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One-dimensional compressible gas flow in ducts

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Fluid Mechanics Steering Group which had the following constitution:

Chairman	
Mr T.V. Lawson	— Bristol University
Members	
Mr K.C. Anthony	— Ove Arup and Partners
Mr E.C. Firman	— Central Electricity Research Laboratories
Mr B.H. Fisher	— Consulting Structural Engineer
Mr L.C. Hickman	— Kellogg International Corporation
Dr G. Hobson	— GEC Turbine Generators Ltd
Mr J.R.C. Pedersen	— Independent
Mr C. Scruton	— Independent
Mr N. Worley	— Babcock and Wilcox (Operations) Ltd.

The Steering Group has benefited from the participation of members from several engineering disciplines. In particular, Mr B.H. Fisher has been appointed to represent the interests of structural engineering as the nominee of the Institution of Structural Engineers, Mr E.C. Firman has been appointed to represent the interests of mechanical engineering as the nominee of the Institution of Mechanical Engineers, and Mr L.C. Hickman has been appointed to represent the interests of chemical engineering as the nominee of the Institution of Chemical Engineers.

The work on this Item was carried out in the Fluid Mechanics and Physical Properties Group of the Engineering Sciences Data Unit under the supervision of Mr N. Thompson, Group Head. The member of staff who undertook the technical work involved in the initial assessment of the available information and the construction and subsequent development of the Item was

Mr B.C. Freeman	— Senior Engineer
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The Fluid Mechanics Steering Group, which first met in April 1970, has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

ONE-DIMENSIONAL COMPRESSIBLE GAS FLOW IN DUCTS

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ONE-DIMENSIONAL COMPRESSIBLE GAS FLOW IN DUCTS

1. NOTATION

Three coherent systems of units are given.

		SI	British	
A	cross-sectional area of duct	m^2	ft^2	ft^2
A^*	cross-sectional area of duct at section where $M = 1$	m^2	ft^2	ft^2
a	local speed of sound in gas, $\sqrt{\gamma RT}$	m/s	ft/s	ft/s
a_t	speed of sound in gas when brought to rest adiabatically*, $\sqrt{\gamma RT_t}$	m/s	ft/s	ft/s
M	Mach number, V/a			
m	mass flow rate of gas	kg/s	lb/s	$slug/s$
p	static pressure	N/m^2	pdl/ft^2	lbf/ft^2
p^*	static pressure at duct section where $M = 1$	N/m^2	pdl/ft^2	lbf/ft^2
p_t	total pressure, pressure which would be achieved if flow were brought to rest isentropically*	N/m^2	pdl/ft^2	lbf/ft^2
R	gas constant [†]	or $J/kg\ K$ m^2/s^2K	$ft\ pdl/lb\ R$ ft^2/s^2R	$ft\ lbf/slug\ K$ ft^2/s^2K
T	static temperature	K	R	K
T^*	static temperature at duct section where $M = 1$	K	R	K
T_t	total temperature, temperature which would be achieved if flow were brought to rest adiabatically*	K	R	K
V	flow velocity, $m/\rho A$	m/s	ft/s	ft/s
γ	ratio of specific heat capacity at constant pressure to that at constant volume			
ρ	density of gas	kg/m^3	lb/ft^3	$slug/ft^3$
ρ_t	density which would be achieved if flow were brought to rest isentropically*	kg/m^3	lb/ft^3	$slug/ft^3$

* See Section 3.2.

[†] $R = (\text{universal gas constant})/(\text{molecular weight of gas})$.
 Universal gas constant = 8314 J/kg-mol K = 1545 ft lbf/lb-mol R
 = 2781 ft lbf/lb-mol K = 89 476 ft lbf/slug-mol K.

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2. INTRODUCTION

2.1 One-dimensional Flow of a Perfect Gas

The application of one-dimensional flow analysis to real duct flow, in which the fluid conditions vary across the duct, is a good approximation provided that the distribution of properties across the duct does not vary markedly from one section to another. This requires approximately fully-developed flow and the conditions that the rate of change of duct area is small and that the radius of curvature of the duct centre-line is large compared with the cross-sectional dimensions.

Most common gases, including air, may be considered to behave as perfect gases* provided that the liquefaction or the dissociation temperatures are not approached.

2.2 One-dimensional Isentropic Flow Applications

Although one-dimensional isentropic flow can only be realised exactly in the flow of a perfect gas with zero viscosity and thermal conductivity, the relations that may be derived for it do find useful application in real gas flows.

