DESIGN FOR MAXIMUM STRUCTURAL EFFICIENCY. ISOTROPIC SANDWICH PANELS UNDER UNIAXIAL COMPRESSION

1. NOTATION

b	width of panel	m	in
E	modulus of elasticity of face plate material	N/m^2	lbf/in^2
E'	$E/(1-\nu^2)$	N/m^2	lbf/in^2
E_c	effective compressive modulus of elasticity of core normal to face plates	N/m^2	lbf/in^2
E_t	tangent modulus of face plate material	N/m^2	lbf/in^2
f	$P/2t$, stress in face plates	N/m^2	lbf/in^2
f_n	stress at which $E_t = ½E$	N/m^2	lbf/in^2
G_c	effective shear modulus of core normal to face plates	N/m^2	lbf/in^2
h	distance between centre planes of face plates	m	in
m	material characteristic for face plate material (see Item No. 76016)		
P	compressive load per unit width applied to panel	N/m	lbf/in
P_b	compressive load per unit width required to cause overall buckling	N/m	lbf/in
P_w	compressive load per unit width required to cause face plate wrinkling	N/m	lbf/in
t	thickness of each face plate	m	in
w	$2t\rho + h\rho_c$, mass per unit area of panel	kg/m^2	lb/in^2
η	plasticity correction factor (see Section 2.2)		
μ	ρ_c/ρ		
ν	Poisson's ratio for face plate material		
ρ	density of face plate material	kg/m^3	lb/in^3
ρ_c	effective density of core	kg/m^3	lb/in^3

Both SI and British units are quoted but any coherent system of units may be used.

Documents
Document Sharing Hub
Full document details: ESDU 75006 1976

2. NOTES

This Item provides charts and calculation procedures for the selection of the thicknesses of the core and face plates of sandwich panels for maximum structural efficiency. The data relate to long rectangular simply-supported sandwich panels loaded in uniaxial in-plane compression on either the long or the short sides; the face plates are isotropic and homogeneous and the core is effectively isotropic in the plane of the sandwich.

Efficient design of sandwich panels in compression is always governed by overall buckling. Face plate wrinkling and constraints upon the face plate stress or the core or face plate thicknesses may also influence the design. This Item presents the data so that the panel of maximum structural efficiency, which is the same as the panel of least weight, may be located within such constraints.

The following two sub-sections describe the use of the charts and calculation procedures provided for locating the design of maximum structural efficiency. The charts are constructed on the assumption that the face plate stress is below the limit of proportionality of the material. It may be advantageous to allow the face plate stress to rise above the limit of proportionality in which case the calculation procedure provided in Section 2.2 may be used.

2.1 Design Below the Limit of Proportionality of the Face Plate Material

Two sets of charts are provided. Figures 1 to 6 are for panels loaded on the short sides and Figures 7 to 12 are for panels loaded on the long sides. Each chart relates to a specific value of $\mu E/G_c$ and is plotted on an abscissa of $w/(\rho_c b)$. The red curves of $P_b/(\mu E b)$ relate to the condition of overall buckling and the blue curves relate to face plate wrinkling, the intersection of the blue curves with the red curves indicating the minimum value of $\mu E_c/E$ required to achieve $P_w = P_b$. For given materials and panel width and with P_b set equal to P the values of $P_b/(\mu E b)$, $\mu E/G_c$ and $\mu E_c/E$ are known. As shown in Sketch 2.1 the region of feasible design is on, or to the right of, the appropriate red curve to ensure that $P_b \geq P$ and at, or below, its intersection with the appropriate blue curve to ensure that $P_w \geq P_b$. In addition lines of constant f/E are provided on the charts. To ensure that the limit of proportionality of the face plate material is not exceeded the appropriate f/E line should therefore be treated as a further boundary. From the abscissa it can be seen that the panel of maximum structural efficiency occurs as far to the left on the appropriate red $P_b/(\mu E b)$ line as the appropriate $\mu E_c/E$ and/or f/E lines allow. The chain dotted lines on the figures and on Sketch 2.1 are the loci of vertical tangency of the $P_b/(\mu E b)$ lines and therefore indicate the maximum structural efficiency with respect to overall buckling. Although the $P_b/(\mu E b)$ lines extend beyond the chain dotted line such extensions are associated with lower structural efficiencies (higher weights) and increased stresses. For the point of minimum weight f/E may be read from the curves provided and the values of t, h and w obtained from the scales on the top, left-hand and bottom of the chart respectively. Limitations on t and/or h may be introduced and treated as additional bounds.

