s - k_i)}{\prod_{i=1}^m (s - l_i)} . \quad (3.1)$$

Two graphs are drawn; in one the logarithm of the amplitude ratio (*i.e.* $\log_{10}|T(i\omega)|$, see Section 5), usually expressed in decibels[†], is plotted against $\log_{10}\omega$ while in the other the phase difference (*i.e.* $\arg T(i\omega)$, see Section 5) is plotted against $\log_{10}\omega$. The logarithm of $|T(i\omega)|$ is taken because contributions from the factors of $N(i\omega)$ and $D(i\omega)$ are then additive; thus, from Equation (3.1),

$$\log_{10}|T(i\omega)| = \log_{10}\lambda + \sum_{i=1}^n \log_{10}|i\omega - k_i| - \sum_{i=1}^m \log_{10}|i\omega - l_i| . \quad (3.2)$$

The contributions of the factors to $\arg T(i\omega)$ are already additive

$$\arg T(i\omega) = \sum_{i=1}^n \arg(i\omega - k_i) - \sum_{i=1}^m \arg(i\omega - l_i) . \quad (3.3)$$

Although each different factor of $T(s)$ makes a different contribution to $\log_{10}|T(i\omega)|$ and $\arg T(i\omega)$, the function $T(i\omega)$ can be expressed in a form that allows the Bode diagram to be drawn quickly for any system from four standard graphs, two for $|T(i\omega)|$ and two for $\arg T(i\omega)$. This process is described in detail in Appendix B.

The degree of stability of a system with feedback may be assessed from the Bode diagram of the open loop using the gain and phase margins defined in Section 2. The Bode diagram is plotted for the open loop as $\log_{10}|G(i\omega)H(i\omega)|$ and $\arg G(i\omega)H(i\omega)$ against $\log_{10}\omega$ as shown in Sketch 3.1.

As illustrated in Sketch 2.4a, the gain margin is the reciprocal of the value of $|G(i\omega)H(i\omega)|$ on the negative real axis, *i.e.* the reciprocal of $|G(i\omega)H(i\omega)|$ at $\arg G(i\omega)H(i\omega) = -180^\circ$, which can be obtained from the ordinate marked in Sketch 3.1a. Similarly, the phase margin is the angle between $\arg G(i\omega)H(i\omega)$ and -180° when $|G(i\omega)H(i\omega)| = 1$ and is given by the ordinate marked in Sketch 3.1b. The system will be unstable if the phase margin is negative, *i.e.* if $\arg G(i\omega)H(i\omega) < -180^\circ$ when $|G(i\omega)H(i\omega)| = 1$, see the Example, Section 3.1. Because of the ease with which a Bode diagram can be built-up for complex systems using the standard curves discussed in Appendix B, it is often quicker to prepare a Bode diagram and convert the logarithmic plot into an appropriate Nyquist diagram where the latter presentation is required. However, such an approach will probably be satisfactory only when a quick qualitative assessment of the system

[†] It is customary to measure the power gain of a system in bels. The gain in bels is defined as

$$\log_{10} \frac{\text{output power}}{\text{input power}} .$$

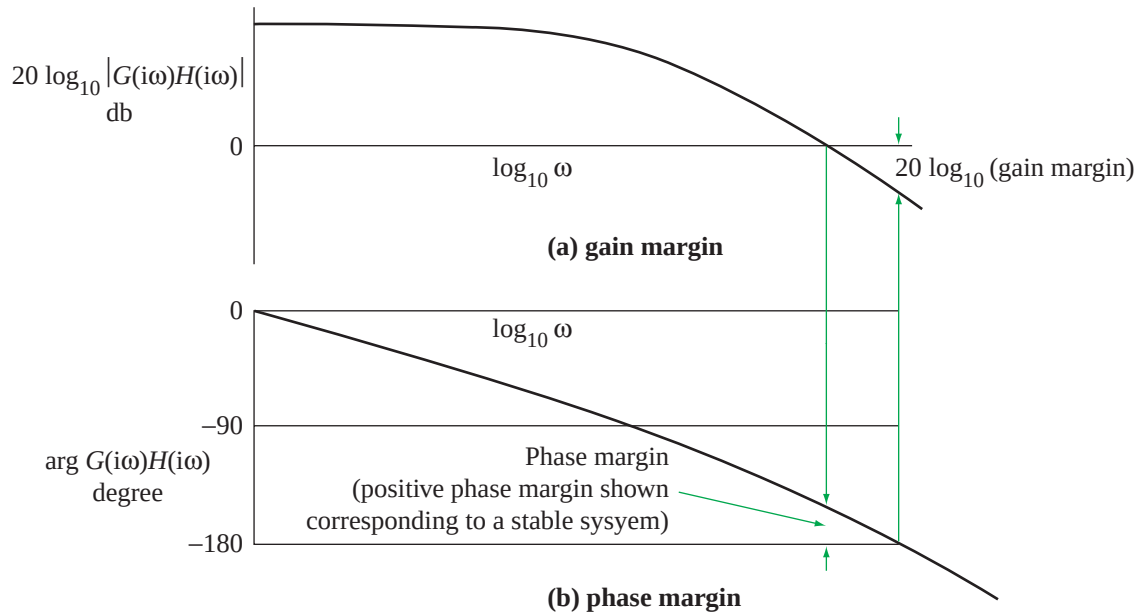
Since the power is proportional to (amplitude)², the amplitude ratio or gain expressed in bels then becomes

$$2 \log_{10} \frac{\text{output amplitude}}{\text{input amplitude}}$$

or, expressed in decibels,

$$20 \log_{10} \frac{\text{output amplitude}}{\text{input amplitude}} .$$

characteristics is required. If accurate values of gain and phase margins are needed, the summation obtained graphically of the contributions due to a number of factors of $T(s)$ will probably involve unacceptable errors and calculation of the Bode diagram, as shown in Section 3.1, will be preferable.



Sketch 3.1 Gain and phase margins

3.1 Example on the Application of the Bode Diagram

For the system defined in Section 2.1 construct the Bode diagrams for $K = 5$ and $\beta = 1, 2, 5$ and 10 to demonstrate the effect on stability of changing β while keeping K constant.

The open-loop transfer function is

$$G(s)H(s) = \frac{20(s+3)}{(s+2)(s^2+2s+6)(s+\beta)}.$$

For the case of $\beta = 5$, Table 2.1 in Section 2.1 already presents the data required for plotting $\log_{10}|G(i\omega)H(i\omega)|$ and $\arg G(i\omega)H(i\omega)$ against $\log_{10}\omega$. Alternatively, if desired the technique of constructing the curves discussed in Appendix B could be used. The Bode diagrams for $\beta = 1, 2, 5$ and 10 are shown in Sketch 3.2 and the values of gain and phase margins read from them are tabulated in Table 3.1.

TABLE 3.1

β	$\log_{10}$ of gain margin	Gain margin	Value of K at instability	Phase margin (degree)
1	UNSTABLE			-23.5
2	UNSTABLE			-3
5	0.43	2.69	13.45	68
10	0.91	8.13	40.65	—

It should be noted that the effect of changing K from the value of 5 to a new value, K' , is merely to add a constant of $20 \log_{10}(K'/5)$ to any value of $|G(i\omega)H(i\omega)|$ considered.