The isentropic flow relations are most commonly applied in the relation between “static” and “total” conditions at a point in a flow since these states can be related through an isentropic† deceleration. Use of these relations does not require the flow to be isentropic since they are applied at a point or, with the assumption of one-dimensional flow, across a single section. Provided that due account is taken of the non-isentropic changes in a real flow, *e.g.* via the total or static pressure losses or the increase in total temperature due to heat addition, the isentropic relations may be used to evaluate conditions at one station from known values at another.

In real gas flows the change in entropy depends not only on the viscosity and thermal conductivity of the gas but also on the velocity gradient and temperature gradient in the flow. In flows at high Reynolds numbers these effects may often be considered to be confined to a thin boundary layer and the flow outside this analysed as isentropic. This is of particular application in external flows. In internal flows the rate of heat transfer is often small enough for the flow to be assumed adiabatic and if the Reynolds number is high enough for viscous effects to be small the flow may be closely approximated by isentropic flow. A particular example of this application is the analysis of flow from a large reservoir to a smaller duct working section (see Section 3.4).

2.3 Mass Flow Functions

From the one-dimensional flow equation of continuity for a perfect gas, functions of the fluid mass flow rate, pressure and temperature and the duct area may be derived that are each a function only of the Mach number and ratio of specific heat capacities. These are commonly known as the “mass flow functions”. An example of the application of the mass flow functions is given in Example 2 in Section 5.

* At very high temperatures, while the perfect gas equation of state ($p = \rho RT$) remains a close approximation, the assumption of constant specific heat capacities may not. Some flow functions for such a “thermally-perfect, calorically-imperfect” gas are given in ESDU Aero. S.00.01.10.

† See Section 3.2.

3. ONE-DIMENSIONAL COMPRESSIBLE FLOW RELATIONS

The following Sections present a number of useful equations from the analysis of one-dimensional compressible gas flow. Further information on their derivation may be obtained from, for example, Derivation 3.

The Equations are presented graphically in the Figures at the end of the Item for a range of values of γ , including $\gamma = 1$ for ease of interpolation, and for, generally, values of M up to about $M = 1$. However, the equations are valid for any value of M provided that the dissociation temperature of the gas is not reached and the gas continues to follow the perfect gas laws*.

Tabulated values of some of the functions may be found in Derivations 1 to 4, the tabulations generally being more complete for $\gamma = 1.4$ than for other values of γ .

3.1 One-dimensional Isentropic Flow

For one-dimensional steady isentropic flow of a perfect gas between two Stations 1 and 2 in a duct the following relations may be derived:

$$p_1/\rho_1^\gamma = p_2/\rho_2^\gamma, \quad (3.1)$$

$$p_1/p_2 = (\rho_1/\rho_2)^\gamma = (T_1/T_2)^{\gamma/(\gamma-1)}, \quad (3.2)$$

$$T_1/T_2 = \left(1 + \frac{\gamma-1}{2}M_2^2\right) / \left(1 + \frac{\gamma-1}{2}M_1^2\right). \quad (3.3)$$

Equation (3.3) is valid for adiabatic flow (*i.e.* with no energy transfer to or from the flow) whether reversible or not, but Equations (3.1) and (3.2) are only valid for reversible adiabatic, or isentropic, flow.

3.2 Static to Total State Relations

The total pressure is defined as the pressure that would be achieved if the flow were brought to rest through an isentropic deceleration. The total temperature is defined as the temperature which would be achieved if the flow were brought to rest adiabatically. However, since an isentropic process is a reversible adiabatic process, the total temperature can also be derived from the isentropic flow relations.

If the velocity at Station 2 in Equations (3.2) and (3.3) is set to zero, so that $M_2 = 0$, the ratios of static to total temperature, pressure and density may be derived as

$$T/T_t = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1} \quad (3.4)$$

$$\text{and} \quad (\rho/\rho_t)^\gamma = p/p_t = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/(1-\gamma)} \quad (3.5)$$

where T and T_t , p and p_t , and ρ and ρ_t now apply at a single point in the flow.

* For conditions in non-isentropic flow through converging-diverging ducts see, for example, Derivation 3.