2.2 Design Above the Limit of Proportionality of the Face Plate Material

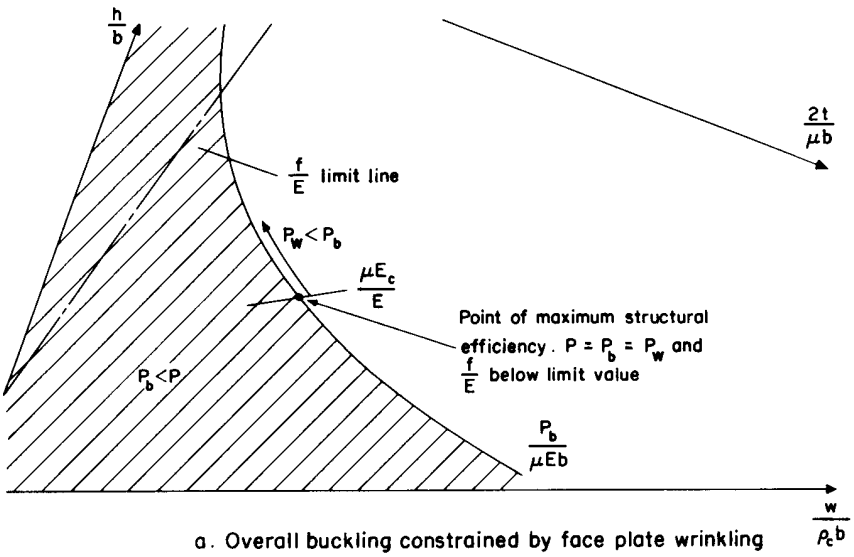
This section describes a simple and rapid calculation procedure for cases where the stress in the face plates is allowed to exceed the limit of proportionality of the face plate material. A range of decreasing values of η should be investigated. Values of η may be taken to be equal to E_t/E and Item No. 76016 "Generalisation of smooth continuous stress-strain curves for metallic materials" can be used to determine the associated values of f/E. Putting $P_b = P$ in the appropriate equations from Table 2.1, the values of h/b, $2t/(\mu b)$ and hence $w/(\rho_c b)$ may be calculated and the minimum value of the latter determined either numerically or graphically. To check for face plate wrinkling P_w may be calculated, using the appropriate value of E_c in the equation provided in Table 2.1. If $P_w < P$ then the stress must be progressively lowered until the equality for E_c/E, given in Table 2.1, is satisfied. In both cases ν may be taken to be constant at 0.3.

Documents
Document Sharing Hub
Full document details: ESDU 75006 1976

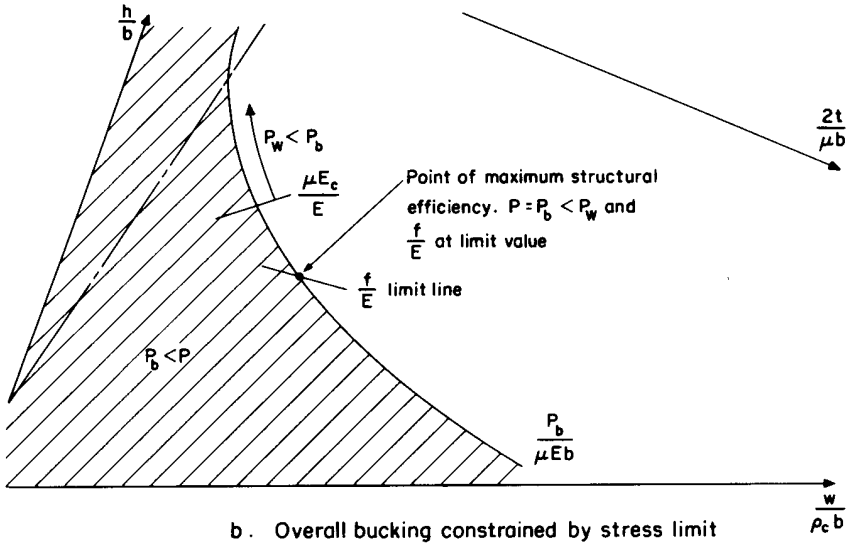
TABLE 2.1

Load on the long sides $$\frac{h}{b} = \left[\left(\frac{P_b}{2G_c b}\right)^2 + \frac{4f}{\pi^2 \eta E'}\right]^{1/2} + \left(\frac{P_b}{2G_c b}\right)$$
Load on the short sides $$\frac{h}{b} = \frac{1 + \frac{4}{\pi}\left(\frac{G_c b}{P_b}\right)\left(\frac{f}{\eta E'}\right)^{1/2}}{4\left(\frac{G_c b}{P_b}\right) - \frac{\pi^2}{4}\left(\frac{\eta E'}{f}\right)\left(\frac{P_b}{G_c b}\right)}$$
Both cases $$\frac{2t}{\mu b} = \frac{P_b}{\mu f b} \qquad \frac{w}{\rho_c b} = \frac{2t}{\mu b} + \frac{h}{b}$$ $$P_w = \left(\frac{8\eta E' E_c t^3}{3h}\right)^{1/2}$$ or $$\frac{E_c}{E} \geq 3\left(\frac{f}{E}\right)^3 \left(\frac{(1-\nu^2)Eb}{\eta P_b}\right)\left(\frac{h}{b}\right)$$

