

FLEXIBILITIES OF AND STRESSES IN UNRESTRAINED PRESSURISED THIN PIPE BENDS SUBJECTED TO IN-PLANE BENDING

1. NOTATION

E	modulus of elasticity of pipe material	N/m ²	lbf/in ²
I	second moment of area of pipe ($\pi r^3 t$ for $r/t \geq 10$)	m ⁴	in ⁴
K	bend flexibility factor defined by $K = \alpha/\beta$ or $K = \alpha EI/MR\psi$ (see Section 2)		
M	applied bending moment in plane of pipe bend	N m	lbf in
p	internal pressure in pipe	N/m ²	lbf/in ²
Q	bend stress factor defined by $Q = 2q_b I/Mr$ (see Section 2)		
q	maximum shear stress due to moment M and pressure p , $q = q_b + q_p$	N/m ²	lbf/in ²
q_b	maximum shear stress in pressurised pipe bend due to moment M	N/m ²	lbf/in ²
q_p	maximum shear stress in pipe bend due to pressure p , given by $pr/2t$	N/m ²	lbf/in ²
R	radius of curvature of unloaded pipe bend axis	m	in
r	mid-wall radius of pipe	m	in
t	pipe wall thickness	m	in
α	change in angle of pipe bend under moment M	rad	rad
β	change in angle of equivalent straight pipe (see Section 2) under moment M	rad	rad
λ	pipe factor defined by $\lambda = tR/r^2$		
ψ	angle of unloaded pipe bend	rad	rad

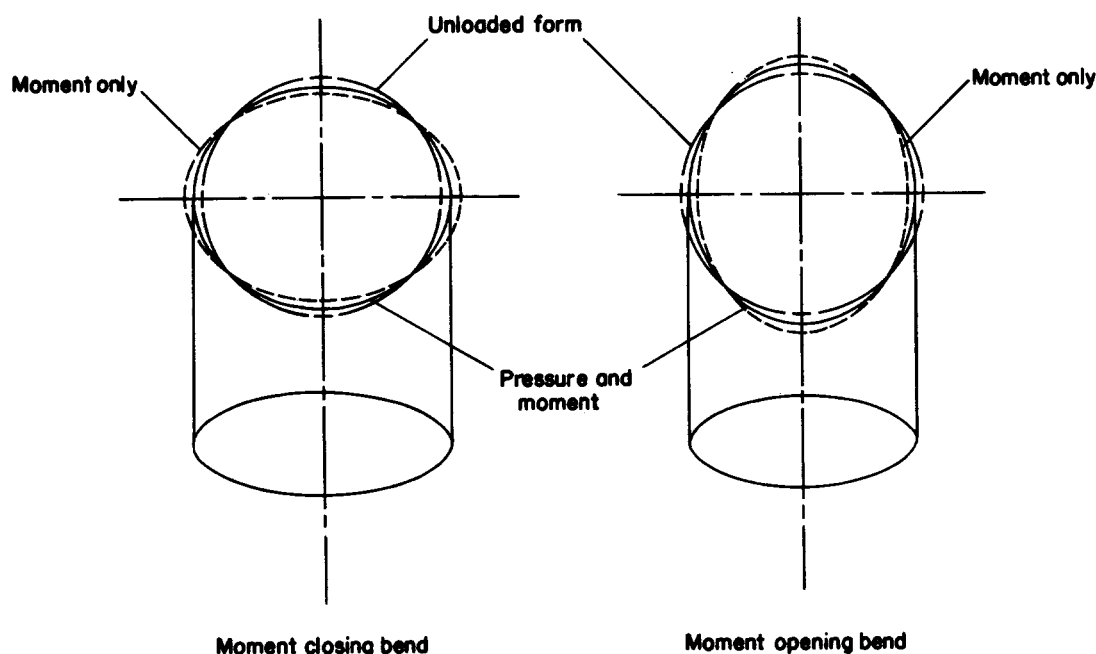
Both SI and British Units are quoted but any coherent system of units may be used.

2. NOTES

Curves are given that enable the flexibilities and maximum elastic shear stresses to be obtained for smooth, unrestrained, pressurised, circular pipe bends subjected to a pure moment applied in the plane of the pipe bend. When a pipe bend is subjected to a bending moment the distribution of the stresses due to the curvature tends to distort the shape of the cross section producing a greater flexibility than that of an equivalent straight pipe. Where there is an internal pressure in the pipe bend this reduces or resists this distortion as

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shown in Sketch 2.1.



Sketch 2.1 Distortion of pipe bend cross section under moment (not to scale)

