

ESTIMATION OF CRUISE RANGE: PROPELLER-DRIVEN AIRCRAFT

1. NOTATION AND UNITS*

		British	SI
C_D	drag coefficient, $D / \frac{\rho}{2} \left(\frac{rV}{3600} \right)^2 S$		
C_L	lift coefficient, $L / \frac{\rho}{2} \left(\frac{rV}{3600} \right)^2 S$		
c	engine specific fuel consumption, Q/F $\begin{cases} \text{weight flow basis} \\ \text{mass flow basis} \end{cases}$	lbf/lbf h slug/lbf h	N/N h kg/N h
c'	engine specific fuel consumption $Q/P, Q/P_e$ $\begin{cases} \text{weight flow basis} \\ \text{mass flow basis} \end{cases}$	lbf/hp h slug/hp h	N/kW h kg/kW h
D	drag	lbf	N
F	total thrust, $(F_P + F_J)$	lbf	N
F_J	thrust component due to exhaust jet	lbf	N
F_P	thrust component due to propeller	lbf	N
g	acceleration due to gravity	ft/s ²	m/s ²
H_p	pressure height	ft	m
K_1, K_2	coefficients in drag equation (Equation (3.4))		
L	lift	lbf	N
m	aircraft mass	slug	kg
m_F	mass of fuel	slug	kg
P	total engine output-shaft power	hp	kW
P_e	engine equivalent output-shaft power, $\left(P + \frac{F_J}{550\eta} \left(\frac{rV}{3600} \right) \right) \text{hp}, P + \frac{F_J}{1000\eta} \left(\frac{rV}{3600} \right) \text{kW}$	hp	kW
Q	total engine fuel consumption $\begin{cases} \text{weight flow basis} \\ \text{mass flow basis} \end{cases}$	lbf/h slug/h	N/h kg/h

* It is customary British practice to work in terms of aircraft weight and fuel weight flow; however, when using SI units aircraft mass and fuel mass flow are used. These practices are followed here and, where numerical differences exist, Equations are presented in both British and SI units.

The SI units have not necessarily received universal acceptance for the particular applications, *i.e.* other multiples and sub-multiples may be used.

R_a	air range	n.mile	km, n.mile
R_{ref}	reference air range (see Equations (4.1) to (4.3))	n.mile	km, n.mile
r	conversion factor (6 076.12 ft/n.mile, 1 000 m/km, 1 852 m/n.mile)	ft/n.mile	m/km, m/n.mile
S	wing area	ft ²	m ²
T	ambient temperature	K	K
t	time, endurance	h	h
V	true airspeed	kn	km/h, kn
W	aircraft weight, mg	lbf	N
W_F	weight of fuel, $m_F g$	lbf	N
y	ratio of airspeed to minimum drag speed, V/V_{md}		
γ_c	climb angle	degree	degree
ε	inclination of total thrust vector to flight path	degree	degree
η	propeller efficiency $\left(\frac{F}{550P} \frac{rV}{3\,600} \right.$ in British units, $\left. \frac{F}{1\,000P} \frac{rV}{3\,600} \right.$ in SI units)		
θ	relative temperature, T/T_0		
ρ	density of air	slug/ft ³	kg/m ³
σ	relative density, ρ/ρ_0		
Suffixes			
f	final condition		
i	initial condition		
max	maximum		
$mean$	mean value during cruise		
mp	minimum thrust-power condition in 1 g level flight		
md	minimum drag condition in 1 g level flight		
0	denotes value at sea level in International Standard Atmosphere (Item Nos 68046 and 72018 give numerical values)		

2. INTRODUCTION

This Item gives methods for estimating the cruise air range of propeller-driven aircraft. It replaces those parts of a previous Item that were not superseded by Item Nos 73018 and 73019 which consider the range performance of turbo-jet and turbo-fan engined aircraft. To avoid repetition of material the user is referred to Item No. 73018 for a general introduction to the subject of range estimation. Note, however, that *important differences* exist between the methods used for jet and propeller-driven aircraft, for example in the customarily used definitions of engine specific fuel consumption and in the need to include the propeller efficiency.

The expressions given for cruise air range are approximate and based on the same simplifying assumptions as those listed in Item No. 73019 Sections 2.1 and 2.3 (excepting the case of fuel flow – see Section 2.1 of this Item). Note that the appropriate Figures of Item No. 73019 may be used to evaluate the expressions for range.

