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Design against fatigue. Basic design calculations

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Stress Analysis and strength of Components Committee. This Committee first met in 1964 and now has the following membership:

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DESIGN AGAINST FATIGUE BASIC DESIGN CALCULATIONS

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DESIGN AGAINST FATIGUE

BASIC DESIGN CALCULATIONS

1. INTRODUCTION

This Data Item gives a method of estimating the fatigue strength of components subjected to loading of constant amplitude with constant mean, or loading that can be regarded as such. The method applies primarily to the design of components in rotating and reciprocating machinery but is generally applicable provided that:

- (a) the component is not subjected to frequent loads higher than the normal running loads, which would invalidate the assumption of constant amplitude loading; the static strength of the component should be checked separately against any high overloads;
- (b) the design endurance is greater than about 10^5 cycles; for shorter endurances the life may be affected by the crack propagation characteristics of the material and of the component geometry.

Fatigue terms used in this Item are explained in ESDU 70016, "Terms and notation for fatigue endurance data" and the principles of design in relation to fatigue are outlined in ESDU 71009, "Design against fatigue. Design principles". Other data, such as elastic stress concentration factors, required for the use of this method may be located by reference to the Engineering Sciences Data Index or the references in this Item.

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2. NOTATION

Both SI and British units are quoted, but any coherent system of units may be used.

a	empirical constant associated with stress gradient (see Section 5.1)	m	in
f	reference stress (<i>i.e.</i> stress calculated from simple theory, based on net or gross section as required, for use with stress concentration factor K_t)	N/m ²	lbf/in ²
f_a	reference alternating stress	N/m ²	lbf/in ²
f_e	von Mises-Hencky equivalent direct stress	N/m ²	lbf/in ²
f_m	reference mean stress	N/m ²	lbf/in ²
f_p	tensile yield strength or 0.2 per cent proof stress of material	N/m ²	lbf/in ²
f_t	tensile strength of material	N/m ²	lbf/in ²
f_x, f_y	direct stresses in arbitrarily chosen orthogonal x - and y -directions respectively	N/m ²	lbf/in ²
K_f	fatigue strength reduction factor due to stress concentration		
K_t	elastic stress concentration factor		
k_{sf}	surface finish factor		
m	index in expression for the effect of mean stress (see Section 7.1)		
N	endurance		
q_{xy}	shear stress in the xy plane	N/m ²	lbf/in ²
r	radius of root of stress concentration	m	in
r_t	peak-to-valley height surface roughness	m	in
$S_{am}(N)$	fatigue strength of material at given mean stress for endurance N , as measured on plain polished laboratory specimens under bending or axial loading	N/m ²	lbf/in ²
$S_{a0}(N)$	value of $S_{am}(N)$ for zero mean stress	N/m ²	lbf/in ²
$S_{a0}(\infty)$	fatigue limit of material at zero mean stress	N/m ²	lbf/in ²

Suffixes

a	alternating component of stress
m	mean component of stress

Sign convention

Tensile stresses are positive. Alternating stresses are always taken as positive.

3. CALCULATION PROCEDURE

The simplified procedure of this section relates to estimating the fatigue strength of a component (which is not subject to fretting) having a stress concentration and subjected to constant amplitude bending and/or direct stress. It is assumed that the elastic stress concentration factor is fully effective on both the mean and alternating stresses. The resulting local stresses are compared with the fatigue properties of the material as represented by Equation (7.1), allowing for the effects of local yield and for deleterious effects of surface finish and treatments.

Modifications to the procedure to account for more complicated loadings and conditions, including alleviation of the effects of the stress concentration factor at very sharp notches, are covered in subsequent sections.

Fatigue strength data for certain types of component (for example, screw threads and thick cylinders) are available in Data Items. Such data can be used directly to estimate fatigue strength.

1. Determine the alternating and mean loads on the component. See Section 4.
2. Locate the most highly stressed region; its location is usually at a stress concentration and therefore obvious, but calculations should be made for all likely critical sections.
3. Refer to the appropriate stress concentration Data Item; this will tell you how to determine f and K_t . Determine f_a and f_m for the respective types of loading, together with the associated values of K_t .
4. Determine the local alternating and mean stresses, $K_t f_a$ and $K_t f_m$.
5. Determine the material properties, f_t , f_p , m (see Section 7.1) and $S_{a0}(N)$. (Take $S_{a0}(N)$ appropriate to the required endurance N , usually $S_{a0}(\infty)$.)

For steel (other than high strength) components designed for unlimited life, in the absence of specific test data, take $S_{a0}(N)/f_t = S_{a0}(\infty)/f_t = 0.4$ and $m = 1$, but note that these are safe values that will under-estimate the fatigue strength. (See ESDU 88008 "Fatigue limit of unnotched steels (related to tensile strength)".)

