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Subsonic lift-dependent drag due to trailing vortex wake for wings without camber or twist

Associated software: ESDUpac A7435

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THE PREPARATION OF THIS DATA ITEM

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SUBSONIC LIFT-DEPENDENT DRAG DUE TO THE TRAILING VORTEX WAKE FOR WINGS WITHOUT CAMBER OR TWIST

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SUBSONIC LIFT-DEPENDENT DRAG DUE TO THE TRAILING VORTEX WAKE FOR WINGS WITHOUT CAMBER OR TWIST

1. NOTATION AND UNITS

		SI	British
A	aspect ratio, b^2/S		
b	wing span	m	ft
C_{Dv}	lift-dependent drag coefficient due to wing trailing vortex wake, $(1 + \delta)C_L^2/\pi A$		
C_L	lift coefficient		
$c(\eta)$	chord at spanwise station η	m	ft
c_r	root chord	m	ft
c_t	tip chord	m	ft
$\bar{c}$	standard (or geometric) mean chord, $\int_0^1 c(\eta)d\eta$		
M	free-stream Mach number		
n	fraction of chord from wing leading-edge		
S	wing planform area	m ²	ft ²
$x_l(\eta)$	streamwise distance of wing leading-edge aft of apex	m	ft
β	compressibility parameter, $(1 - M^2)^{1/2}$		
δ	$(1 + \delta)$ is lift-dependent drag factor due to trailing vortex wake		
Λ_n	sweepback of n -chord line	deg	deg
$\Lambda_{1/2}$	sweepback of mid-chord line	deg	deg
$\Lambda_{1/2M}$	$\tan^{-1}((\tan \Lambda_{1/2})/\beta)$	deg	deg
η	spanwise distance from wing centre-line as fraction of wing semispan		
λ	taper ratio, c_t/c_r		
Suffix			
<i>eff</i>	denotes an effective value		

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2. GENERAL

The lift-dependent drag coefficient due to the wing trailing vortex wake at subsonic speeds is given by

$$C_D = \frac{1 + \delta}{\pi A} C_L^2. \quad (2.1)$$

Figures 1 to 7 present $1 + \delta$ for uncambered, untwisted wings with straight leading- and trailing-edges and streamwise tips, as a function of βA and $A \tan \Lambda_{1/2}$ for values of $\lambda = 0, 0.05, 0.125, 0.25, 0.5, 0.75$ and 1 .

Except for low values of βA , Figures 1 to 7 are based on Equations (23) and (24) of Derivation 1 with spanwise loadings calculated by means of linearised lifting-surface theory. The bulk of the spanwise loading data used came from the sixty-four planforms considered in Derivation 1 and cover the range $1.5 \leq \beta A \leq 8$. Some additional cases were calculated, via Derivation 2 for the spanwise loading, to extend the reduced aspect ratio, βA , up to 12. The values for $\beta A = 0$ were obtained by extrapolation for $A \tan \Lambda_{1/2} > 2$ and from slender wing theory (predicting elliptic loading, with $1 + \delta = 1$) for $A \tan \Lambda_{1/2} \leq 2$.

When the planform taper ratio differs from all the selected values in Figures 1 to 7 interpolation from a cross plot of $1 + \delta$ against λ is recommended (see Section 5). As an aid in drawing the cross-plot Figure 8 provides the value of λ , for a given combination of $A \tan \Lambda_{1/2}$ and βA , at which $1 + \delta$ is a minimum.

The data in this Item are given in terms of the sweepback of the mid-chord line, $\Lambda_{1/2}$, which is related to the sweepback of the n -chord line, Λ_n , via the expression

$$A \tan \Lambda_{1/2} = A \tan \Lambda_n + 2(2n - 1) \frac{1 - \lambda}{1 + \lambda}. \quad (2.2)$$

A computer program, ESDUpac A7435, that predicts $1 + \delta$ is described in Appendix A.

3. APPLICABILITY

The theory from which the data were derived is linearised and relates to inviscid flow. These restrictions are known to be relatively unimportant for most practical purposes in calculating the lift and lift-dependent drag due to the trailing vortex wake **provided** the flow remains fully attached over the wing surface. In particular, this condition is not likely to apply, at other than low incidences, to wings with highly swept leading-edges.

Although the Figures were derived from calculations for straight-tapered planforms with streamwise tips they may be applied to other types of planform via the following expressions giving equivalent values of the required planform parameters:

$$(A \tan \Lambda_{1/2})_{eff} = \frac{12}{\bar{c}} \int_0^1 \left[x_l(\eta) + \frac{c(\eta)}{2} \right] (2\eta - 1) d\eta \quad (3.1)$$

$$\text{and} \quad \lambda_{eff} = \int_0^1 c(\eta)(3\eta - 1) d\eta \bigg/ \int_0^1 c(\eta)(2 - 3\eta) d\eta. \quad (3.2)$$

Caution should be exercised in using data obtained from this Item via Equations (3.1) and (3.2) for planforms that depart significantly from a straight-tapered layout. For such cases the trailing vortex drag predictions may be unduly optimistic.

In applying data from this Item it should be borne in mind that camber and twist and the presence of a body, engine nacelles *etc.* all affect trailing vortex drag to a greater or lesser degree. Of those mentioned the body effect is usually the most important and is generally detrimental owing to the distortion of the spanwise loading in its vicinity.

4. DERIVATION

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. GARNER, H.C.
INCH, S.M. Subsonic theoretical lift-curve slope, aerodynamic centre and spanwise loading for arbitrary aspect ratio, taper ratio and sweepback. ARC CP 1137, 1970.
2. LEHRMAN, D.E.
GARNER, H.C. Theoretical calculation of generalised forces and load distribution on wings oscillating at general frequency in a subsonic stream. ARC R&M 3710, 1971.

5. EXAMPLE

Estimate the lift-dependent drag coefficient due to the trailing vortex wake at $M = 0.6$ and $C_L = 0.2$ for a wing with the following planform geometry:

$$A = 3, \Lambda_{1/2} = 60 \text{ degrees}, \lambda = 0.6.$$

For this wing $A \tan \Lambda_{1/2} = 3 \times \tan 60^\circ = 5.2$ and $\beta A = (1 - 0.6^2)^{1/2} \times 3 = 0.8 \times 3 = 2.4$.

Use of Figures 1 to 7 with $\beta A = 2.4$ and $A \tan \Lambda_{1/2} = 5.2$ yields the following table of $1 + \delta$ values:

λ	0	0.05	0.125	0.25	0.5	0.75	1
$1 + \delta$	1.020	1.009	1.010	1.020	1.050	1.081	1.103

From Figure 8 the minimum value of $1 + \delta$ for this case occurs at $\lambda = 0.075$.

The data are plotted in the following graph which gives, for $\lambda = 0.6$, $1 + \delta = 1.063$.