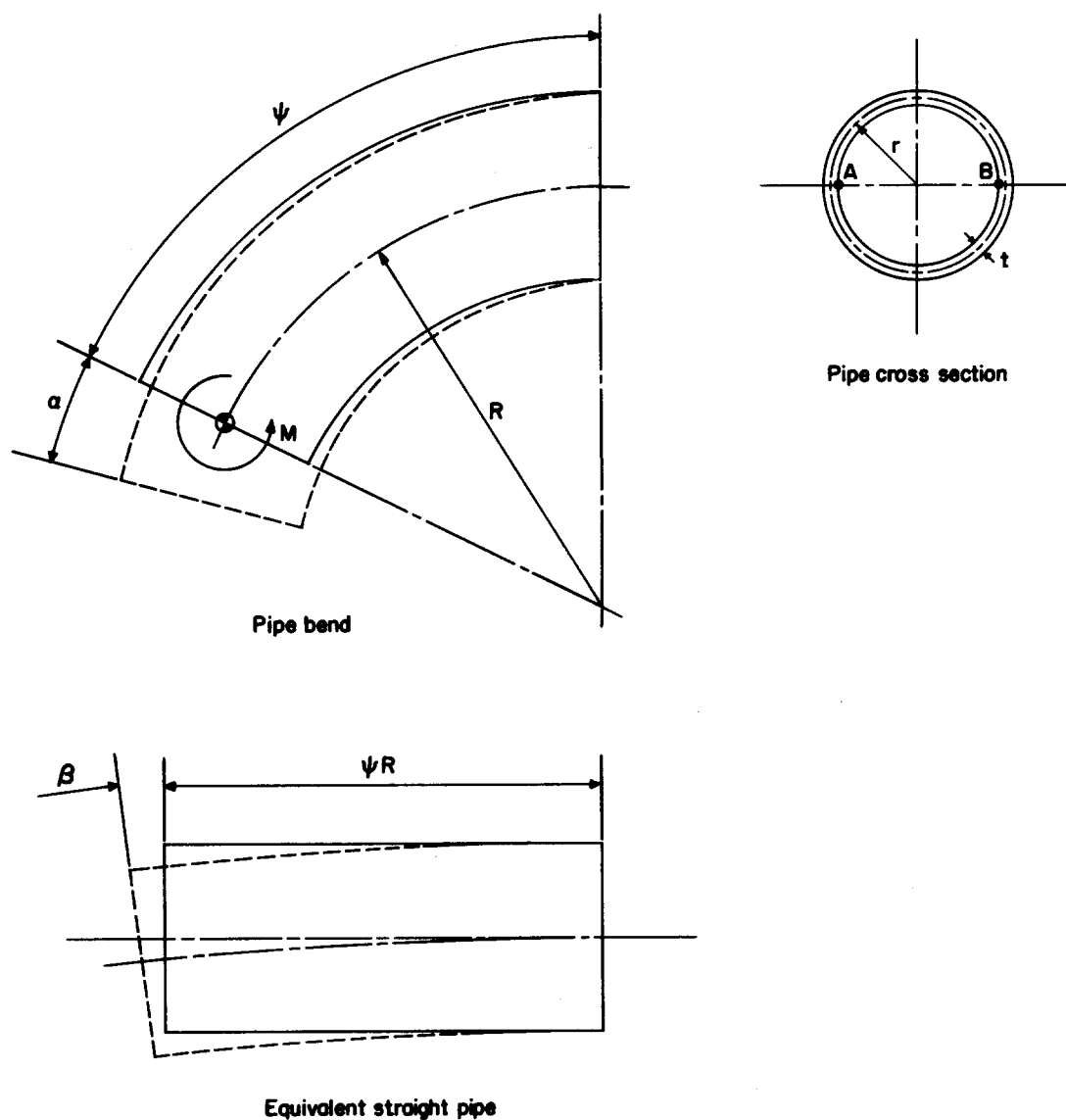
In Figure 1 the flexibilities of pipe bends under in-plane moments are presented in terms of K plotted against λ for various values of the pressure parameter pR^2/Ert . The bend flexibility factor K is the ratio of the flexibility of the bend to the flexibility of an equivalent straight pipe, which is a straight pipe of the same cross section and material properties under the same loading conditions having a length equal to ψR (see Sketch 2.2).

In Figures 2 and 3 the maximum shear stresses due to the applied moment, q_b , for such pipe bends are presented in terms of the stress factor Q plotted against λ for various values of pR^2/Ert . The stress factor Q is the ratio of twice the maximum shear stress q_b (due to moment M) in the pressurised bend to the maximum bending stress of an equivalent straight pipe (as previously defined). There is a further slight variation in the value of Q with R/r particularly when $R/r < 10$. Figures 2 and 3 give values of Q for $R/r = 2$ and $R/r \geq 10$, respectively. The stress q_b due to the applied moment M , obtained from Q , may be added to the stress q_p due to pressure alone to provide the maximum shear stress q . According to the Tresca criterion yield will occur when $2q$ equals the uniaxial yield stress (see Item Nos 73004 or Struct 02.00.00).

For most common configurations of pressurised thin pipe bends the maximum stresses are closely associated with the circumferential bending stresses which are a maximum at, or close to, the points A and B on the sides of the pipe as shown on Sketch 2.2. In these circumstances as the radial stress is considered negligible (see next paragraph) the maximum circumferential bending stress is equal to $2q_b$. For cases associated with a combination of high values of R/r , r/t or p the influence of the longitudinal bending stresses increases and the maximum shear stress due to bending is no longer closely associated with the maximum circumferential bending stress and will move from points A and B. In such cases the use of the term q_p as given will overestimate the true stress due to pressure which in extreme cases may be reduced to $pr/4t$.

The curves are valid for thin pipes with $r/t \geq 10$ and have been calculated for a value of Poisson's ratio of 0.28. It is assumed that the pipe bend is smooth with no irregularities and that it is free from restraint or discontinuity effects at the ends of the bends. The theoretical basis of the curves assumes that the moment

is a pure moment (opening or closing the bend) applied in the plane of the bend as shown in Sketch 2.2 and that during bending plane sections normal to the plane of bending remain plane. The radial stress through the thickness of the pipe due to the diffusion of the pressure into the pipe wall is assumed to be negligible. The Item is applicable to pipe bends connected to long straight pipes (say length $\geq 3 \times$ diameter) where pure bending unaffected by end restraints may be assumed. Where adjacent straight sections of the pipe are comparatively short or where flanges or other pipe bends are adjacent the effect of these restraints on the behaviour of the pipe bend should be considered.



Sketch 2.2 (Not to scale)

It should be noted that under the action of M pipe bends may buckle at stresses below the yield stress, particularly for bends with small values of λ . The Item does not apply to pipe bends subjected to external pressure.