

FLEXIBILITIES OF AND STRESSES IN THIN PIPE BENDS UNDER IN-PLANE BENDING: INFLUENCE OF BEND ANGLE AND TANGENT PIPE ENDS

1. NOTATION

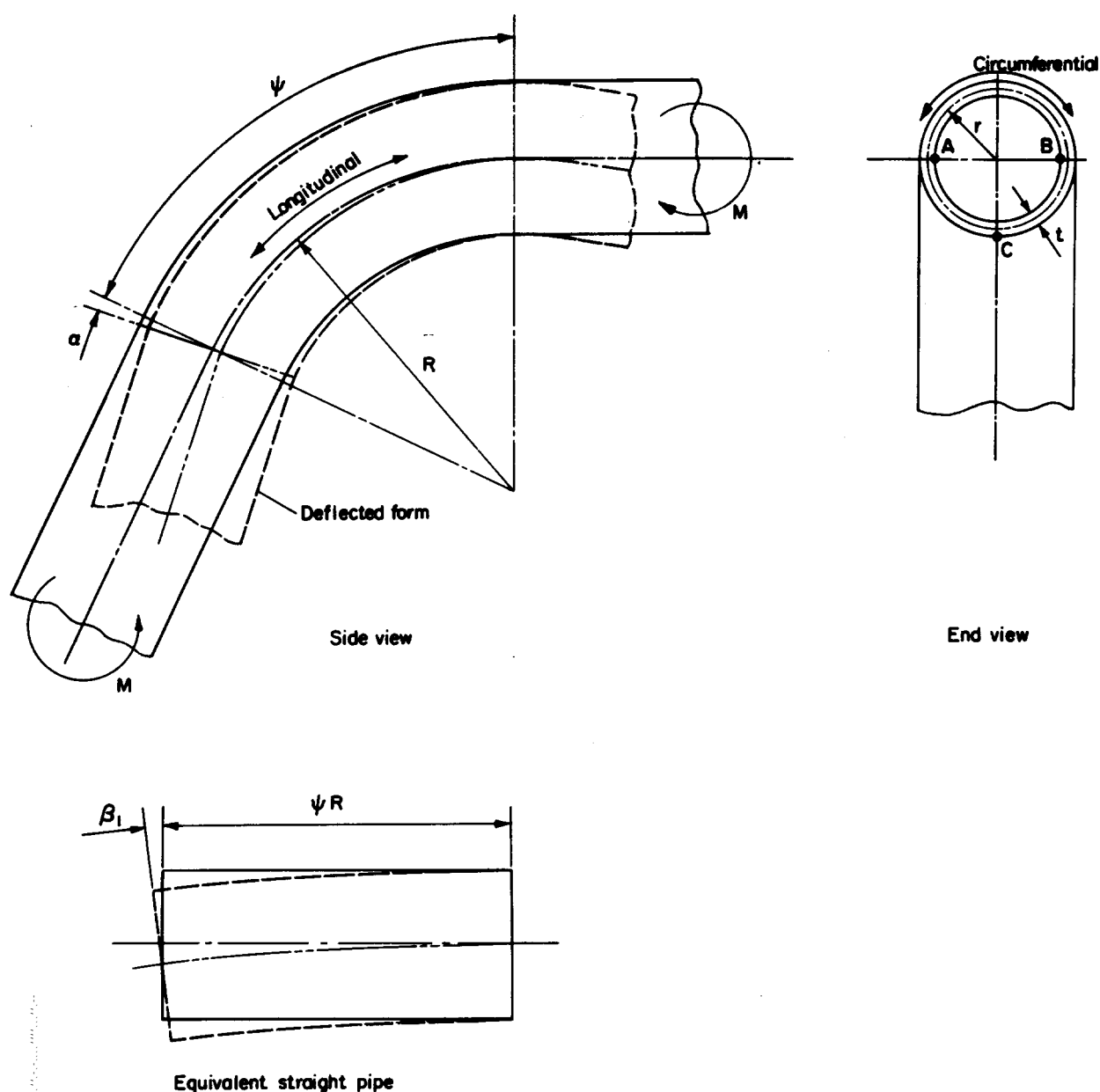
E	modulus of elasticity of pipe material	N/m ²	lbf/in ²
F_c	bend stress factor defined by $F_c = f_c I / Mr$ (see Section 2)		
F_e	bend stress factor defined by $F_e = f_e I / Mr$ (see Section 2)		
f_c	maximum circumferential stress in pipe bend due to moment M	N/m ²	lbf/in ²
f_e	maximum equivalent stress due to moment M (based on von Mises-Hencky criterion, see Section 2)	N/m ²	lbf/in ²
I	second moment of area of pipe ($\pi r^3 t$ for $r/t \geq 10$)	m ⁴	in ⁴
K	bend flexibility factor defined by $K = \alpha / \beta_1$ or $K = \alpha EI / MR \psi$ (see Section 2)		
L	combined length of adjacent straight pipes	m	in
M	applied bending moment in plane of pipe bend	N m	lbf in
R	radius of curvature of bend axis of unloaded pipe bend	m	in
r	mid-wall radius of unloaded pipe	m	in
t	pipe wall thickness	m	in
α	change in angle of pipe bend under moment M (see Sketch 2.1)	rad	rad
β_1	change in angle of equivalent straight pipe under moment M (see Sketch 2.1)	rad	rad
β_2	change in angle for straight pipe of length L under moment M	rad	rad
λ	pipe factor defined by $\lambda = tR/r^2$		
ψ	angle subtended by ends of unloaded pipe bend (see Sketch 2.1)	rad	rad

Both SI and British Units are quoted but any coherent system of units may be used.

