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Friction losses for fully-developed flow in straight pipes of constant cross section – subsonic compressible flow of gases

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes ESDU Aero. 00.03.24, was monitored and guided by the Aerodynamics Committee and the Fluid Mechanics Steering Group. The Aerodynamics Committee first met in March 1942, and the Fluid Mechanics Steering Group first met in April 1970. The Aerodynamics Committee had the following membership:

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FRICTION LOSSES FOR FULLY-DEVELOPED FLOW IN STRAIGHT PIPES OF CONSTANT CROSS-SECTION - SUBSONIC COMPRESSIBLE FLOW OF GASES

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FRICTION LOSSES FOR FULLY-DEVELOPED FLOW IN STRAIGHT PIPES OF CONSTANT CROSS-SECTION - SUBSONIC COMPRESSIBLE FLOW OF GASES

1. NOTATION AND UNITS

Three coherent systems of units are given below.

		SI		*British*
A	cross-sectional area of duct	m^2	ft^2	ft^2
a	local speed of sound in fluid, $\sqrt{\gamma RT}$	m/s	ft/s	ft/s
D_E	equivalent or hydraulic diameter of duct, $4A/P$	m	ft	ft
f	Fanning friction factor	–	–	–
$\bar{f}$	mean value of friction factor in adiabatic flow (see Section 3.4)	–	–	–
L_A	length of duct required for flow to reach $M = 1$ in adiabatic flow	m	ft	ft
L_I	length of duct required for flow to reach $M = 1/\sqrt{\gamma}$ in isothermal flow	m	ft	ft
M	flow Mach number, V/a	–	–	–
m	mass flow rate	kg/s	lb/s	slug/s
P	wetted perimeter of duct	m	ft	ft
p	static pressure	N/m^2	pdl/ft^2	lbf/ft^2
p_t	total pressure, pressure that would be achieved if flow were brought to rest isentropically	N/m^2	pdl/ft^2	lbf/ft^2
R	gas constant*	J/kg K or m^2/s^2 K	ft pdl/lb R ft^2/s^2 R	ft lbf/slug K ft^2/s^2 K
Re	Reynolds number, $\rho VD_E/\mu$ or $mD_E/(A\mu)$	–	–	–
T	static temperature	K	R	K
T_t	total temperature, temperature that would be achieved if flow were brought to rest adiabatically	K	R	K
V	mean flow velocity, $m/\rho A$	m/s	ft/s	ft/s

For footnote see end of Notation Section.

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x	distance measured along duct axis	m	ft	ft
γ	ratio of specific heat capacity at constant pressure to that at constant volume for fluid	–	–	–
μ	dynamic viscosity of fluid	N s/m^2	pdl s/ft^2	lbf s/ft^2
ρ	density of fluid	kg/m^3	lb/ft^3	slug/ft^3

* The gas constant R = (universal gas constant)/(molecular weight of fluid).
The universal gas constant = 8314 J/kg-mol K = 1545 ft lbf/lb-mol R = 2781 ft lbf/lb-mol K = 89 476 ft lb/slug-mol K.

Suffixes

1	conditions at upstream point of interest in duct
2	conditions at downstream point of interest in duct
A , *I*	see L_A and L_I

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2. INTRODUCTION AND USE OF THE ITEM

This Item extends the application of the friction factor data in ESDU 66027 (Reference 2) to two cases of subsonic compressible flow. Whilst the flow of liquids may almost always be considered incompressible (and the friction losses therefore calculated as in ESDU 66027), the flow of gases may only be considered incompressible if the Mach number is low (M less than about 0.2) and if the pressure drop is also a small proportion of the initial static pressure (less than about 5 per cent) so that changes in density are small.

Two methods of using this Item are presented. Firstly, a hand calculation approach is considered using the graphs provided and Section 5 gives two worked examples using this method. Secondly, Appendix B presents a Fortran 77 computer program designed to solve the full equations as derived in Appendix C. The same examples from Section 5, along with an additional example to illustrate choking, are then solved using these equations.

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3. BASIS OF THE ITEM

ESDU 66027 presents information on the calculation of friction losses for incompressible flow in straight, constant area ducts for which ρ (and hence V) is by definition considered constant along the duct. It has been found that the friction factor data are equally applicable to compressible flows and the present Data Item extends the application of these data to two cases of subsonic compressible flow, in which ρ and V vary along the duct. In principle it is possible to derive the compressible flow losses in terms of a correction to the incompressible flow values but it is more convenient to analyse the flow in terms of the Mach number variation along the duct which leads to the calculation methods presented in this Item.

Notes are given here on some aspects of the idealised flow on which the Item is based and on its application to real flows. A brief summary of the analysis is given in Appendix C which includes the derivation of approximate working equations which may be applied in certain conditions.

