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Natural frequencies of rectangular flat plates with various edge conditions

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item, which supersedes Item No. 66019, was monitored and guided by the Acoustic Fatigue Panel, which first met in 1966 and had the following membership:

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(Continued on inside back cover)

NATURAL FREQUENCIES OF RECTANGULAR FLAT PLATES WITH VARIOUS
EDGE CONDITIONS

CONTENTS

	Page
1. NOTATION	1
2. NOTES	2
2.1 General	2
2.2 Mode Numbers	3
2.3 Effect of Poisson's Ratio	3
2.4 Accuracy and Limitations on Data	4
3. EFFECT OF IN-PLANE LOADS	4
4. DERIVATION	5
5. EXAMPLES	5
APPENDIX A ENERGY METHOD FOR ESTIMATING CRITICAL BUCKLING LOAD	50
A1. ENERGY METHOD FOR ESTIMATING CRITICAL BUCKLING LOAD	50
A2. NOTES	52
A3. EXAMPLE	52
APPENDIX B COMPARISON OF NATURAL FREQUENCIES ESTIMATED USING THE PROCEDURES IN THIS ITEM WITH THOSE COMPUTED USING ESDUPAC A9016	53
FIGURES 1 TO 43	7 TO 49

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NATURAL FREQUENCIES OF RECTANGULAR FLAT PLATES WITH VARIOUS EDGE CONDITIONS

1. NOTATION

a	length of longer side of plate	m	in
b	length of shorter side of plate	m	in
C	indicates clamped edge		
c	$(E/\rho)^{1/2}$ for plate material*	m/s	in/s
E	Young's modulus for plate material	N/m ²	lbf/in ²
F	indicates free edge		
F_a	frequency coefficient in $f = F_a ct/a^2$		
F_b	frequency coefficient in $f = F_b ct/b^2$		
F_{ab}	frequency coefficient in $f = F_{ab} ct/(ab)$		
f	natural frequency of vibration of plate without in-plane loading	Hz	Hz
f_N	natural frequency of vibration of plate under biaxial in-plane loads N_x and N_y	Hz	Hz
m	mode number in x -direction		
N_x	in-plane load per unit length acting parallel to x -axis, positive tensile	N/m	lbf/in
N_y	in-plane load per unit length acting parallel to y -axis, positive tensile	N/m	lbf/in
N_{xcr}	critical buckling value of N_x when plate is subject to biaxial loading in ratio N_x/N_y	N/m	lbf/in
N_{xy}	shear force per unit length acting along plate sides, positive as shown in Sketch 1.2	N/m	lbf/in
n	mode number in y -direction		
S	indicates simply-supported edge		
t	plate thickness	m	in

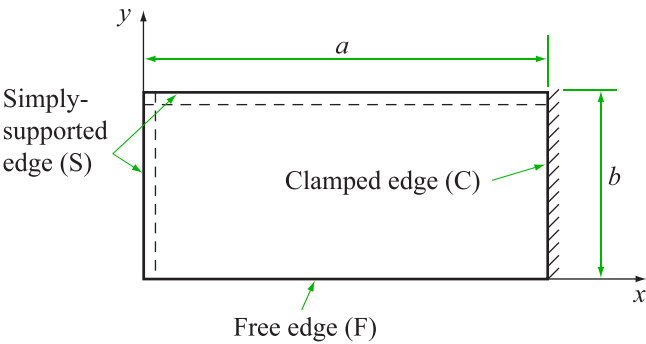
*For footnote see end of Notation.

x, y	orthogonal coordinate system defined in Sketch 1.1	
ν	Poisson's ratio of plate material	
ρ	density of plate material	kg/m^3 †

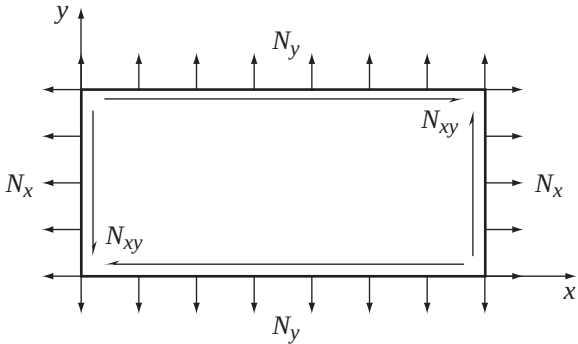
* For all common structural metallic materials the value of c is 5080 m/s (200 000 in/s).