2. NOTES

Curves are given that enable the flexibilities, the maximum von Mises-Hencky equivalent stresses and the maximum circumferential stresses to be obtained for thin smooth circular, unpressurised pipe bends subjected to pure bending moments in the plane of the bend. Data are provided for any angle of bend and relate to bends with long straight pipes attached tangentially to both ends.

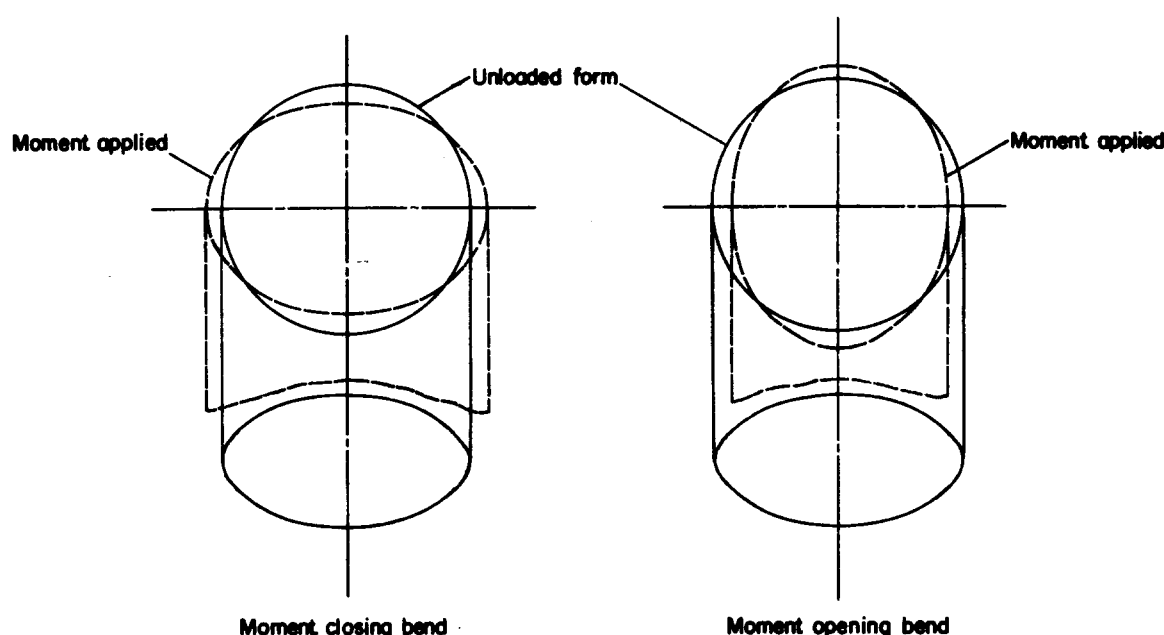
Figure 1 gives the flexibilities for the pipe bends in terms of the bend flexibility factor K plotted against the bend angle for various values of the pipe factor. The bend flexibility factor is the ratio of the flexibility of the bend to the flexibility of an equivalent straight pipe, which is a straight pipe of the same cross section and material properties under the same loading condition having a length equal to ψR , see Sketch 2.1. The factor K applies to the bend only but takes account of the interaction of the bend with the attached straight pipes and their influences on each other. The straight pipes should be treated along their entire length as ordinary straight pipes in any overall analysis as shown in the example.



Sketch 2.1 (not to scale)

In Figures 2 and 3 the stress factors F_e and F_c , respectively, are given on a similar basis to the flexibility factors. The stress factors are ratios of the maximum von Mises-Hencky equivalent stress and the maximum circumferential stress to the maximum (longitudinal) bending stress in an equivalent straight pipe (as previously defined).

For thin pipes with a bend angle greater than about 30° f_e is in the neighbourhood of the maximum circumferential stress, f_c , which occurs close to the points A and B on the sides of the pipe as shown in Sketch 2.1. The stress f_c is predominantly a through-wall bending stress arising from the distortion of the pipe cross section which relieves the longitudinal stresses directly induced by the applied moment (see Sketch 2.2). For pipes with bend angles of less than about 30° f_e is more closely associated with the maximum longitudinal stress, the then dominant stress, which occurs close to point C on the intrados of the pipe bend, as shown in Sketch 2.1. According to the von Mises-Hencky criterion yield will occur when f_e equals the uniaxial yield stress (see Item Nos. 73004 or Struct 02.00.00).