3.1 One-dimensional Flow of Perfect Gases

The Item is based on the analysis of one-dimensional flows in which the flow properties are taken to be constant across the duct section. In practice, in flow in straight ducts, generally only the static pressure is constant across the section. The flow properties at a section in an equivalent one-dimensional flow (having the same mass flow rate as the actual flow) are taken as a set of mean flow properties for that section of the actual flow. Provided that the variation in flow properties across the duct is similar from one section to another, as in the fully-developed flow case considered here, the values predicted at other sections by the one-dimensional flow model will represent a realistic variation of the mean flow properties along the duct.

Most common gases, including air, may be considered to behave as perfect gases over the variations in temperature attainable in subsonic flow.

3.2 Isothermal and Adiabatic Flow

Two standard theoretical cases of one-dimensional flow of a perfect gas are considered; isothermal flow for which energy is transferred to or from the gas at a rate which maintains the static temperature constant, and adiabatic flow for which no energy is transferred between the gas and its surroundings so that the total temperature remains constant.

For subsonic flow to remain isothermal, heat must be transferred to the flow at a rate which increases rapidly with increase in flow velocity. In the case of low velocity gas flow in long, uninsulated pipelines it is possible for an equilibrium to be established with the surroundings which allows sufficient heat transfer to maintain approximately isothermal flow. With shorter ducts or higher velocities, deliberately imposed heat transfer will generally be necessary. Adiabatic flow is approximated when the rate of heat transfer to or from the duct is small compared with the rate of energy flow along it. Such conditions frequently occur in high velocity or high temperature flows in short ducts, especially if the duct is insulated. Generally, unless heat transfer is deliberately imposed, practical situations tend to be nearer to adiabatic flow than to isothermal flow.

3.3 The Reference Lengths L_I and L_A

From the analysis of isothermal and adiabatic flows, two convenient reference lengths, L_I and L_A respectively, may be derived. They are the duct lengths at which a flow would reach the limiting Mach number from a given value of the entry Mach number. In Appendix C, Sections C.1.1 and C.1.2, the limiting values are shown to be $M = 1/\sqrt{\gamma}$ for isothermal flow and $M = 1$ for adiabatic flow. The mass flow rate

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would then be the maximum possible for that section of duct, *i.e.* the flow would be choked and, if the duct length were increased beyond L_I or L_A, the mass flow rate would be reduced to maintain the required exit conditions.

In practice variations in flow properties across the duct would prevent these lengths (or the corresponding mass flow rates) from being achieved. In the isothermal flow case the very high heat transfer rates required as M_2 increases (tending towards infinite heat transfer as M_2 tends to $1/\sqrt{\gamma}$) make it impossible to approach closely a duct length L_I. In the adiabatic case longer ducts lead to the possibility of heat transfer between the gas and the duct and hence loss of adiabatic conditions but the limiting length can be more closely approached. Thus, if the actual duct length approaches L_I or L_A, the results given by this Item should be used with caution.

3.4 The Friction Factors f and $\bar{f}$

For isothermal flow, the friction factor f is constant along the duct since, because (ρV) and μ are constant, the Reynolds number, $Re = (\rho V)D_E/\mu)$, on which the value of f depends, is constant.

For adiabatic flow however, since T (and hence μ) varies along the duct, the Reynolds number, and hence the friction factor, will vary along the duct.

It is therefore theoretically necessary to use a mean friction factor, $\bar{f}$, defined by Equation (C.7) in Appendix C. However, in practice the variations in f which can arise in a subsonic adiabatic flow are small enough for the error in using a value of f_1, corresponding to temperature T_1, to be small. The error depends on γ and on the variation of μ with T. For air, with $\gamma = 1.4$, the maximum error in p_1/p_2, is about 1½ per cent in turbulent flow and about 3 per cent in laminar flow, but generally the errors will be very much less than those. If it is not possible initially to determine T_1, the total temperature T_t should be used, with subsequent correction only being necessary at high Mach numbers.

Friction factor data are given in Reference 2 and information on the variation of viscosity with temperature in References 3 and 4.

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4. REFERENCES

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2. ESDU Friction losses for fully-developed flow in straight pipes. Engineering Sciences Data ESDU 66027, 1966.
3. WATSON, J.T.R. *Viscosity of gases in metric units*. HMSO, 1972.
4. ESDU Reynolds number, speed of sound, dynamic viscosity, kinetic pressure and total pressure coefficient in air. Engineering Sciences Data ESDU 73017, 1973.

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5. EXAMPLES

(1) Air passes through a steel pipe 100 m long and 0.15 m diameter at a rate of 2.5 kg/s. Assuming fully-developed isothermal flow at a temperature of 288 K, what is the pressure drop if the outlet absolute pressure is 110×10^3 N/m^2? (For air $R = 287$ J/kg K.)