† A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before being used to calculate c . (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s².) For example, if the material density is 0.097 lb/in³, the value input should be 2.510×10^{-4} lb s²/in⁴.

Both SI and British units are quoted by any coherent system of units may be used.



Sketch 1.1 Plate geometry and edge condition codes



Sketch 1.2 Positive loading system

2. NOTES

2.1 General

This Data Item provides data for calculating the natural frequencies of flat rectangular plates. A procedure to account for the effect of uniform, steady in-plane loading on the plate natural frequencies is also given. The main natural frequency data provided in this Item are presented in graphical form. A computer program, referred to as ESDUpac A9016, may be used to evaluate flat plate natural frequencies with, or without, in-plane loading. The use of that program is described in Data Item No. 90016*.

Data are provided for all the combinations of free, simply-supported and clamped edge conditions. For each particular case the edge condition is denoted by a sequence of four letters. The first pair of letters indicates the edge conditions on the plate longer side and the second pair the short side edge conditions. In order to provide a unique edge condition identification each pair of edge condition letters, for both the long and short sides, is written in order of increasing edge fixity. Using this notation the plate in Sketch 1.1 has edge conditions defined by FS-SC; this sketch also defines a diagrammatic coding for plate edge conditions.

* Natural frequencies of isotropic and orthotropic rectangular plates under static in-plane loading (including shear loading).

The natural frequencies of the unloaded plate are obtained from one of the expressions

$$f = F_a c \frac{t}{a^2},$$

$$f = F_b c \frac{t}{b^2},$$

or
$$f = F_{ab} c \frac{t}{ab}.$$

The data are presented in terms of the coefficient most convenient to use from these three expressions, for each set of edge conditions and modes. Frequency coefficients are plotted against b/a in Figures 2 to 43. The figure required for a particular edge condition may be found by reference to the guide in Figure 1.

Frequency coefficient F_a is used for modes where bending occurs only about axes parallel to the y -axis. This condition arises in the cases of plates with both long sides free when $n = 1$. Frequency coefficient F_{ab} is used for pure torsion modes which occur for plates with both long sides free when $n = 2$, and for plates with one long side free and the other simply-supported when $n = 1$. Frequency coefficient F_b is used in all other cases.

2.2 Mode Numbers

For any mode of vibration, mode numbers m and n may be related to the nodal lines provided that these lines are parallel, or approximately parallel, to the plate sides. For these cases, m and n are one plus the number of nodal lines crossing any line parallel to the x -axis and y -axis, respectively. In this definition plate edges, supported or free, are not counted as nodal lines.

The condition of nodal lines parallel, or approximately parallel, to the plate sides does not apply in all cases for either square plates or plates having two adjoining edges free. For square plates, nodal lines are not parallel to the plate edges unless the edge conditions are only singly-symmetric about a line parallel to a plate edge. For plates having two adjoining free edges nodal lines may not be parallel to a plate edge unless a third edge is free. Further guidance on the nodal patterns for special cases is provided in Item No. 90016.

2.3 Effect of Poisson's Ratio

In calculating frequency and buckling coefficients presented in this Item a value of $\nu = 0.3$ was used. For plates without a free edge the frequency is inversely proportional to $\sqrt{1 - \nu^2}$; however, the relationship is more complicated when one or more plate edges are free.

Investigations over a range of values of b/a for the four edge conditions FF-SS, SS-FF, FS-SS and SS-FS have shown that differences in natural frequencies for $\nu = 0$ and $\nu = 0.5$ from the values for $\nu = 0.3$ are generally less than 10 per cent. For all cases investigated, for plates with a uniform edge condition along each plate side, frequencies increase with increasing ν . The effect of varying ν may be investigated using the computer program provided in ESDUpac A9016.