Some other useful relations that can be derived are,

$$V/a_t = M \sqrt{T/T_t}, \quad (3.6)$$

$$\frac{1}{2} \rho V^2 / p_t = \frac{1}{2} \gamma M^2 (p/p_t), \quad (3.7)$$

$$\text{and} \quad (p_t - p) / (\frac{1}{2} \rho V^2) = (1 - p/p_t) / [\frac{1}{2} \gamma M^2 (p/p_t)] \quad (3.8)$$

Equations (3.4) to (3.8) may be used generally to evaluate conditions at a point in a flow when some conditions are known and are presented graphically in Figures 1 to 5.

3.3 Critical Pressure Ratio, Expansion through a Throat

If the pressure ratio p_2/p_{t1} across an isentropic gas flow in a converging duct is decreased, the exit Mach number, M_2 , and the mass flow rate increases. Eventually a condition is reached where $M_2 = 1$ and the duct is then choked, with the maximum possible mass flow passing. The pressure p_2 then equals p^* and the pressure ratio equals the critical pressure ratio given by

$$\frac{p^*}{p_{t1}} = \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma - 1)}. \quad (3.9)$$

The corresponding temperature ratio is given by

$$\frac{T^*}{T_{t1}} = \frac{2}{\gamma + 1}. \quad (3.10)$$

For air, with $\gamma = 1.4$, these ratios have the values 0.528 and 0.833 respectively.

The local conditions at a point in the duct may then be expressed in terms of the minimum area, or throat, values as functions of the local Mach number. Thus

$$\frac{p}{p^*} = \left[\frac{\gamma + 1}{2 \left(1 + \frac{\gamma - 1}{2} M^2 \right)} \right]^{\gamma/(\gamma - 1)}, \quad (3.11)$$

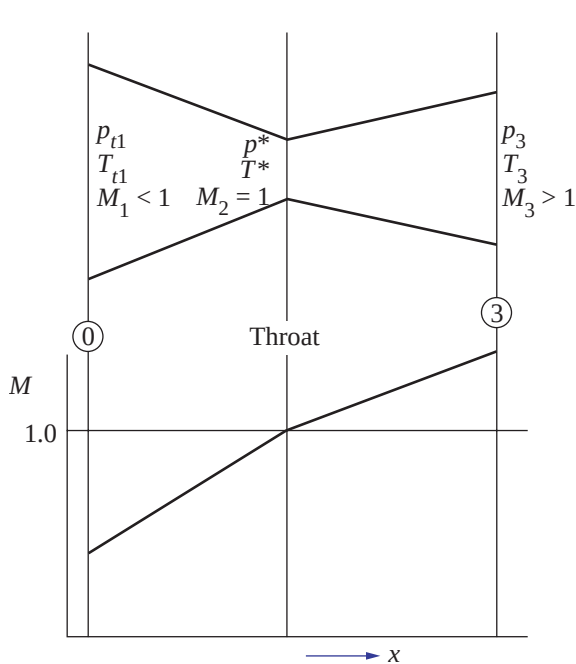
$$\frac{T}{T^*} = \frac{\gamma + 1}{2 \left(1 + \frac{\gamma - 1}{2} M^2 \right)} \quad (3.12)$$

and the corresponding required area ratio is

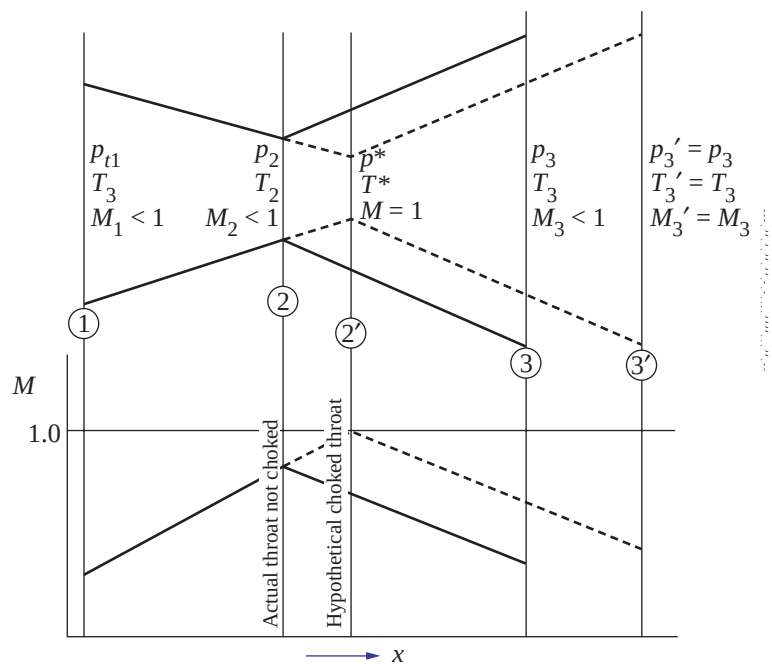
$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{\gamma + 1}{2 \left(1 + \frac{\gamma - 1}{2} M^2 \right)} \right]^{(\gamma + 1)/[2(\gamma - 1)]}. \quad (3.13)$$