It is intended to issue an addendum to Item No. 74018 to extend to propeller-driven aircraft the method of that Item for estimating lost range due to climb and descent.

2.1 Definition of Engine Specific Fuel Consumption

For turbo-jet and turbo-fan engines it is convenient to define the specific fuel consumption, c , as the fuel flow per unit thrust since, at a given cruise height and engine setting, the thrust and fuel flow do not vary rapidly with forward speed. Consequently, when calculating the cruise range of aircraft fitted with such engines it is permissible to assume that, to a first approximation, c is constant.

For piston engines the shaft power produced at a given height and a fixed setting of the engine controls is approximately constant so that the propeller thrust varies inversely with forward speed. In this case it is therefore more appropriate to define the specific fuel consumption, c' , as the fuel flow per unit engine output-shaft power, *i.e.*

$$c' = Q/P \quad (2.1)$$

since, to a first approximation, this may be assumed constant in a calculation of cruise range.

In the case of the turbo-prop engine, the use of a specific fuel consumption, c' , based on engine equivalent output-shaft power, P_e , is less satisfactory because at a given engine setting the power developed tends to increase with forward speed while the fuel flow remains essentially constant. Consequently, the value of c' tends to decrease with increased forward speed. Nevertheless, the variation of c' with forward speed is not as rapid as would be the case for the corresponding specific fuel consumption defined on a thrust basis.

On balance, for propeller-driven aircraft, it is usually more convenient to define specific fuel consumption, c' , on the basis of the engine output-shaft power (or equivalent power) and to assume that c' remains constant during cruise. This is the fuel flow law used in the range-estimation methods given in this Item.

Further information regarding engine fuel consumption is given in Appendix A. The Appendix describes the forms in which engine data are usually available and indicates methods that can be used to obtain an analytical representation of specific fuel consumption for cases where the assumption of a constant value is considered inadequate. For the case where the specific fuel consumption is assumed to vary with forward speed, the method given is similar to that developed in Item No. 73019.

2.2 Representation of Fuel Flow for Propeller-driven Aircraft

From Equation (2.1) and following the format of Item No. 73018, Section 3.2, the fuel consumption of propeller-driven aircraft may be expressed:

(i) in British Units, with c' in lbf/hp h, as

$$Q = \frac{c'}{550\eta} \frac{rV}{3600} W \left[\frac{\frac{D}{L} \cos \gamma_c + \frac{1}{g} \frac{dV}{dt} + \sin \gamma_c}{\cos \epsilon + \frac{D}{L} \sin \epsilon} \right] \text{ lbf/h}, \quad (2.2)$$

(ii) in SI Units, with c' in kg/kW h, as

$$Q = \frac{c'}{1000\eta} \frac{rV}{3600} mg \left[\frac{\frac{D}{L} \cos \gamma_c + \frac{1}{g} \frac{dV}{dt} + \sin \gamma_c}{\cos \epsilon + \frac{D}{L} \sin \epsilon} \right] \text{ kg/h}. \quad (2.3)$$

In Equation (2.2) the factor,

$$\frac{550 \times 3600}{r} = 325.87 \text{ kn lbf/hp};$$

in Equation (2.3),

$$\frac{1000 \times 3600}{r} = 3600 \text{ N km/kW h if } V \text{ is in km/h}$$

or $1943.84 \text{ kn N/kW if } V \text{ is in kn.}$

3. SPECIFIC AIR RANGE AND ENDURANCE

If the assumptions listed in Section 2.1 of Item No. 73019 are applied to Equation (4.1) of Item No. 73018 and Equations (2.2) and (2.3) of this Item then the specific air range is, in British units,

$$-\frac{dR_a}{dW} = \frac{V}{Q} = \frac{V}{c'P} = \frac{325.87 \eta L}{W c' D} \text{ n.mile/lbf} \quad (3.1)$$

and in SI units,

$$-\frac{dR_a}{dm} = \frac{3600 \eta L}{mg c' D} \text{ km/kg} \quad (3.2)$$

and

$$-\frac{dR_a}{dm} = \frac{1943.84 \eta L}{mg c' D} \text{ n.mile/kg}. \quad (3.3)$$

From Equations (3.1) to (3.3) the condition for maximum specific air range at any given value of W or m is that $(\eta/c')(L/D)$ be a maximum. If, for cruising flight, it is permissible to assume constant values for η and c' then the condition for maximum specific air range is that $L/D = (L/D)_{\max}$. For an aircraft for which

the relationship between the cruise lift and drag coefficients is

$$C_D = K_1 + K_2 C_L^2, \quad * (3.4)$$

the corresponding speed for maximum specific air range is such that $y = (V/V_{md}) = 1$ (see Appendix A of Item No. 73019). However, in practice, the values of η and c' are not constant and the actual value of y for maximum specific air range is usually greater than unity. For calculation purposes values of η may be obtained from sources such as those given in Section 5.2; values of c' must be obtained from the engine manufacturer – see Appendix A.

