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Pressure loss during crossflow of fluids with heat transfer over plain tube banks without baffles

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(Continued on inside front cover)

PRESSURE LOSS DURING CROSSFLOW OF FLUIDS WITH HEAT TRANSFER OVER PLAIN TUBE BANKS WITHOUT BAFFLES

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PRESSURE LOSS DURING CROSSFLOW OF FLUIDS WITH HEAT TRANSFER OVER PLAIN TUBE BANKS WITHOUT BAFFLES

1. NOTATION AND UNITS

Three sets of units are given below. The SI set is part of a completely coherent system of units. The British units are not.

		Units		
		SI	British	
A_{bp}	minimum flow area in by-pass normal to local ideal flow direction, see Sketch 5.1	m ²	ft ²	ft ²
A_{min}	minimum cross-sectional area between tubes normal to local idea flow direction, see Section 3.1 step (ii)	m ²	ft ²	ft ²
A'_{min}	A_{min} associated with ideal portion of bank, see Sketch 5.1	m ²	ft ²	ft ²
B	transverse span of duct, see Sketch 5.1	m	ft	ft
B'	transverse span of ideal portion of duct, see Sketch 5.1	m	ft	ft
b	tube span by-pass dimension, see Sketch 5.1	m	ft	ft
C	function of X_t used to obtain K'_1 in Figure 1b	—	—	—
D	tube outside diameter	m	ft	ft
f_R	surface roughness Reynolds number factor	—	—	—
f_T	inlet turbulence Reynolds number factor	—	—	—
G_{max}	mass flow per unit cross-sectional area calculated between tubes, W/A_{min}	kg/(s m ²)	lb/(h ft ²)	slug/(s ft ²)
g_c	force conversion constant equal to 1.0 for SI units and equal to 4.17×10^8 and 1.0 for the British units respectively	— (1.0)	ft/h ² (4.17×10^8)	— (1.0)
K_{bp}	by-pass pressure loss coefficient	—	—	—
K'_1	pressure loss coefficient for single rows at reference conditions	—	—	—
K'_{10}	pressure loss coefficient for tube banks of ten or more rows at reference conditions	—	—	—
K'_B	K'_{10} associated with Re_B , see Figures 5a and 5b	—	—	—

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$\bar{K}$	general overall pressure loss coefficient for tube banks	—	—	—
$\bar{K}_1$	overall pressure loss coefficient for single rows	—	—	—
$\bar{K}_{10}$	overall pressure loss coefficient for tube banks of ten or more rows	—	—	—
$\bar{K}_N$	overall pressure loss coefficient for tube banks of between two and nine rows inclusive	—	—	—
k	interpolation factor used in Equation (6.1)	—	—	—
L	tube length, see insert on Figure 7	m	ft	ft
L'	length of tube in ideal portion of bank, see Sketch 5.1	m	ft	ft
l	tube end by-pass dimension, see Sketch 5.1	m	ft	ft
M	average number of tubes in each transverse row	—	—	—
N	number of tube rows in direction of flow	—	—	—
N_ϕ	number of major restrictions in direction of flow, see Section 3.1, step (vi)	—	—	—
n	tube row number in direction of flow	—	—	—
p	static pressure	N/m ²	lbf/ft ²	lbf/ft ²
Δp	overall static-pressure loss caused by bank in absence of effects due to gravity	N/m ²	lbf/ft ²	lbf/ft ²
q	function of X_l , used in Figures 6c, 6d and 6e to obtain K_{bp}	—	—	—
Re	Reynolds number, $G_{max}D/\mu_b$	—	—	—
Re_A	lower limit of Reynolds number in regime where tube surface roughness effects become significant, see Figures 5a and 5b	—	—	—
Re_B	Reynolds number marking approach of regime where tube surface roughness effects become independent of Re , see Figures 5a and 5b	—	—	—
s	general pitch dimension between adjacent tubes	m	ft	ft
s_l	longitudinal tube pitch measured normal to tube axes, see Sketch 3.1 and insert on Figure 7	m	ft	ft
s_t	transverse tube pitch, see Sketch 3.1	m	ft	ft
T	static temperature	K	°R	K

