

EXAMPLE OF THE USE OF DATA ITEMS ON RANGE PERFORMANCE

1. NOTATION AND UNITS*

		British	SI
a	speed of sound	kn	km/h, kn
C_D	drag coefficient, $D / \left[\frac{\rho}{2} \left(\frac{rV}{3600} \right)^2 S \right]$		
C_L	lift coefficient, $L / \left[\frac{\rho}{2} \left(\frac{rV}{3600} \right)^2 S \right]$		
c	specific fuel consumption, Q/F_N	$\left\{ \begin{array}{l} \text{weight flow basis} \\ \text{mass flow basis} \end{array} \right.$	$\left\{ \begin{array}{l} \text{N/N h} \\ \text{kg/N h} \end{array} \right.$
D	drag	lbf	N
F_N	total net thrust (see Item No. 69006 for thrust definitions)	lbf	N
g	acceleration due to gravity	ft/s ²	m/s ²
H	geopotential height	ft	m
H_p	geopotential pressure height	ft	m
h_e	energy height, $H + \left[\frac{rV}{3600} \right]^2 / 2g_0$	ft	m
K_1, K_2	coefficients in drag equation		
L	lift	lbf	N
M	Mach number, V/a		
m	aircraft mass	slug	kg
Q	total engine fuel consumption	$\left\{ \begin{array}{l} \text{weight flow basis} \\ \text{mass flow basis} \end{array} \right.$	$\left\{ \begin{array}{l} \text{N/h} \\ \text{kg/h} \end{array} \right.$
R_a	air range	n.mile	km, n.mile
R_{ref}	reference air range (see Item No. 73019, Equation (4.1))	n.mile	km, n.mile
r	conversion factor (6 076.12 ft/n.mile, 1 000 m/km, 1 852 m/n.mile)	ft/n.mile	m/km, m/n.mile

* The example in this Item is presented in British units and follows the customary British practice of working in terms of aircraft weight and fuel weight flow.

S	wing area	ft ²	m ²
t	time	h	h
V	true airspeed	kn	km/h, kn
W	aircraft weight, mg	lbf	N
W_F	weight of fuel	lbf	N
y	ratio of airspeed to minimum drag speed, V/V_{md}		
Δh_e	change in energy height	ft	m
ΔR_a	lost range	n.mile	km, n.mile
δR_a	contribution to lost range due to change of energy height	n.mile	km, n.mile
Δt	lost time	h	h
ΔW_F	lost fuel	lbf	N
δW_F	contribution to lost fuel due to change of energy height	lbf	N
ρ	density of air	slug/ft ³	kg/m ³

Suffixes

cl	climb
cr	cruise
d	descent
f	final condition
i	initial condition
max	maximum
md	minimum drag condition in 1g level flight
$mean$	mean value
$stage$	stage
$take-off$	take-off
0	denotes value at sea level in International Standard Atmosphere (Item No. 68046 gives numerical values)

2. INTRODUCTION

This Item provides an illustrated explanation, by means of a worked example, of the combined use of Item No. 73019, "Approximate methods for estimation of cruise range and endurance: aircraft with turbo-jet and turbo-fan engines" and Item No. 74018, "Lost range, fuel and time due to climb and descent: aircraft with turbo-jet and turbo-fan engines". A general introduction to the subject of range estimation and the derivation of the basic equations is given in Item No. 73018.

3. EXAMPLE

Estimate the take-off weights and fuel loads required for a large subsonic transport aircraft to carry a payload of 80 000 lbf over stage lengths of 500, 1 000, 1 500 and 2 000 n.mile in still air ISA conditions for a suitable range of cruise heights. Find also the value of cruise height for each of these stage lengths which gives the minimum overall fuel consumed, subject to the quoted limits, and the associated flight times.

The aircraft has an operating weight, empty, of 240 000 lbf and a wing area of 3 500 ft². It may be assumed that the aircraft cruises at $M = 0.82$ and that the following information applies:

- (i) the drag at $M = 0.82$ may be expressed in the form:

$$C_D = K_1 + K_2 C_L^2,$$

where the values $K_1 = 0.018$ and $K_2 = 0.05$ are valid up to a cruise lift coefficient of $C_L = 0.55$,

- (ii) the variation of the installed engine specific fuel consumption and the maximum permitted cruise thrust, $F_{N_{cr,max}}$, with cruise height at $M = 0.82$ is as given in the following table,

H_p ft	25 000	30 000	35 000	40 000
c lbf/lbf h	0.710	0.685	0.675	0.675
$F_{N_{cr,max}}$ lbf	34 500	30 000	25 500	19 500

- (iii) for the purposes of this Example,

fuel reserve in all cases 22 000 lbf,

fuel allowance for take-off to 1 000 ft,
250 kn calibrated airspeed 1 000 lbf,

fuel allowance for final approach from
1 000 ft, 250 kn calibrated airspeed,
landing and taxi 1 000 lbf.

4. METHOD OF SOLUTION

It is not possible to solve this problem directly without iteration; the range performance is therefore calculated for a number of values of fuel weight and the fuel loads required for the specified stage lengths are then deduced graphically. The principal steps in the calculation are as follows:

- (i) Using the method of calculation described in Item No. 73019, the relationship is found between still-air cruise range and fuel weight at $M = 0.82$ at heights of 25 000, 30 000, 35 000 and 40 000 ft (see Section 4.1 in the present Item).
- (ii) Using the method of calculation described in Item No. 74018, the lost range and lost time are calculated for each cruise case considered (see Section 4.2 in the present Item).
- (iii) For each cruise case the lost range is subtracted from the cruise air range to give a relationship between stage length, stage fuel weight and cruise height (see Section 4.2.3).
- (iv) For each of a range of values of cruise height, the aircraft weight and stage fuel weight are calculated which correspond to the specified lift coefficient limit and maximum permitted cruise thrust limit (see Section 4.3).
- (v) Using the results of (iii) and (iv) above, the values of stage fuel weight and hence take-off weight are found for each of the required stage lengths and cruise heights, subject to the specified lift coefficient limit and maximum permitted cruise thrust limit (see Section 4.4). The associated flight times and mean flight speeds are subsequently deduced and presented in Section 5.