From Reference 4, for $T = 288$ K, $\mu = 18.3 \times 10^{-6}$ kg/m s (N s/m^2).

The Reynolds number
$$Re = \frac{m}{A}\frac{D_E}{\mu} = \frac{2.5}{\frac{\pi}{4}(0.15)^2}\,\frac{0.15}{18.3 \times 10^{-6}}$$
$$= 1.160 \times 10^6 .$$

It will be necessary to make an assumption as to the state of the pipe surface and, assuming a new smooth surface, Table 9.2 of ESDU 66027 indicates that the effective roughness height, ε, will be about 5×10^{-5} m and hence $\varepsilon/D_E \approx 0.000\,333$. From Figure 1 of ESDU 66027, for $\varepsilon/D_E = 0.000333$ and $Re = 1.160 \times 10^6$, $f = 0.0039$.

For a duct length $x_2 - x_1 = 100$ m, $4f(x_2 - x_1)/D_E = 4 \times 0.0039 \times 100/0.15$
$$= 10.40.$$

The downstream pressure, p_2, is known so that
$$\frac{m\sqrt{RT}}{Ap_2} = \frac{2.5\sqrt{287 \times 288}}{\frac{\pi}{4}(0.15)^2 \times 110 \times 10^3}$$
$$= 0.370.$$

The mass flow function $(m\sqrt{RT})/(Ap_2)$ is greater than 0.1 and, for $4f\,(x_2 - x_1)/D_E = 10.4$ and $(m\sqrt{RT})/(Ap_2) = 0.370$, p_1/p_2 is outside the range of Figure 1. From Figure 2, $(4fL_I/D_E)_2 = 4.31$.

From Equation (A.4),
$$(4fL_I/D_E)_1 = (4fL_I/D_E)_2 + 4f\,(x_2 - x_1)/D_E$$
$$= 14.71.$$

From Figure 2, for $(4fL_I/D_E)_1 = 14.71$,
$$(m\sqrt{RT})/(Ap_1) = 0.230.$$

Hence
$$p_1 = p_2 \times \frac{(m\sqrt{RT})/(Ap_2)}{(m\sqrt{RT})/(Ap_1)} = 177 \times 10^3 \text{ N/m}^2,$$

from which the pressure drop $(177 - 110) \times 10^3$ N/m^2 = 67×10^3 N/m^2.

This example is repeated in Appendix B where it is solved by use of a computer program.

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(2) Two pressure tappings, 300 ft apart, were taken from a smooth insulated duct, 3 in square in cross section, carrying compressed air. The pressure readings were 12 000 lbf/ft^2 and 2995 lbf/ft^2 and the total temperature was found to be constant at 330 K. Assuming the flow to be fully developed, determine the mass flow rate through the duct. (For air $\gamma = 1.4$ and $R = 89476/28.96 = 3090 =$ ft lbf/slug K.)

Since T_t is constant the flow is adiabatic.

Since T_1 is not known, the viscosity at temperature T_t will be used and, from Reference 4, for a value of T_t of 330 K, $\mu = 4.8 \times 10^{-7}$ lbf s/ft^2.

Since m is not known, Re cannot be calculated. It will be necessary to assume a set of values of m (or more conveniently, of m/A) and determine the corresponding values of Re. The friction factor $\bar{f}$ is then found from ESDU 66027 for each value of Re. The following table gives the results of the calculations.

$\frac{m}{A}$	m	$Re = \frac{m}{A}\frac{D_E}{\mu}$	$\bar{f}$
2.5	0.156	1.30×10^6	0.00279
3.0	0.188	1.56×10^6	0.00270
3.5	0.219	1.82×10^6	0.00263

Since p_1 and p_2 are both known, $(p_1/p_2) = 12000/2995 = 4.007$. It is now necessary to calculate $(m\sqrt{RT_t})/(Ap_2)$ which is found in each case to be greater than 0.1. Since $p_1/p_2 > 1.3$, $(m\sqrt{RT_t})/(Ap_1)$ is also required. From Figures 4a and 4b, $(4\bar{f}\,L_A/D_E)_1$ and $(4\bar{f}\,L_A/D_E)_2$ can be obtained and hence $4\bar{f}\,(x_2 - x_1)/D_E$ from which, since $(x_2 - x_1) = 300$ ft, f is obtained. These steps are summarised in the following table for the three chosen values of m/A.