$\left(\frac{\overline{u^2}}{u^2}\right)^{1/2} / V_\infty$	turbulence intensity; ratio of standard deviation of longitudinal fluctuating velocity in free stream upstream of bank to mean free stream velocity, see Table 12.2	—	—	—
W	mass flow rate of fluid	kg/s	lb/h	slug/s
X_l	longitudinal tube pitch to diameter ratio, s_l/D	—	—	—
X_t	transverse tube pitch to diameter ratio, s_t/D	—	—	—
α_1	fluid property variation factor for single rows	—	—	—
α_2	by-pass factor for single rows	—	—	—
α_3	tube inclination factor for single rows	—	—	—
β_1	fluid property variation factor for ten or more rows	—	—	—
β_2	by-pass factor for ten or more rows	—	—	—
β_3	tube inclination factor for ten or more rows	—	—	—
β_4	tube surface roughness factor for ten or more rows	—	—	—
ε	effective surface roughness height	m	ft	ft
θ	inclination of tube axis to duct centre line, see insert on Figure 7	degree	degree	degree
Λ	ratio of number of major restrictions in tube end by-pass to N_ϕ	—	—	—
μ	dynamic viscosity	kg/(m s)	lb/(ft h)	slug/(ft s)
ρ	density	kg/m ³	lb/ft ³	slug/ft ³
σ	bank effective porosity defined in Section 5.3.1	—	—	—
σ'	porosity of ideal portion of bank, see Section 5.3.1	—	—	—
$\Delta\sigma$	effective increase in porosity due to by-pass, $\sigma - \sigma'$	—	—	—
ϕ	tube end by-pass average contraction ratio, $\bar{A}_{bp}/(Bl\sin\theta)$	—	—	—
ω	parameter used in Figure 8 to calculate β_4	—	—	—

Suffixes

<i>b</i>	appropriate to bulk conditions of crossflow (averaged at transverse section)
<i>bm</i>	appropriate to conditions at crossflow fluid bulk mean temperature, see Section 7.2
<i>w</i>	appropriate to tube wall conditions (averaged at transverse section)
<i>i</i>	appropriate to conditions at crossflow inlet of tube bank, or group of rows
<i>o</i>	appropriate to conditions at crossflow outlet of tube bank, or group of rows

2. INTRODUCTION

Pressure loss data are presented for single-phase Newtonian* fluid flow normal to banks of plain tube rows. The data cover a wide range of regular arrays of in-line and staggered tubes and allowances are given for known temperature variations through the bank. However, for the particular types of tube arrangements shown in Table 3.1, ESDU 79034 (Reference 4) may be preferred because it is easier and more consistent to use due to the more detailed treatment given to such tube geometries. ESDU 79034, however, does not consider the effects of by-passing for which this Item may be used. It should be noted that the notation and method of this Item are different from those of ESDU 79034 but the relationships that link the two notations are given in Appendix D.

Complementary to this Item on pressure loss is ESDU 73031 (Reference 3) which can be used to calculate the heat transfer data; the two Items are compatible in terminology and notation.

The flow around a bank of tubes is a complex phenomenon† and there are therefore many factors which influence the pressure loss. Some of these factors can be allowed for systematically by the methods described in Sections 4 and 5. Other factors which are known to be significant but unquantifiable are discussed in Section 8. The comprehensive calculation process takes into account the following effects:

- (i) fluid property variations due to temperature variations,
- (ii) by-passing,
- (iii) tube inclination,
- (iv) tube outside surface roughness,
- (v) upstream turbulence intensity,
- (vi) number of tube rows in direction of crossflow.

For some applications, the refinements given by considering all the above effects may not be necessary. Section 3.3 therefore offers guidance on simplifying the full calculation procedure and enables checks to be made on any consequent losses of accuracy. Section 9 gives details on the uncertainty and limitations of the data.

* Most pure liquids and simple mixtures of pure gases are Newtonian; non-Newtonian fluids include paints, polymer solutions, etc.

† Appendix C offers simple physical explanations of the flow around the tubes and may help the user to recognise and to deal with situations where the determination of pressure loss is difficult.

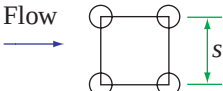
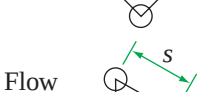
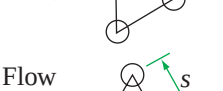
3. GUIDANCE ON CALCULATION OF PRESSURE LOSS

3.1 Input Data

The method requires the evaluation of the following parameters.

- (i) $X_t = s_t/D$ and $X_l = s_l/D$, the transverse and longitudinal tube pitch to diameter ratios respectively. For the conversion of common pitch to diameter ratios, the following table* is provided.

TABLE 3.1 Geometry for Common Tube Arrangements (Arrangements Used in ESDU 79034)