Note that in an operational context, it is usual to schedule a “long range” cruise condition at the higher of the two speeds corresponding to $0.99(V/Q)_{max}$ since this gives a significant increase in speed (typically 5 to 10 kn) for a negligible loss of range.

The specific endurance, $-dt/dW$ or $-dt/dm$, is proportional to $1/Q$ (the constants of proportionality being the same as in Equations (3.1) to (3.3)). Thus the condition for maximum specific endurance is that the fuel flow, and hence the engine output-shaft power, be minimised. If it is assumed that η and c' are constant, the condition for maximum specific endurance is that DV be a minimum and, provided C_L and C_D are related as in Equation (3.4), the corresponding minimum power speed[†], V_{mp} , obtained such that

$$\frac{V_{mp}}{V_{md}} = \frac{1}{4\sqrt{3}} = \frac{1}{1.316} = 0.7589 \quad (3.5)$$

(see Item No. 73019, Appendix A).

In some instances the minimum-power flight condition has another significance since for a given engine-propeller-airframe configuration it is the speed for minimum external noise generation (see Reference 19).

3.1 Practical Limitations on Speeds for Range and Endurance

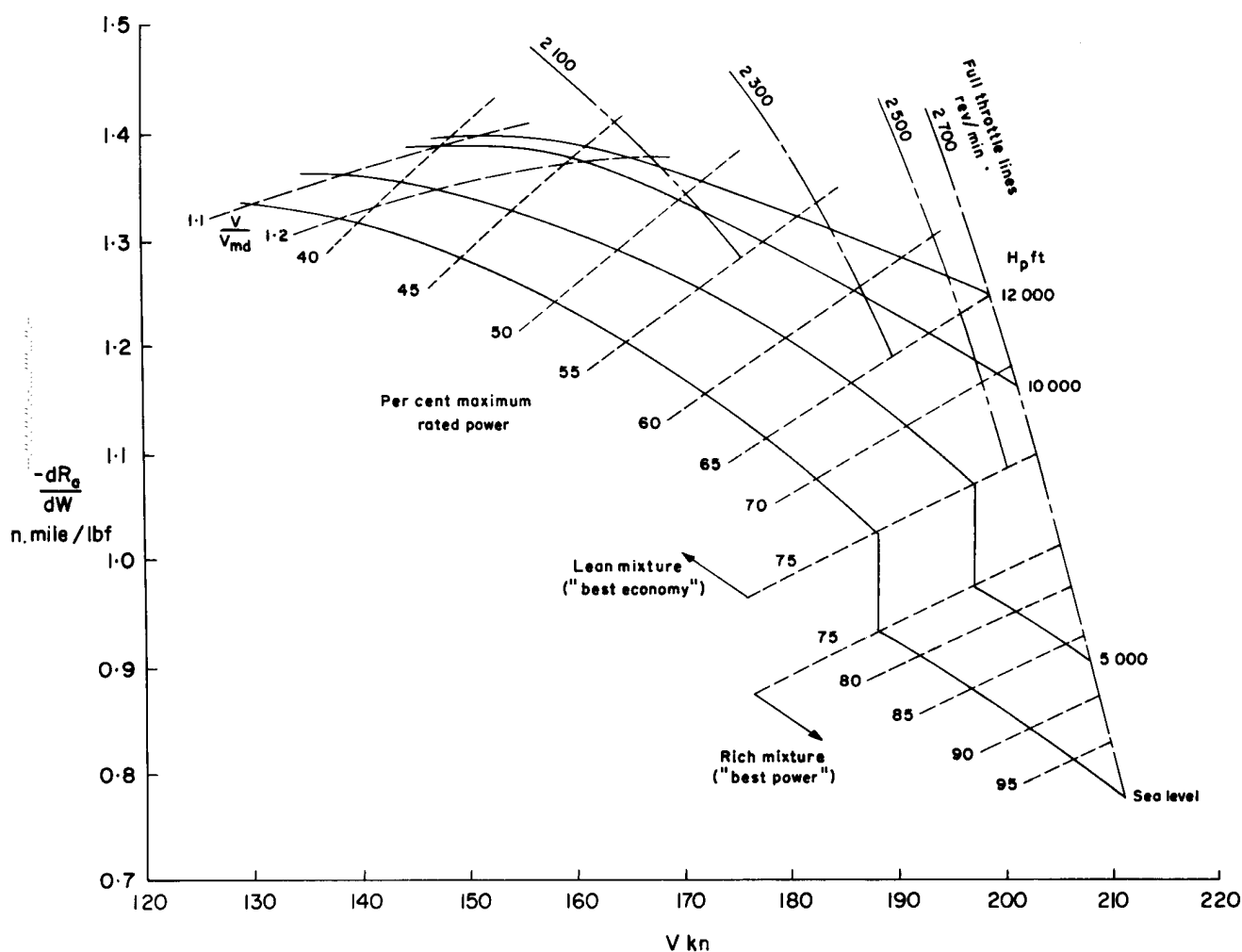
The task of maintaining a steady speed and height at a fixed power setting becomes more difficult as the required forward speed is decreased and this may set a lower limit on the speeds that may be used when maximum range or endurance is required. This is because the excess of power available over power required, following a small reduction in speed, decreases as the power setting corresponding to V_{mp} is approached. Consequently there is an increase in the time taken to regain the nominal height and/or speed following an atmospheric disturbance or control input and a *minimum speed for continuous cruising* may be defined which is considerably greater than V_{mp} . The difference between this minimum cruising speed and V_{mp} is affected by the aircraft's drag characteristics and the variation (if any) with forward speed of propeller efficiency[†] and of engine output-shaft power. In practice, the main effect is to set a lower limit on the speeds that can be used to obtain maximum endurance and it is usual to find scheduled speeds for “holding” in the range $y = 0.9$ to 1.05 (or, approximately, $V/V_{mp} = 1.2$ to 1.4).