$\frac{m}{A}$	$\frac{m\sqrt{RT_t}}{Ap_2}$	$\left(\frac{4\bar{f}\,L_A}{D_E}\right)_2$	$\frac{m\sqrt{RT_t}}{Ap_1}$	$\left(\frac{4\bar{f}\,L_A}{D_E}\right)_1$	$\frac{4\bar{f}\,(x_2 - x_1)}{D_E}$	$\bar{f}$
2.5	0.843	0.24	0.210	18.8	18.56	0.00387
3.0	1.012	0.069	0.252	12.6	12.53	0.00261
3.5	1.180	≈ 0	0.294	8.6	8.6	0.00179

It will generally be necessary at this stage to plot the two sets of values obtained for $\bar{f}$ against m/A. However, in the present case by comparing the values of $\bar{f}$ obtained in both tables, it is evident by inspection that the required value of m/A is approximately 3.0 and the mass flow rate is approximately 0.185 slug/s.

A much simpler method of solution to this problem is to use the computer program of Appendix B, which performs all the necessary iterations.

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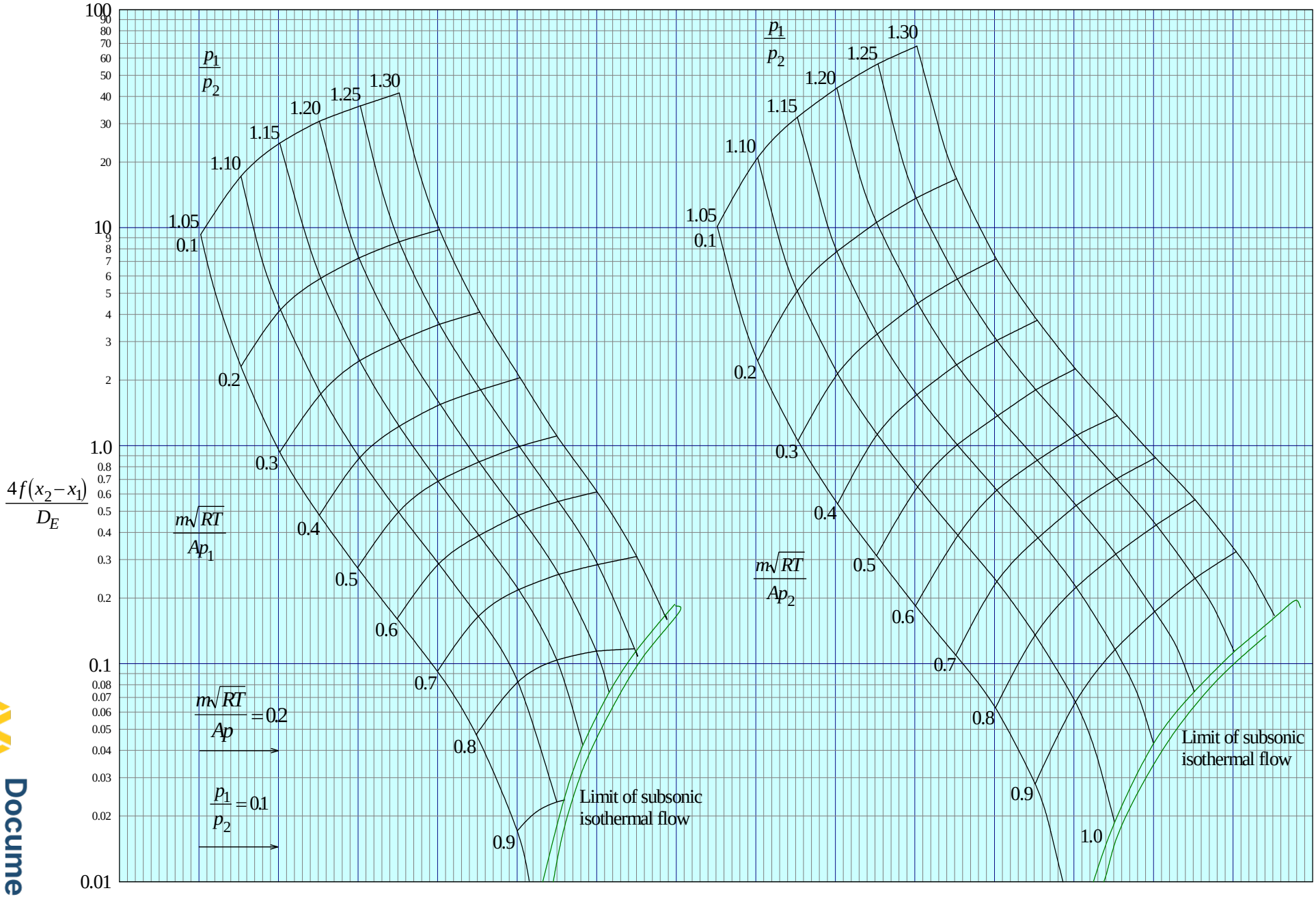

FIGURE 1 ISOTHERMAL FLOW (All γ)

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