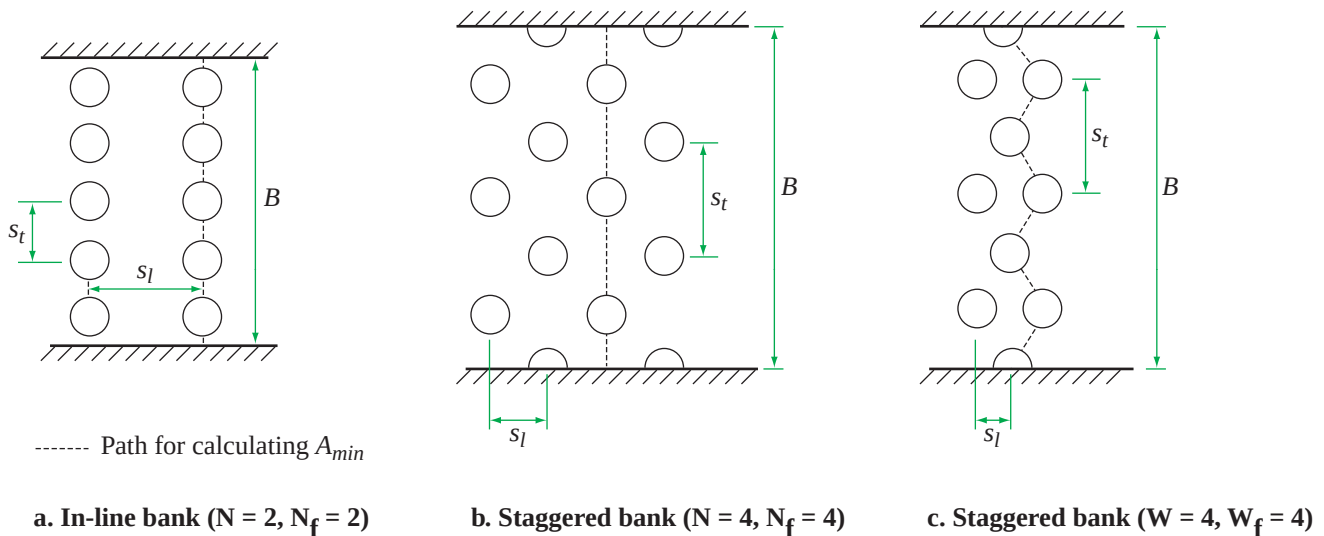
Conversion of Common Arrangements to X_t and X_l				
Tube arrangement	Bank type	To obtain X_t , multiply s/D by:	To obtain X_l , multiply s/D by:	
	Square in-line	In-line	1.0	1.0
	Rotated square	Staggered	1.414	0.707
	Equilateral triangle	Staggered	1.0	0.866
	Rotated equilateral triangle	Staggered	1.732	0.5

Where the tube axes are inclined towards the cross flow, see insert on Figure 7 for definition of s_l .

- (ii) A_{min} , the minimum cross-sectional area between tubes normal to the local ideal flow direction[†] (see Sketch 3.1).

* Pressure loss data for the common pitch to diameter ratios illustrated in the table may also be obtained from equations given in ESDU 79034⁴.

[†] Note that for staggered banks (illustrated in Sketches 3.1b and 3.1c), A_{min} may be calculated either in a transverse plane or along a staggered path whichever gives the lower value. The curve index for staggered banks (insert on Figures 5a and 5b) presents a line above which A_{min} should be calculated in a transverse plane and below which A_{min} should be calculated along a staggered path.



Sketch 3.1

For banks where the effects of a by-pass are to be taken into account, see Sketch 5.1.

In cases where the tube axes are inclined towards the crossflow, A_{min} is the minimum cross-sectional area, between the inclined tubes, resolved normal to the crossflow.

- (iii) $G_{max} = W/A_{min}$.
- (iv) $Re = G_{max}D/\mu_b$ where μ_b may be taken initially as the arithmetic mean of the bank inlet and outlet viscosities. Note that where μ_b varies significantly through the bank (*i.e.* for most liquids in non-isothermal crossflow) and where $Re < 3 \times 10^3$, it may be necessary to divide the bank into groups of tubes within which the variation of μ_b is comparatively small. Section 7.1 explains how to do this.
- (v) ρ_{bm} , the bulk mean density of the crossflow fluid. When the fluid densities between the bank inlet and outlet vary by less than 20 per cent, ρ_{bm} may be taken as the arithmetic mean of these values. When the variation of ρ_b is greater than 20 per cent, Section 7.2 gives guidance on the procedures to follow.
- (vi) N_ϕ , the number of major restrictions through the bank. For in-line banks and for staggered banks where A_{min} is calculated in a transverse plane (illustrated in Sketches 3.1a and 3.1b), $N_\phi = N$, the number of rows. For staggered banks where A_{min} is calculated along a staggered path (illustrated in Sketch 3.1c), $N_\phi = N - 1$.

3.2 General Pressure Loss Equation

The overall static pressure drop caused by a tube bank within a constant area duct^{*}, in the absence of gravity effects, is given by Equation (3.1) which consists of the sum of two terms.

The first term[†], which is always positive, deals with the irrecoverable component of static-pressure loss and depends upon a value of K which may be obtained from the data in this Item.

^{*} For calculating the pressure drop in a shell-and-tube heat exchanger, the correlations given in ESDU 79034⁴ will be easier to apply.