6. Refer to Figure 1a, 1b or 1c according to the value of m and determine $S_{am}(N)$, the fatigue strength of the material corresponding to the local mean stress $K_t f_m$ or to the ratio $K_t f_m / K_t f_a$. (Section 4.1 explains the reasons for the alternatives and also the conditions under which each is appropriate.)
7. Determine the surface finish factor k_{sf} . See Section 8.
8. Compare the factored local alternating stress $K_t f_a$ with the factored fatigue strength of the material, $S_{am}(N)k_{sf}$, to ensure that there is a sufficient margin between stress and strength.

4. ESTIMATION OF LOADS

The first essential of satisfactory fatigue design calculations is a realistic evaluation of the loading conditions, remembering that most structures and machine components can be subject to fatigue loading conditions that are often complex in character with significant loads at more than one frequency of occurrence.

Three basic approaches to load estimations are:

- (a) calculation from first principles, based on normal running loads, taking account of inertial, frictional, out-of-balance loads, vibration, *et cetera*,
- (b) calculation as in (a) but modified by analysis of failures and successful operation in previous similar designs,
- (c) measurement of loads on a test-rig or in service.

The first approach is usually practical only on relatively simple designs or on those that incorporate load-limiting devices such as fluid couplings. It must be borne in mind that most power sources can deliver considerably greater than the rated torque when subjected to a sudden increase of load. For example some electric motors, even if fitted with an overload trip, can deliver twice the rated torque momentarily before tripping-out, to which must be added the torque needed to decelerate the motor, armature and drive. In the case of a sudden stall this torque can be many times the rated torque. In machinery where such loading occurs in normal use, or misuse, it can be very damaging, but the magnitudes of the loads and particularly their frequency of occurrence can be very difficult to estimate as they are dependent on the operator and operating conditions.

The third approach, measurement of the loads, though desirable, is not always practicable in service conditions. In the absence of measurement the best that can be done is to follow the second approach and work from experience with similar machinery. From analysis of past failures and of successful designs (using nominal loads and consistent methods of stress analysis) it is possible to derive a suitable factor to apply to the nominal calculated loads in future similar designs. A method of doing this is outlined in ESDU SS1, Section 5. The factors so obtained vary widely between different types of machinery and applications and depend also on the sophistication of the methods of analysis, for example, if the analysis made specific allowance for vibration, a lower factor would be expected than if vibration had been ignored. Accordingly, it would be misleading to quote typical values of such factors in this Data Item.

Care should be taken in choosing any additional factor, discussed in Section 10, to allow for approximations, manufacturing tolerances and unforeseen circumstances, so as to ensure a balanced design that is neither too expensive nor too weak.

4.1 Alternating and Mean Loads

This Data Item applies strictly to fatigue loading of constant amplitude and constant mean. In practice the amplitude and often the mean stresses vary or are uncertainly determined, while still legitimately being regarded as of constant amplitude for the purpose of analysis. Such variations or uncertainties are allowed for in the margin between the alternating stresses in the component and the estimated fatigue strength of the component discussed in Section 10. However, the magnitude of this margin can depend on whether the mean stress or the ratio of mean to alternating stress is regarded as fixed when determining the value of the fatigue strength of the material.

For example, where the mean stress arises from some steady load on which are superimposed stresses due

to vibration, the mean stress may be regarded as fixed and the fatigue strength determined accordingly, since a margin of safety is required on the less certainly estimated vibratory stresses.

On the other hand in a machine such as a press where the loads return essentially to zero between each operation, but the maximum of the load in each operation varies or is uncertain, the mean of the load cycle also varies or is uncertain. However the ratio of mean to alternating load remains unity. In this case it is appropriate to determine the fatigue strength for a fixed ratio of mean to alternating stress.

5. EFFECT OF STRESS CONCENTRATION

A fatigue crack will be initiated at any region in a component at which the local alternating stress exceeds the fatigue limit of the material. Accordingly fatigue calculations are based on a stress analysis that takes account of high local stresses in the vicinity of a discontinuity by applying the appropriate elastic stress concentration factors to the reference stresses calculated from simple theory.

5.1 Small Components and Sharp Notches

In the case of very localised stresses, such as occur at sharp discontinuities in small components such as the specimens usually used for fatigue testing, an insufficient volume of material may be stressed to the fatigue limit to start a significant fatigue crack. In addition, beneficial compressive residual stress may be introduced by machining with a pointed tool. In such cases application of K_t to the reference stress may lead to an underestimate of fatigue strength, and the use of an empirically determined fatigue strength reduction factor K_f in place of K_t is more realistic.