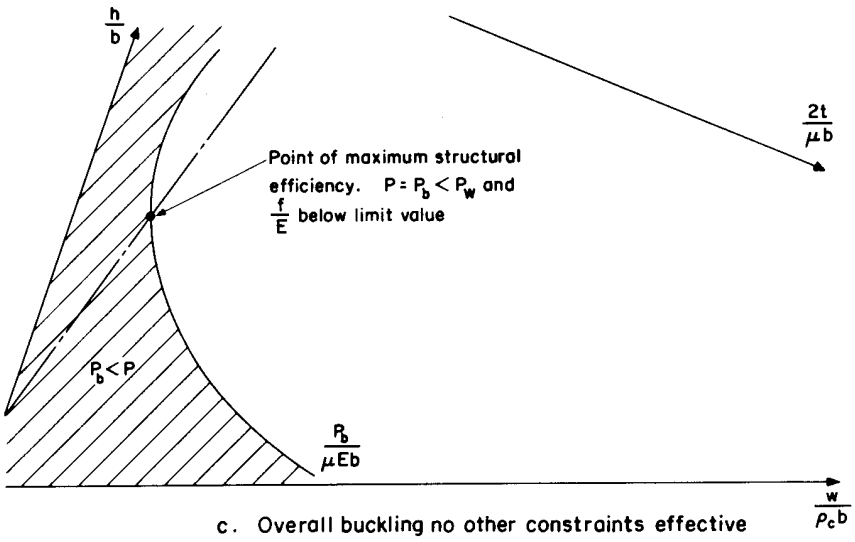
Documents
Document Sharing Hub
Full document details: ESDU 75006 1976



a. Overall buckling constrained by face plate wrinkling



b. Overall bucking constrained by stress limit



c. Overall buckling no other constraints effective

Sketch 2.1

Documents
Document Sharing Hub
Full document details: ESDU 75006 1976

3. DESIGN PHILOSOPHY

Design philosophy with regard to achieving maximum structural efficiency is often governed by practical factors which are beyond the scope of this Item. However, in all cases account should be taken of the factors noted in the following sections. The design charts and equations can only be used for known sets of material properties. For most designs the choice of core is open and it is recommended that several cores be investigated so that the most favourable can be determined.

3.1 Effect of Variations from Geometry and Variations in Material Properties

Design for maximum structural efficiency results in a panel in which buckling occurs at the design load. It is therefore sensitive to deviations from the assumed material properties and the prescribed geometry.

3.1.1 Face plate materials

The design of maximum structural efficiency is sensitive to the shape of the material stress-strain curve and attention should therefore be paid to Sections 4 and 5 of Item No. 76016 concerning the accuracy of calculation of f_n and m. Experimental evidence suggests that $\eta = E_t/E$ is a conservative relationship and in consequence its use will provide some small insurance against material variations above the limit of proportionality.

3.1.2 Core materials

The properties of most cores are available from the material manufacturers. For cores manufactured from organic materials both the shear and compressive moduli may be subject to wide variations. For honeycomb cores G_c may be estimated by means of Item No. 67022 "Modulus of rigidity of sandwich panels with hexagonal cell cores". For cores that behave orthotropically in the plane of the sandwich the geometric mean of the moduli may be used in place of G_c. Errors in overall buckling loads arising from this approach may be determined from Item No. 67025 "Buckling loads under combined longitudinal and transverse direct stress of flat sandwich panels". An estimate of the value of E_c for honeycomb type cores is given by the product of the modulus of elasticity for the core material and the area of core material per unit area of the panel.

4. GENERAL LIMITATIONS

It has been assumed that the core offers no resistance to loads in the plane of the panel. Poisson's ratio for the face plate material has been taken to be 0.3 in all cases. The data are valid only if t is sufficiently small in relation to h for the local second moment of area of the face plates to be negligible, that is, $t^3/12 \ll h^2 t/4$.