[†] If the bank is divided into a series of groups of tubes (to allow for large variations in the fluid physical properties with changes in temperature through the bank) the first term becomes a summation over the separate groups.

The second term can be positive or negative and is derived from momentum considerations allowing for density changes in the crossflow fluid between the inlet and outlet of the bank. Usually these will only affect the pressure loss significantly if the density variation is greater than 15 per cent.

$$\Delta p = \frac{N_{\phi} \bar{K} G_{max}^2}{2 \rho_{bm} g_c} + \frac{1}{g_c} \left[\frac{A_{min} G_{max}}{B L \sin \theta} \right]^2 \left[\frac{1}{\rho_o} - \frac{1}{\rho_i} \right]. \quad (3.1)$$

Additional terms due to gravity should be considered wherever the crossflow fluid changes vertical height. Also in open systems, such as in some fan blown air heaters, buoyancy effects should be allowed for when calculating the overall pressure loss.

The overall pressure loss coefficient $\bar{K}$ will assume a suffix 1, 10 or N as appropriate to the conditions set out in the following table.

TABLE 3.2 Definitions of $\bar{K}$

$\bar{K}$	Appropriate conditions	Section
$\bar{K}_1$	Applicable to a tube bank with only one row, illustrated in Sketch 4.1	4
$\bar{K}_{10}$	Applicable to a tube bank with ten or more rows	5
$\bar{K}_N$	Applicable to a tube bank with between 2 and 9 rows inclusive	6

The overall pressure loss coefficients in Table 3.2 are themselves the product of a reference condition loss coefficient K' (applicable to some typical but fixed conditions only) and up to four correction factors (given in Sections 4 and 5) which allow for departures from the reference conditions.

3.3 Guidance on the Simplification of the Full Calculation Procedure

In some cases approximate estimates only of the pressure loss may be required as, for example, in comparing initial design studies of banks of different geometries. In these cases approximate values can be found by equating some of the parameters or correction factors (allowing for departures from the reference conditions) to unity. Table 12.4 indicates the possible magnitudes of the resulting errors that can occur when the parameters are simplified in this way and those parameters that can lead to an underestimate of pressure loss when simplified are in brackets.

The assumption that the by-pass factors, α_2 and β_2 , are unity simplifies the calculation procedure considerably and will generally lead to an estimate of a higher pressure loss than would be predicted using the full calculation procedure.

4. OVERALL PRESSURE LOSS COEFFICIENT, $\bar{K}_1$, FOR SINGLE ROWS

The value of $\bar{K}_1$, for a single row is given by

$$\bar{K}_1 = K'_1 \alpha_1 \alpha_2 \alpha_3, \quad (4.1)$$

where K'_1 , obtained from Section 4.1, is the pressure loss coefficient at the following reference conditions.

- (i) There are no temperature differences between the bulk fluid and the tube walls.
- (ii) There is no by-passing, see Sketch 4.1.
- (iii) The flow is normal to the plane of the tube axes, *i.e.* $\theta = 90^\circ$.

Note that, unlike the case for ten or more rows, K'_1 itself includes the effects of tube roughness.

The factors α_1 , α_2 , and α_3 that account for the effects on $\bar{K}_1$ of departures from the reference conditions are given in Table 4.1.

TABLE 4.1 Correction Factors for $\bar{K}_1$

Factor	Description	Section	Figure or Equation
α_1	Allows for variation of fluid properties due to $T_w \neq T_b$	4.2	Figure 2
α_2	Allows for by-passing between tubes and duct walls	4.3	Equation (4.2)
α_3	Allows for tube inclination to crossflow	4.4	Figure 3

Other effects are discussed in Section 8.

4.1 Calculation of Pressure Loss Coefficient, K'_1 , at Reference Conditions for Single Rows

For Re less than 10^4 , Figure 1a gives values of K'_1 for single rows as a function of Reynolds number and pitch to diameter ratio, X_t .

For Re greater than 10^4 , Figure 1b gives values of C (which may be found from the insert) and K'_1/C which, for $75^\circ \leq \theta \leq 90^\circ$, is given as a function of the following:

- (i) Reynolds number,
- (ii) surface roughness Reynolds number factor*, f_R , which may be evaluated through Table 12.1 and Figure 1c and
- (iii) inlet turbulence Reynolds number factor*, f_T , which may be obtained through Table 12.2 and Figure 1d.

* The factors f_R and f_T may be taken as unity for $Re \leq 3 \times 10^4$ because this marks the lower limit of the Reynolds number regime which is affected by roughness and turbulence. Any consequent discontinuity in the value of f_T will not cause K'_1 to vary discontinuously.

For $\theta < 75^\circ$ with $Re > 3 \times 10^4$, it is recommended that K'_1/C is equated to 1.05, irrespective of the value of $f_R f_T Re$; this is likely to yield an overestimate, rather than an underestimate, of pressure loss.