Experience suggests that for flights of long duration the minimum speed for *comfortable* continuous cruising may be higher still. For conventional propeller-driven aircraft this speed (when quoted) is at about

* For air-cooled piston engines the drag due to the (open) cowl flap (if variable) plus cooling-air momentum losses can be of the order of ten per cent of cruise drag. The cooling-air required depends on the power output, the fuel-air mixture ratio and the ambient conditions; to represent the associated drag accurately would require an extra term in Equation (3.4). For an example of specific range performance including the effects of a variable cooling requirement see Reference 6. Reference 28 reports work on cooling requirements of current engines and presents some example calculations.

† For speeds of the order of V_{mp} , the propeller efficiency is likely to be increasing with increase in speed and to result in the speed for minimum *output-shaft power* and hence minimum fuel flow being greater than V_{mp} .

$y = 1.05$, i.e. $V/V_{mp} = 1.4$ approximately. (Data from Reference 5 indicate values of between $y = 1.0$ and 1.2 for most of the twenty aircraft types considered.)



Sketch 3.1 Specific air range for a normally-aspirated twin-engined aircraft in ISA conditions at $W = 5\,300$ lbf

3.2 Piston-Engined Aircraft

Sketch 3.1 presents a summary of the specific air range performance at a single weight of a typical, current, twin piston-engined aircraft. The several features of Sketch 3.1 serve to introduce a number of points of significance in the range performance of piston-engined aircraft.

- (i) Each of the four curves of $-dR_a/dW$ against V is an envelope obtained by fairing-in segments of curves obtained at constant engine speed (see the Examples in Item No. Perf RP1/3). Curves of specific air range at constant engine speed are obtainable whenever a constant-speed propeller is fitted and further examples of such curves are given in References 2, 4, 5, 6, 9 and 10. When a fixed-pitch propeller is fitted, each value of V corresponds to a particular engine speed and a single curve of $-dR_a/dW$ against V is obtained (for fixed aircraft weight and ambient conditions) see Reference 1.
- (ii) The power settings are represented by the broken lines as percentages of the engine normal rated power at sea level in ISA conditions. The power required at any given engine speed is obtained by varying the throttle setting.