Since the data relate to long panels their application to finite panels will provide conservative combinations of t and h. The data for panels loaded on the short sides may be used for panels having a length to width ratio as low as 2.6 with an error in P_b of less than 2½ per cent. The error will not exceed 5 per cent provided that the length to width ratio is greater than 1.6. The error in applying the data to finite panels and to panels with orthotropic cores may be estimated from Item No. 67025. The data for panels loaded on the long sides are based upon the wide strut. In this case errors in P_b may be significant if the length to width ratio is less than about 5. As shown by Equation (5.3) of Section 5 values of P_w are unaffected by panel overall size.

Documents
Document Sharing Hub
Full document details: ESDU 75006 1976

5. DERIVATION

1. FLUGGE, W. The optimum problem of the sandwich plate. *J. appl. Mech.*, Vol. 19, No. 1, pp. 104-108, March 1952.

2. HEATH, W.G. Sandwich construction. Part I. The strength of flat sandwich panels. *Aircr. Engng*, pp. 186-191, July 1960.

The following equations were used to produce the charts and equations.

For overall buckling of a long panel loaded on the long sides,

$$P_b = \frac{\pi^2 E'(h^2 t/2)}{b^2 + \dfrac{\pi^2 E'(h^2 t/2)}{G_c h}} . \tag{5.1}$$

For overall buckling of a long panel loaded on the short sides,

$$P_b = \frac{\pi^2 E'(h^2 t/2)}{\dfrac{b^2}{2} + \dfrac{\pi^2 E'(h^2 t/2)}{2 G_c h} + \left[\dfrac{\pi^2 E'(h^2 t/2)}{2 G_c b h}\right]^2} . \tag{5.2}$$

For face plate wrinkling,

$$P_w = \left[\frac{8 E' E_c t^3}{3h}\right]^{1/2} . \tag{5.3}$$

6. EXAMPLES

6.1 Design Below the Limit of Proportionality

It is required to design a long simply-supported sandwich panel 400 mm wide to carry a compressive load on the long sides of 292 kN/m in which the face plate stress does not exceed the limit of proportionality of the face plate material. The properties of the materials to be used are given below.

Material properties:

the core, $\rho_c = 56$ kg/m^3, $G_c = 146$ MN/m^2 and $E_c = 1460$ MN/m^2,

the face plates, $\rho = 2800$ kg/m^3, $E = 73\,000$ MN/m^2, $\nu = 0.3$

and the limit of proportionality is taken as 295 MN/m^2.

Documents
Document Sharing Hub
Full document details: ESDU 75006 1976

From the given data, setting $P_b = P$,

$$\mu = \frac{56}{2800} = 0.02 ,$$

$$\frac{\mu E}{G_c} = \frac{0.02 \times 73\,000 \times 10^6}{146 \times 10^6} = 10 ,$$

$$\frac{\mu E_c}{E} = \frac{0.02 \times 1460 \times 10^6}{73\,000 \times 10^6} = 400 \times 10^{-6}$$

and

$$\frac{P_b}{\mu E b} = \frac{292 \times 10^3}{0.02 \times 73\,000 \times 10^6 \times 0.4} = 500 \times 10^{-6} .$$

To ensure that the limit of proportionality is not exceeded we require

$$\frac{f}{E} \leq \frac{295 \times 10^6}{73\,000 \times 10^6} = 4 \times 10^{-3} .$$

From Figure 8, for $P_b/(\mu E b) = 500 \times 10^{-6}$, the lowest value of $w/(\rho_c b)$ that can be achieved is restricted by the $f/E = 4 \times 10^{-3}$ value, the $\mu E_c/E = 400 \times 10^{-6}$ being further up the $P_b/(\mu E b)$ line. Thus, at the f/E limit the following values are read from Figure 8.

$$\frac{h}{b} = 0.041 \quad \text{giving} \quad h = 16.4 \text{ mm} ,$$

$$\frac{2t}{\mu b} = 0.125 \quad \text{giving} \quad t = 0.5 \text{ mm}$$

and

$$\frac{w}{\rho_c b} = 0.166 \quad \text{giving} \quad w = 3.72 \text{ kg/m}^2 .$$

6.2 Design Above the Limit of Proportionality

The panel of Example 6.1 is to be redesigned with no restriction on the face plate stress. The face plate material properties above the limit of proportionality are described by $f_n = 365$ MN/m^2 and $m = 15$.

Since Example 6.1 shows that increasing f/E over 4×10^{-3} would enable a greater efficiency to be achieved the procedure described in Section 2.2 will be employed. Thus, $P_b/(2G_c b) = 2.5 \times 10^{-3}$ and $P_b/(\mu b) = 36.4 \times 10^6$ N/m^2. Selecting a range of values of η and using Figure 3 of Item No. 76016 to calculate the related value of f/E the equations of Table 2.1 are evaluated and the following table is constructed.

Documents
Document Sharing Hub
Full document details: ESDU 75006 1976