4.2 Factor α_1 Allowing for Change of Fluid Properties for Single Rows

For gases*, $\alpha_1 = 1$.

For liquids, α_1 is given by Figure 2 as a function of Reynolds number and the ratio of the viscosity of the crossflow fluid at the mean tube wall temperature to that at the mean crossflow fluid temperature.

4.3 Factor α_2 Allowing for By-passing in Single Rows

By-passes for single rows are illustrated in Sketch 4.1. No data have been found for by-pass effects on single rows but, for reasons given in Appendix A, it can be assumed that at low Reynolds numbers ($Re < 100$) these will be similar to those for multi-row banks. At high Reynolds numbers ($Re > 10^3$), by-passing may be treated as a modification of blockage ratio. Between these low and high Reynolds number regions, the high Reynolds number method given in Section 4.3.2 yields an overestimate of pressure loss.

4.3.1 By-pass factor α_2 for $Re < 100$

In this regime, α_2 may be taken as equal to β_2 given in Section 5.3.1 provided that X_l is assumed equal to X_t .

4.3.2 By-pass factor α_2 for $Re > 10^3$

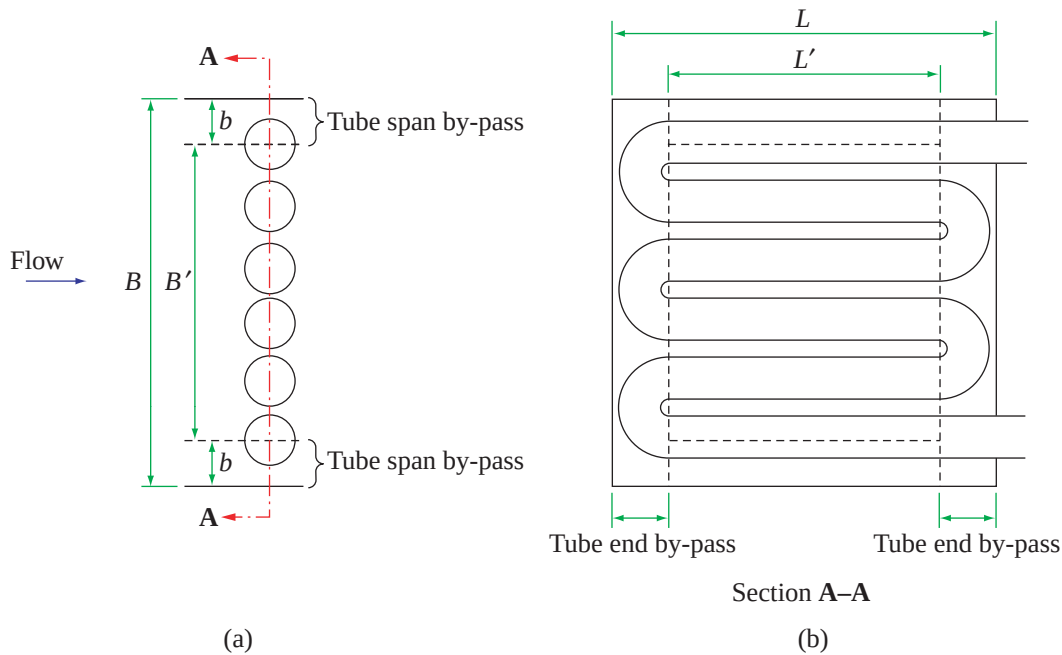
In this regime, α_2 is given by

$$\alpha_2 = X_t \left(1 - \frac{A_{min}}{BL \sin \theta} \right) \quad (4.2)$$

and may be applied to both tube span by-passes and to tube end by-passes (see Sketch 4.1).

Note that in single rows with no tube end by-passes and where the duct walls are half a transverse pitch ($s_t/2$) away from the centre-lines of the outer tubes, $\alpha_2 = 1$.

* Data reported in Derivation 36 suggest that there is a small effect on the pressure loss behaviour of air, cooled in crossflow, which may be accounted for by factoring the Reynolds number in a similar way to roughness and turbulence. However, too few data are available to assess this reliably.



Sketch 4.1 Single row by-pass notation (tubes not inclined)

4.4 Factor α_3 Allowing for Tube Inclination to the Crossflow for Single Rows

For Reynolds numbers, Re , less than 50, α_3 may be taken as unity for all values of θ . At higher Reynolds numbers ($Re > 300$), α_3 may be found from Figure 3 where it is given as a function of the tube inclination, θ , and the transverse pitch ratio, X_t . For Reynolds numbers between 50 and 300, Section 9.5 may help the user to estimate a value for α_3 . Note that for $\theta < 75^\circ$ at very high Reynolds numbers ($f_R f_T Re > 3 \times 10^5$), because the effect of tube inclination on the reference condition loss coefficient, K'_1 , is uncertain, it is recommended that the value of K'_1/C (Figure 1b) should be taken as 1.05 irrespective of the true value of $f_R f_T Re$; this is likely to yield an overestimate, rather than an underestimate, of pressure loss.