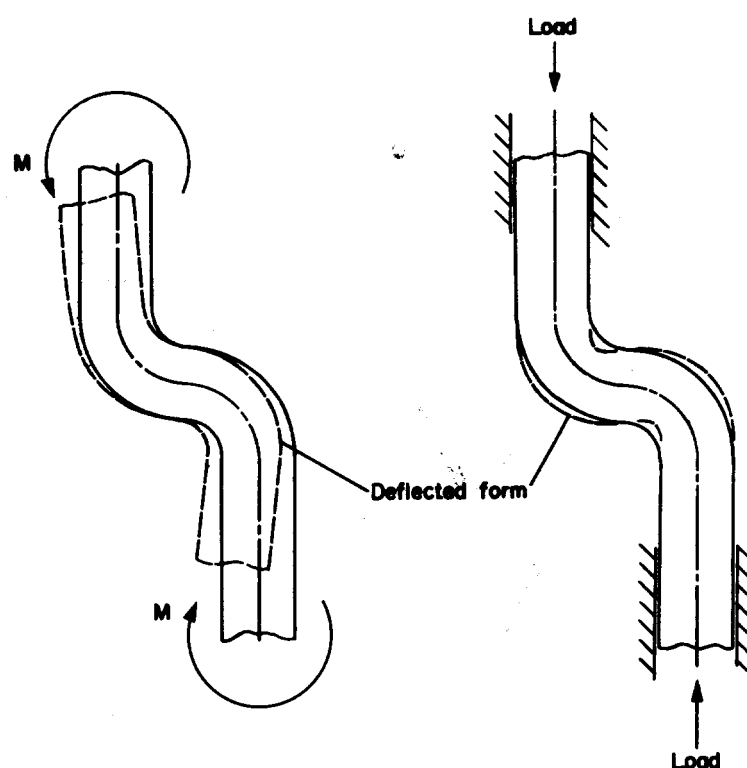
Sketch 2.2 Distortion of pipe bend cross section under moment. (not to scale)

The data provided relate strictly to pipe bends in which $r/t \geq 10$ and $R/r = 2$ and have been calculated for a value of Poisson's ratio of 0.30. The flexibility factors for bends with $r/t \geq 10$ and $R/r > 2$ are sensibly independent of R/r so that values taken from Figure 1 may be applied provided $R/r \geq 2$. The stress factors of Figures 2 and 3 are slightly reduced for values of $R/r > 2$. They may therefore be treated as upper limits and the degree to which they overestimate the stresses for $R/r > 2$ may be judged by reference to Item No. 74043 "Flexibilities of and stresses in unrestrained pressurised thin pipe bends subjected to in-plane bending".

It is assumed that the bending moment, which may either tend to close or to open the bend, is applied in the plane of the bend to the tangent-bend-tangent combination and is constant throughout the entire length of the composite unit as shown in Sketch 2.1. It is also assumed that the pipe is smooth with no initial irregularities, that the bend is attached at each end to straight pipes of the same cross section and material properties and that radial stresses normal to the middle surface of the pipe wall are negligibly small.

The data on which the curves are based have been obtained from the general three-dimensional thin shell finite element computer program of Derivations 6 and 7 and are applicable to pipe bends connected to long straight pipes each of length greater than about $6r$. The analysis used assumes linear elastic relationships hold and does not consider elastic instability or plastic collapse. For certain materials with low values of E and extreme geometries high local deformations may occur at elastic stresses and consideration should be given to the possibility of elastic instability. In such cases it would be inadvisable to work to high elastic stresses. Limited guidance on plastic collapse of pipe bends (unrestrained) which should occur at higher values of M than those considered here may be obtained in References 11 and 12.

Where the adjacent straight sections of the pipe are short (length less than $6r$) or where flanges or other bends are adjacent, the behaviour of the pipe bend is modified. The effect of using adjacent straight pipes less than $6r$ long between the bend and some other part of the piping system will be to reduce both the flexibility of the bend and the stresses below the values given on the figures. Similarly the introduction of flanges closer than about $6r$ to the bend will also reduce the flexibility and the stresses. The amount of the reduction will, to a certain extent, depend on the distance from the bend and the stiffness of the flanges. The effect becomes more marked as the bend angle becomes smaller (ψ less than about 45°). For such bends the longitudinal stress in the vicinity of the flange can exceed the circumferential stress in the bend.



Sketch 2.3 Effects of moment and point loading on S-bend pipes (not to scale)

Where bends of opposite directions are attached to each other (S bends) and are subjected to a constant moment along their length, the effect is to close one bend and open the other; this reduces the flexibility factors of the two segments considered individually. For example with two adjacent opposed 90° bends K for the whole assembly will be about 15 per cent less than K for one of the 90° bends and the reduction in F_e or F_c will be greater than 10 per cent when $0.1 < \lambda < 0.2$. However, when the moment on an S bend results from point or end loading the effects will be different as the individual segments may act in conjunction (see Sketch 2.3). Two bends in the same direction separated by a long straight pipe (length $> 6r$) should be treated as separate bends. When the straight pipe between such bends is short the overall flexibility of the system should approach that for a bend of the combined angle. Derivations 1,