Figure 2 indicates approximate values of K_f that may be expected in cases where the stress amplitudes approach the fatigue limit. It is based on the equation

$$K_f = \frac{K_t}{1 + 2 \left(\frac{K_t - 1}{K_t} \right) \left(\frac{a}{r} \right)^{1/2}}$$

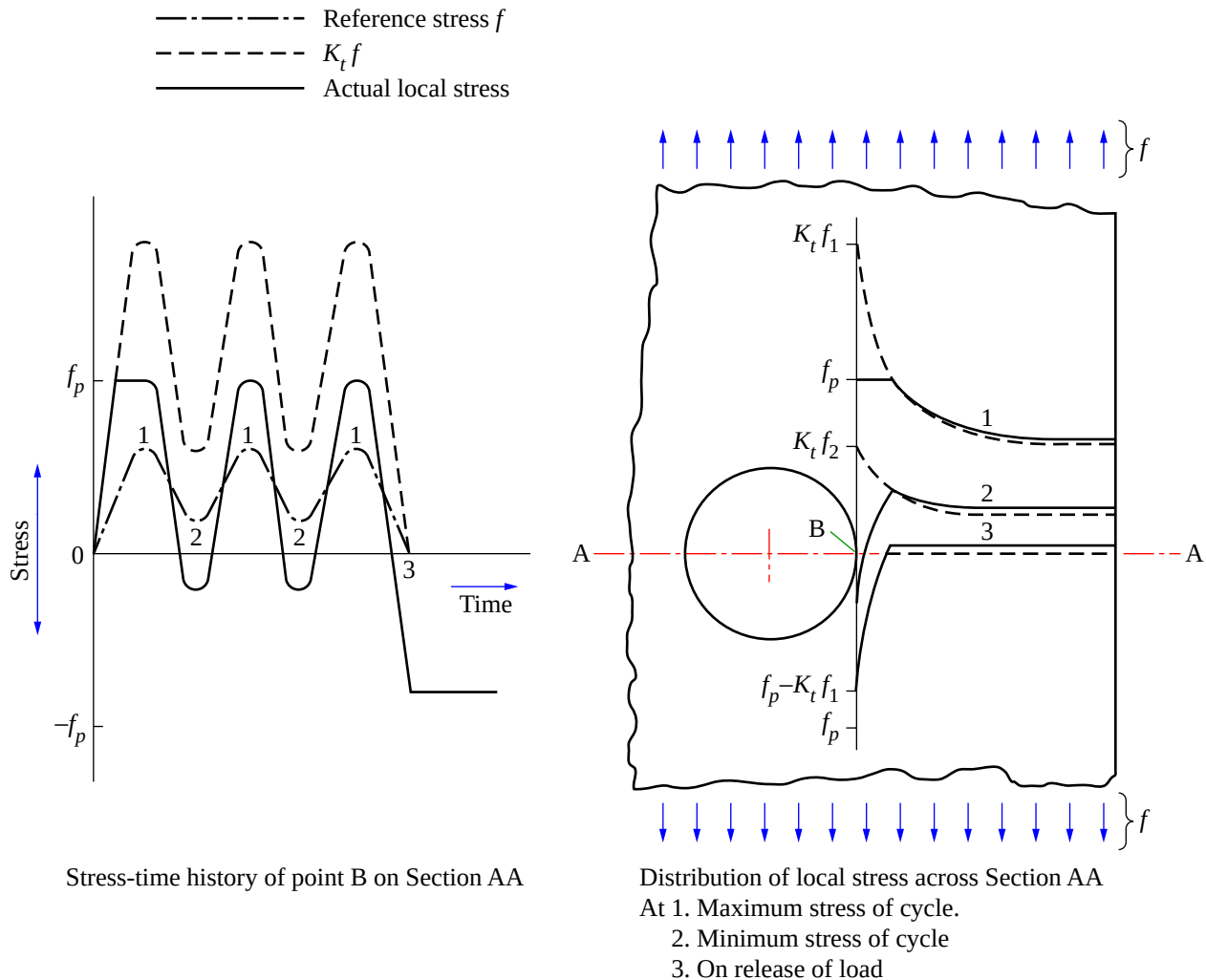
with the proviso that K_f is never less than unity. Values of the empirical constant a , based on the results of fatigue tests on notched specimens, are given on Figure 2 but substantial variations from these values can occur in particular cases.

5.2 Residual Stresses at a Stress Concentration

The manufacturing process and subsequent treatment at a stress concentration often give rise to an initial residual stress in the surface material at the critical region on which are superimposed the stresses applied to the section, thus modifying the local mean stress and significantly affecting the fatigue strength.

Such initial residual stresses are difficult to measure or to predict with accuracy but their effects can be assessed qualitatively; a compressive residual stress tends to increase fatigue strength and is therefore beneficial while a tensile residual stress reduces fatigue strength. ESDU 71009 gives examples of processes that give rise to each type of residual stress. The maximum local stress cannot exceed the yield stress of a material that has a sharp yield, so that limits to the effect of residual stress can be assessed approximately by assuming the presence of a local mean stress of up to $\pm f_p$ at the surface of the critical section.