5. OVERALL PRESSURE LOSS COEFFICIENT, $\bar{K}_{10}$, FOR BANKS OF TEN OR MORE ROWS*

The value of $\bar{K}_{10}$ for a bank of ten or more rows is given by

$$\bar{K}_{10} = K'_{10} \beta_1 \beta_2 \beta_3 \beta_4, \quad (5.1)$$

where K'_{10} , obtained from Section 5.1, is the pressure loss coefficient at the following reference conditions.

- (i) There are no temperature differences between the bulk fluid and the tube walls.
- (ii) There is no by-passing, see Sketch 5.1.
- (iii) The flow is normal to the plane of the tube axes, *i.e.* $\theta = 90^\circ$.
- (iv) Tubes are assumed to have a nominal roughness ratio of $\varepsilon/D = 5 \times 10^{-5}$ (equivalent to 30 mm diameter drawn metal tube).

The factors β_1 , β_2 , β_3 and β_4 that account for the effects on K'_{10} of departures from the reference conditions are given in the following table.

TABLE 5.1 Correction Factors for $\bar{K}_{10}$

Factor	Description	Section	Figure or Equation
β_1	Allows for variation of fluid properties due to $T_w \neq T_b$	5.2	Figure 2
β_2	Allows for by-passing between tubes and duct walls	5.3	Figures 6a to 6e Equations (5.2), (5.3) and (5.4)
β_3	Allows for tube inclination to crossflow	5.4	Figure 7
β_4	Allows for tube roughness ratios different from $\varepsilon/D = 5 \times 10^{-5}$	5.5	Figure 8 Equations (5.2a), (5.5b) and (5.5c)

Other effects are discussed in Section 8.

5.1 Pressure Loss Coefficient, K'_{10} , at Reference Conditions for Banks of Ten or More Rows

Values of K'_{10} are given in Figures 4a and 4b, and 5a and 5b, for in-line and staggered banks respectively. Indexes to the curves are inset on the Figures to which they apply and, to obtain K'_{10} for particular values of X_l and X_t , the appropriate curve is selected from the Index according to the value of X_l and X_t required. Sometimes, however, it may be necessary to interpolate between values of K'_{10} given by the nearest values of X_l and X_t for which curves are available; the user may find the method applied in Example 1 (Section 10.1) helpful.

5.2 Factor β_1 Allowing for Change of Fluid Properties for Banks of Ten or More Rows

For gases, $\beta_1 = 1.0$.

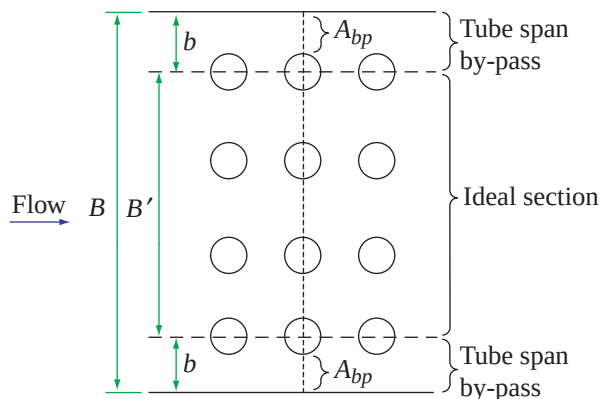
* See Section 6 for banks of between 2 and 9 rows.

For liquids, $\beta_1 = \alpha_1$ and is given by Figure 2. The ratio μ_w/μ_b required for Figure 2 should be evaluated at the viscosities appropriate to the crossflow fluid at the mean tube wall temperature at a given row and the mean crossflow fluid temperature at that same row*.

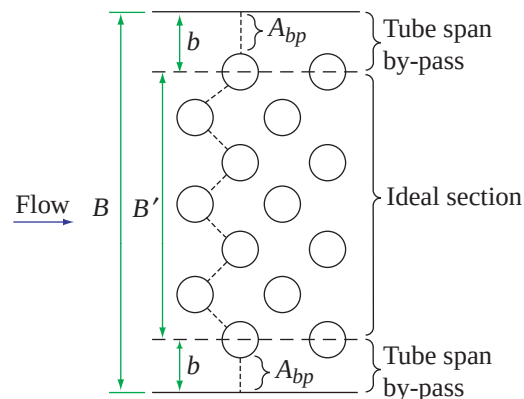
5.3 Factor β_2 Allowing for By-passing in Banks of Ten or More Rows

Various by-passes are illustrated in Sketch 5.1. The treatment of by-passing varies according to the Reynolds number regime in which the bank is operating. Data on the effect of by-passing for $Re < 100$ are given in Section 5.3.1 and for $Re \geq 500$ are given in Section 5.3.3. Between these regimes, see Section 5.3.2 for guidance.