If a diverging duct is attached downstream of the converging choked duct and the overall pressure ratio p_3/p_{t1} (where p_3 is the pressure at the exit) reduced below the critical value, the flow conditions up to the throat remain unchanged at those for the choked flow whilst downstream of the throat the flow accelerates to $M_3 > 1$ (Sketch 3.1). For isentropic flow* throughout, the local conditions in the downstream section are given by Equations (3.11) to (3.13) where now M is greater than unity. In particular Equation (3.13) may be used to determine the required outlet area to achieve a given Mach number flow.

If, in a converging-diverging duct, the flow conditions are such that critical flow does not occur, i.e. $M < 1$ at the throat, the flow will accelerate up to the throat and then decelerate subsonically downstream. Equations (3.9) to (3.13) may be applied by assuming a hypothetical throat at which the flow would choke as a reference condition as in Sketch 3.2. In that case M is a maximum at Station 2 and A/A^* cannot exceed A_2/A^* .



Sketch 3.1 Choked flow



Sketch 3.2 Wholly subsonic flow

Equations (3.11) to (3.13) are presented graphically in Figures 6 to 8b.

3.4 Isentropic Expansion from Rest

In the case of expansion of a gas from a large reservoir to a small duct the flow has been found to follow quite closely an isentropic process. In this case the “total” conditions (which remain constant) are the same as the static conditions for the gas in the reservoir and the static conditions at the duct section may be derived from Equations (3.4) and (3.5). These may be expressed in terms of the pressure ratio. Thus,

$$T_2/T_1 = (p_2/p_1)^{(\gamma-1)/\gamma} \quad (3.14)$$

$$\rho_2/\rho_1 = (p_2/p_1)^{1/\gamma} \quad (3.15)$$

* For conditions in non-isentropic flow through converging-diverging ducts see, for example, Derivation 3.

$$V_2/a_2 = M_2 = \left\{ \frac{2}{\gamma - 1} \left[\left(\frac{p_2}{p_1} \right)^{(1-\gamma)/\gamma} - 1 \right] \right\}^{1/2} \quad (3.16)$$

and

$$V_2/a_{t2} = \left\{ \frac{2}{\gamma - 1} \left[1 - \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2} . \quad (3.17)$$

Equations (3.14) to (3.17) are presented graphically for a value of $\gamma = 1.4$, which is appropriate for air, in Figure 9. They may be determined in terms of M and γ from Equations (3.4) to (3.6) where condition “1” is taken to correspond to the “total” condition “t” and condition “2” to the unsuffixed static condition. If the gas is to reach supersonic Mach numbers, as on the left of Figure 9, the duct must be of converging-diverging shape (see Section 3.3). Equations (3.9) to (3.13) may also be applied in that case, being of particular use in the duct section downstream of a throat.

3.5 Mass Flow Functions

From the steady one-dimensional flow equation of continuity,

$$m = \rho AV \quad (3.18)$$

and the equation of state for a perfect gas,

$$p = \rho RT , \quad (3.19)$$

three mass flow functions may be derived in terms of different total and static pressure and temperature combinations.

They are

$$\frac{m \sqrt{RT_t}}{Ap_t} = \sqrt{\gamma} M \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-(\gamma + 1)/[2(\gamma - 1)]} , \quad (3.20)$$

$$\frac{m \sqrt{RT_t}}{Ap} = \sqrt{\gamma} M \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{1/2} \quad (3.21)$$

and

$$\frac{m \sqrt{RT}}{Ap} = \sqrt{\gamma} M . \quad (3.22)$$

These functions are plotted in Figures 10 to 12 as functions of γ and M .

Alternative versions of these functions are sometimes used in which the weight flow rather than the mass flow is used or in which the gas constant is omitted.