Frequency coefficients are calculated assuming fully effective anticlastic bending except in cases where both the plate longer sides are free, $n = 1$ and b/a is nearly 0. These latter cases are those for which F_a is provided. The assumption of fully effective anticlastic bending may not be valid when one or more long sides of the plate are free and b/a approaches 0. In these cases the frequency coefficients F_{ab} and F_b must be treated with caution.

2.4 Accuracy and Limitations on Data

Exact values of natural frequency coefficient are plotted for all cases where exact values are calculable; such cases are those where two opposite sides of the plate are simply-supported. For all other cases the frequency coefficients were obtained using a Rayleigh-Ritz analysis method that utilised deflection mode shapes composed of 36-term beam functions. For these latter cases the frequency coefficients are upper bound values. The accuracy of these values decreases with increase in mode number or the number of free edges, but for cases of practical engineering interest the data presented are, in general, within 3 per cent of the exact values.

In calculating the frequency coefficients, effects of shear deformation and rotary inertia were neglected. The contribution of these effects to the frequency coefficient becomes increasingly significant for higher order modes.

Data provided are limited to plates having b/t ratios greater than about 10.

3. EFFECT OF IN-PLANE LOADS

The natural frequencies of vibration for a plate subjected to uniform steady biaxial in-plane loads N_x and N_y may be estimated from

$$f_N = f \left(1 - \frac{N_x}{N_{xcr}} \right)^{1/2} . \quad (3.1)$$

In using Equation (3.1) the values of f , f_N and N_{xcr} must be all for the same mode number in both the x - and y -directions. If the required values of N_{xcr} are not available from other sources they may be calculated, for some cases, from Item No. 72019 or by the approximate method suggested in Appendix A. Alternatively, natural frequencies under in-plane loading may be evaluated using ESDUpac A9016 which does not require an independent evaluation of N_{xcr} .

Equation (3.1) applies strictly only to plates simply-supported at all edges, to long narrow plates loaded length-wise or to any other configuration in which the mode shape in buckling is the same as in vibration. However, for other cases that have been analysed, the difference in frequency found using the equation is less than 10 per cent for all loads, and less than 2 per cent under tensile loading up to twice the buckling load.

Equation (3.1) applies to initially perfectly flat plates. Any slight initial curvature in the plate of similar form to the compressive buckling mode reduces considerably the effect of compressive in-plane loads. Tests have shown frequencies at loads approaching buckling remaining at about 80 per cent of the zero-load frequency instead of decreasing to near zero in accordance with simply theory. Moreover, for compressive loads approaching the buckling load, large deflection effects become important and the method does not apply. For this reason the results should be treated with caution with compressive loads.

It is expected that the effect of shear load, N_{xy} , on the natural frequency in the (1, 1) mode will be small. Shear loads can have a significant effect, however, on the frequency of higher modes, introducing combinations of modes $(m_1, n_1) + (m_2, n_2)$ in which $m_1 \pm m_2$ and $n_1 \pm n_2$ are odd numbers. More guidance on the effect of shear loading, including combined mode effects, is provided in Item No. 90016.

It should also be noted that when loads are compressive the lowest frequency does not always correspond to the (1, 1) mode because when b/a is small higher order buckling modes become critical.

4. DERIVATION

1. TIMOSHENKO, S. *Vibration problems in engineering*. Second edition, Van Nostrand, New York, 1937.
2. WARBURTON, G.B. The vibration of rectangular plates. *Proc. Instn mech. Engrs*, Vol. 168, No. 12, pp. 371-384, 1954.
3. TIMOSHENKO, S. *Theory of plates and shells*. Second edition, McGraw Hill, New York, 1959.
4. WITTRICK, W.H. General sinusoidal stiffness matrices for buckling and vibration analyses of thin flat walled structures. *Int. J. mech. Sci.*, Vol. 10, pp. 949-966, 1966.
5. LEISSA, A.W. Vibration of plates. NASA SP-160, 1969.
6. LEISSA, A.W. The free vibration of rectangular plates. *J. Sound Vib.*, Vol. 31, No. 3, pp. 257-293, December 1973.