In general, the section of the bank that is geometrically ideal, *i.e.* in which the flow can be considered two dimensional, must be treated separately from the complete bank†, see Sketch 5.1. The value of β_2 then depends on the flow regime as defined by the Reynolds number, and the three cases are treated in the following sections.

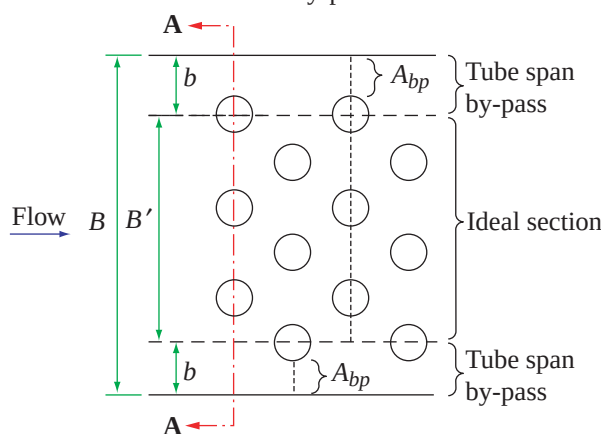


(a) In-line bank by-pass notation

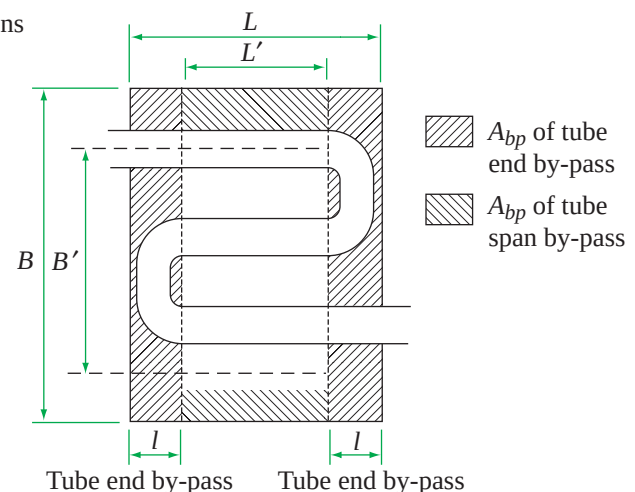


(c) Staggered bank by-pass notation where $A_{c_{min}}$ is calculated along a staggered path

- Paths taken for calculating A_{min}
- - - - - Boundaries between by-pass sections and ideal sections



(c) Staggered bank by-pass notation where A_{bp} for one tube span by-pass is offset



(d) Section through A A on Sketch (5.1 c)

Sketch 5.1 Illustration of by-pass notation appropriate to non-inclined tubes only

* Where μ_w/μ_b varies significantly through the bank, a stage-by-stage calculation of pressure loss must be made, see Section 7.1.

† Appendix A, which explains the derivation of the by-pass factors, also gives simple equations enabling estimates of the different flow rates in each section of the bank to be made.

5.3.1 By-pass factor, β_2 , for $Re < 100$

For $Re < 100$, an approximate allowance for by-passing can be made by considering the effective increase in overall porosity which the by-pass gives to the bank. Figure 6a gives values for the by-pass factor, β_2 , as a function of the effective porosity increase, $\Delta\sigma$, which is defined as follows

$$\Delta\sigma = \sigma - \sigma' \quad (5.2)$$

where^{*}
$$\sigma = 1 - \frac{\text{Volume occupied by active tubes in bank}}{\text{Active volume of containing duct} (=NDX_1BL \sin \theta)}$$

and

$$\sigma' = 1 - \frac{\text{Volume occupied by tubes in ideal section}}{\text{Volume of ideal containing duct} (=NDX_1B'L' \sin \theta)}$$

Where there is a high rate of heat transfer in the bank, by-passing can lead to large transverse variations in the crossflow fluid viscosity. Especially at very low Reynolds numbers ($Re < 10$), this will exaggerate the flow distortion which will in turn result in higher pressure losses than would be predicted by using β_2 (typically 50 per cent greater).

5.3.2 By-pass factor, β_2 , for $100 < Re < 500$

In this regime, judgement must be used to interpolate between values of β_2 for $Re = 100$ given by Section 5.3.1 and $Re = 500$ given by Section 5.3.3. Where an overestimate of pressure loss is preferable, however, it is recommended that the method given in Section 5.3.3 is adopted and for the purposes of applying Equation (5.4), values of K_{bp} and K'_{10} must be appropriate to $Re = 500$.