The calculations for this Example were carried out to a greater number of significant figures than are used in the text and Tables of the Item. The values presented were obtained by rounding after the calculation was completed and this has resulted in a few instances of apparent discrepancies between adjacent columns in some of the Tables.

In order to reduce the size of the Item some of the detail calculations are presented for a cruise height of 30 000 ft only, however the text should be read as if the results were given for all of the heights considered.

Except where stated otherwise, values of atmospheric parameters throughout this Item are taken from Item No. 68046.

4.1 Calculation of Cruise Air Range for Given Fuel Loads

It is assumed that all the stage fuel, $W_{F, stage}$, is burnt at the cruise height and speed and the resulting values of cruise air range are obtained by following the calculation sequence described in Item No. 73019, Sections 4 and 4.2.1, with values of R_a/R_{ref} read from Figure 6 of that Item. The range of cruise heights for which the calculations are performed is 25 000, 30 000, 35 000 and 40 000 ft*; the calculations for 30 000 ft are presented in Table 4.1. For each cruise height values of cruise air range are calculated for values of $W_{F, stage}$ starting at 10 000 lbf and increasing in steps of 10 000 lbf until a cruise air range well in excess of 2 000 n.miles is obtained. From the given information, the take-off weight is

$$W_{take-off} = 240\,000 + 80\,000 + 22\,000 + 1\,000 + 1\,000 + W_{F, stage} = 344\,000 + W_{F, stage} \text{ lbf.}$$

* Nearly all cruise operations of aircraft of the type considered in this example take place well within this height range.

Thus, by omitting the 1 000 lbf fuel for take-off to 1 000 ft, the initial weight, W_i , for this cruise calculation is

$$W_i = 343\,000 + W_{F,stage} \text{ lbf.}$$

The following additional information is derived from Item No. 73019, Appendix A:

(i) from Equation (A1.7) of Item No. 73019,

$$V_{md} = \frac{3\,600}{r} \left[\frac{2W}{\rho S} \right]^{1/2} \left[\frac{K_2}{K_1} \right]^{1/4} = \frac{3\,600}{6\,076.12} \left[\frac{2}{3500} \right]^{1/2} \left[\frac{0.05}{0.018} \right]^{1/4} \left[\frac{W}{\rho} \right]^{1/2} = 0.018\,28 \left[\frac{W}{\rho} \right]^{1/2} \text{ kn,}$$

(ii) from Equation (A1.6) of Item No. 73019,

$$\left[\frac{L}{D} \right]_{max} = \frac{1}{2} \left[\frac{1}{K_1 K_2} \right]^{1/2} = \frac{1}{2} \left[\frac{1}{0.018 \times 0.05} \right]^{1/2} = 16.67,$$

(iii) from Equation (A1.8) of Item No. 73019,

$$\frac{L}{D} = \frac{(L/D)_{max}}{\frac{1}{2} \left[y^2 + \frac{1}{y^2} \right]} = \frac{33.33}{y^2 + \frac{1}{y^2}}.$$

Values of cruise air range calculated as in Table 4.1 are plotted in Figure 1 as lines of constant fuel weight and constant pressure height.

TABLE 4.1 Cruise Air Range at $M = 0.82$, $H_p = 30\,000$ ft for Given Fuel Weights
 $[\rho = 0.000\,889\,3 \text{ slug/ft}^3, V_{cr} = 483.2 \text{ kn}, c = 0.685 \text{ lbf/lbf h}]$

$W_{F,stage}$ lbf	W_i lbf	$V_{md,i}$ kn	y_i	$\left[\frac{L}{D} \right]_i$	$\left[\frac{V_{cr} L}{c D} \right]_i$ n.mile	$\frac{W_F}{W_i}$	$^* \log_e \left[\frac{1}{1 - \frac{W_F}{W_i}} \right]$	$R_{ref} =$ $\left[\frac{V_{cr} L}{c D} \right]_i \log_e \left[\frac{1}{1 - \frac{W_F}{W_i}} \right]$ n.mile	$\frac{R_a}{R_{ref}}$ from Item No. 73019, Figure 6	R_a n.mile
0	343 000	359.1	1.346	14.10	9 950					
10 000	353 000	364.3	1.327	14.32	10 100	0.028 33	0.028 74	290.3	0.993 0	288.2
20 000	363 000	369.4	1.308	14.52	10 240	0.055 10	0.056 67	580.5	0.985 5	572.1
30 000	373 000	374.4	1.291	14.71	10 380	0.080 43	0.083 80	870.1	0.980 5	853.2
40 000	383 000	379.4	1.274	14.89	10 500	0.104 4	0.110 3	1 159	0.975 0	1 130
50 000	393 000	384.3	1.257	15.06	10 620	0.127 2	0.136 1	1 445	0.970 0	1 402
60 000	403 000	389.2	1.242	15.22	10 740	0.148 9	0.161 2	1 731	0.965 5	1 671
70 000	413 000	394.0	1.227	15.37	10 840	0.169 5	0.185 7	2 013	0.961 5	1 936
80 000	423 000	398.7	1.212	15.51	10 940	0.189 1	0.209 6	2 293	0.958 0	2 197
90 000	433 000	403.4	1.198	15.64	11 030	0.207 9	0.233 0	2 570	0.954 0	2 452

* This quantity could have been read from Figure 2 of Item No. 73019.