Residual stresses may also be induced, deliberately or otherwise, through loading a section with a stress concentration such that the local stress exceeds the yield stress while the bulk of the section remains elastic. The effect is illustrated in Sketch 5.1, and is taken into account in Section 7. When it occurs it eliminates the effect in the critical region of any initial residual stress.

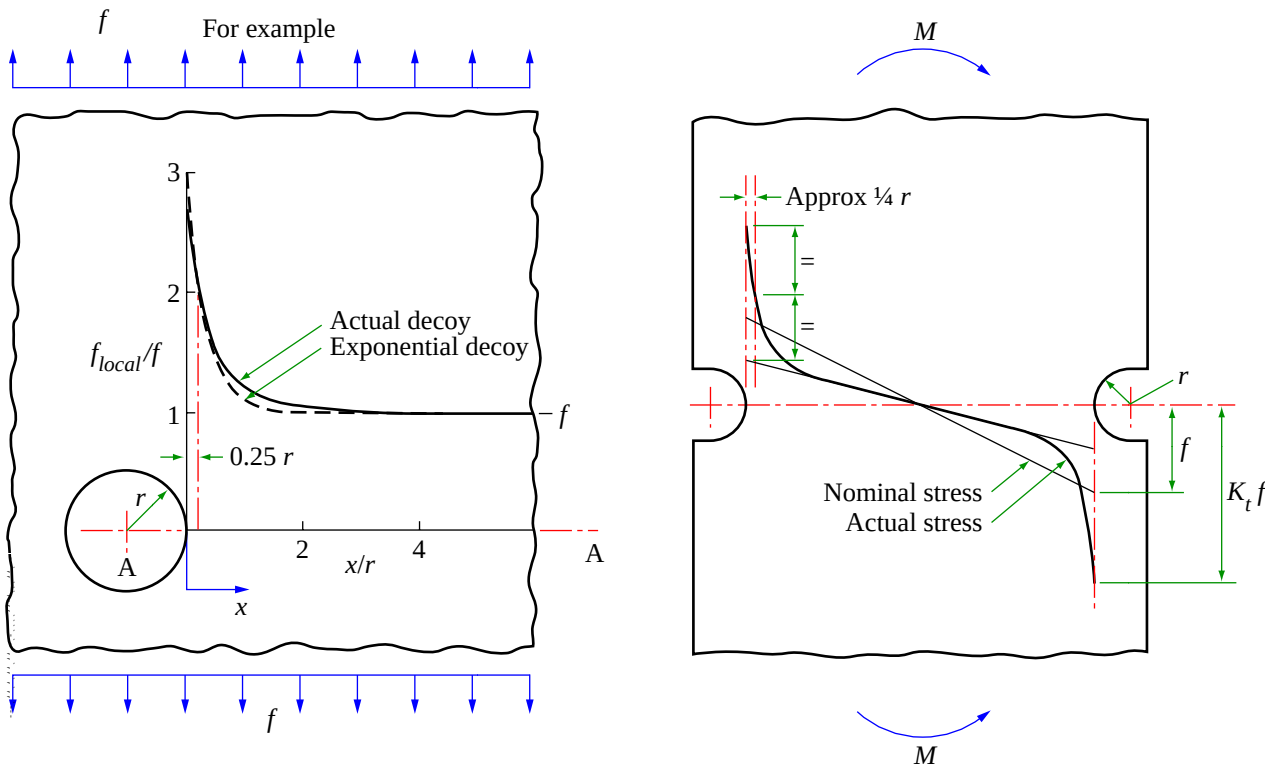


Sketch 5.1

It also follows that even a single compressive overload can cause a residual tensile stress that may seriously reduce fatigue strength under subsequent cycles; on the other hand a tensile overload, unless repeated sufficiently frequently to be a fatigue hazard itself, can improve fatigue strength under subsequent normal running loads.

5.3 Adjacent and Superimposed Stress Concentrations

Allowance should be made for interactions between coincident and adjacent stress concentrations. The Items on stress concentrations provide data on a number of adjacent stress concentrations. In other cases the maximum stress may be estimated by increasing either the reference stress or the maximum stress, calculated for an isolated stress concentration, by the additional local stress due to the presence of other stress concentrations. As a rule-of-thumb it can be assumed that the stress decays to the value remote from the stress concentration along an approximately exponential curve, such that the difference between the peak stress and the stress remote from the concentration halves in a distance of about $0.25r$ normal to, and $0.5r$ parallel to the principal stress direction. For example, see Sketch 5.2.



Sketch 5.2

It should be noted that the interaction between stress concentrations often produces an entirely new stress pattern that can be less severe than that around an isolated stress concentration, as in the case of multiple grooves and notches. Examples are illustrated in ESDU 71009.