5.3.3 By-pass factor, β_2 , for $Re \geq 500$

In this regime, if the ratio of the sum of the minimum areas, ΣA_{bp} , of the by-pass sections to A_{min} is less than 0.05, by-passing is unlikely to reduce the pressure drop by more than 15 per cent. If, however, $\Sigma A_{bp}/A_{min}$ exceeds 0.05 or a more reliable criterion is required, the value of β_2 must be found from Equation (5.4) using the following procedure.

- (i) Divide the bank into sections as shown in Sketch 5.1.
- (ii) Evaluate A'_{min} for the ideal portion and A_{bp} separately for each by-pass section. Note that in the case of tube span by-passing in staggered banks, the path chosen for the calculation of A_{min} is not necessarily in a single transverse plane, as shown in Sketch 5.1c; check that A_{min} satisfies the following relation:

$$A_{min} = A'_{min} + \Sigma A_{bp} \quad (5.3)$$

- (iii) Evaluate K_{bp} for each by-pass section as follows.
 - (a) For tube end by-passes, the value of K_{bp} may be found from Figure 6b the derivation of which is explained in Appendix A. As a rough guide, it is suggested that the contraction ratio[†], ϕ , of the

* The explanation of the use of the term 'active' is as follows. If a tube span by-pass is blocked either, for example, by sealing strips or by the outer tubes touching the duct wall, i.e. $b = D/2$, the fluid in the by-pass will be virtually stagnant. In that case, therefore, the active volume of the containing duct will be less than the geometric volume by $NDX_1bL \sin \theta$ for each tube span by-pass so affected; similarly, the active volume of the tubes will be less than their geometric volume by that volume of the tubes immersed in the stagnant fluid within the by-pass(es).

by-pass under consideration is calculated and that a value of K_{bp}/Λ is obtained from Figure 6b. This quantity should then be multiplied by Λ to obtain K_{bp} , where Λ is the ratio of the number of contractions* in the by-pass to N_ϕ .

- (b) For tube span by-passes, values of K_{bp}/q , q and hence K_{bp} may be found from Figures 6c to 6e as functions of the by-pass width ratio, b/D , Reynolds number and the ideal portion longitudinal pitch ratio, X_l . Note that values of K_{bp}/q for in-line banks are twice those for staggered banks because, for any given number of rows, there are twice as many major restrictions through an in-line bank by-pass as through a staggered bank by-pass.
- (iv) Insert A_{min} and the values of A_{bp} with the appropriate values of K_{bp} into Equation (5.4) together with values of K'_{10} and A'_{min} for the ideal portion. Evaluate β_2 from

$$\beta_2 = \frac{1}{K'_{10}} \left[\frac{A_{min}}{\frac{A_{min}}{\sqrt{K'_{10}}} + \sum \frac{A_{bp}}{\sqrt{K_{bp}}}} \right]^2. \quad (5.4)$$

Note that, except for very large by-passes, β_2 will generally lie between 0.7 and 1.1.

5.4 Factor β_3 Allowing for Tube Inclination to the Crossflow in Banks of Ten or More Rows

For $Re < 50$, β_3 may be taken as unity for $30^\circ \leq \theta \leq 90^\circ$.

Figure 7 gives approximate values for β_3 as a function of the inclination angle, θ , for Reynolds numbers greater than 10^3 .

For a Reynolds number between 50 and 10^3 , a proportional interpolation between the curves in Figure 7 is unreliable. However, Section 9.5 may help the user to select an approximate value for β_3 in this Reynolds number range. Alternatively, for the geometries illustrated in Table 3.1, ESDU 79034 may be used to evaluate the effect of inclination. Note, however, the different notation which is related to that in this Item in Appendix D.

5.5 Factor β_4 Allowing for the Effect of Roughness on Banks of Ten or More Rows

For in-line banks, no consistent effect of roughness on the pressure loss coefficient could be found when the roughness either increased or decreased from the reference condition $\varepsilon/D = 5 \times 10^{-5}$. Variations in $\bar{K}_{10}$ of about ± 20 per cent were typical but, in the absence of other information, β_4 should be equated to unity.

For staggered banks roughness can usually be allowed for as set out in Section 5.5.1.

† In the case of tube end by-passes, it is convenient to define the contraction ratio as the ratio of the minimum free flow area around the obstructions, A_{bp} , to the unobstructed cross-sectional area of the by-pass channel, $B \sin \theta$. Note that where this ratio varies from contraction to contraction, an average value should be acceptable, and where the plane of the obstruction is inclined to the flow direction, the area around the obstruction projected normal to the flow should be used for the calculations.

* The user must decide to what extent the tube end by-pass contractions will interfere and thus whether or not to give Λ as high a value as implied by pure geometric considerations.