5. EXAMPLES

- 5.1 It is required to estimate the lowest four natural frequencies, for which $n = 1$, for a plate with long sides clamped and short sides simply-supported. The plate is unloaded and has the following dimensions and physical properties:

$$a = 450 \text{ mm}, \quad b = 225 \text{ mm}, \quad t = 1.5 \text{ mm},$$

$$E = 70\,000 \text{ MN/m}^2, \quad \rho = 2710 \text{ kg/m}^3, \quad \nu = 0.3.$$

Firstly $\frac{a}{b} = \frac{225}{450} = 0.5$

and $c = \sqrt{\frac{70\,000 \times 10^6}{2710}} = 5082 \text{ m/s}.$

The edge conditions of this plate are coded CC-SS and from Figure 1 the frequency coefficients are given on Figure 41.

From Figure 41, for $b/a = 0.5$ the values of F_b for modes (1, 1), (2, 1), (3, 1) and (4, 1) are 1.15, 1.39, 1.88 and 2.64, respectively.

The natural frequencies are obtained from

$$f = F_b c \frac{t}{b^2}.$$

Now $\frac{t}{b^2} = \frac{1.5 \times 10^{-3}}{(225 \times 10^{-3})^2} = 0.0296 \text{ /m}.$

so that the required frequencies are

$$\begin{aligned} \text{mode 1, 1} \quad f &= 1.15 \times 5082 \times 0.0296 = 173 \text{ Hz}, \\ \text{mode 2, 1} \quad f &= 1.39 \times 5082 \times 0.0296 = 209 \text{ Hz}, \\ \text{mode 3, 1} \quad f &= 1.88 \times 5082 \times 0.0296 = 283 \text{ Hz}, \\ \text{mode 4, 1} \quad f &= 2.64 \times 5082 \times 0.0296 = 398 \text{ Hz}. \end{aligned}$$

- 5.2 If the panel of Section 5.1 is subjected to biaxial loads of $N_x = 30 \text{ kN/m}$ and $N_y = 15 \text{ kN/m}$, estimate the four natural frequencies.

For this case the values of N_{xcr} are estimated using the approximate method given in Appendix A.

The values of f_N are then estimated from

$$f_N = f \left(1 - \frac{N_x}{N_{xcr}} \right)^{1/2}.$$

The frequencies are tabulated as follows.

Mode (m, n)	N_x/N_{xcr}	f (Hz)	f_N (Hz)
1, 1	-1.076	173	249
2, 1	-1.317	209	318
3, 1	-1.265	286	426
4, 1	-1.037	398	568

A comparison of the above natural frequencies with those evaluated using the computer program in ESDUpac A9016 is given in Appendix B.

	Free edge		Simply-supported edge		Clamped edge
FF–FF Figures 2 and 8	FS–FF Figures 11 and 17	FC–FF Figure 20	SS–FF Figure 26	SC–FF Figure 32	CC–FF Figure 38
FF–FS Figures 3 and 9	FS–FS Figures 12 and 18	FC–FS Figure 21	SS–FS Figure 27	SC–FS Figure 33	CC–FS Figure 39
FF–FC Figures 4 and 9	FS–FC Figures 13 and 18	FC–FC Figure 22	SS–FC Figure 28	SC–FC Figure 34	CC–FC Figure 40
FF–SS Figures 5 and 10	FS–SS Figures 14 and 19	FC–SS Figure 23	SS–SS Figure 29	SC–SS Figure 35	CC–SS Figure 41
FF–SC Figures 6 and 10	FS–SC Figures 15 and 19	FC–SC Figure 24	SS–SC Figure 30	SC–SC Figure 36	CC–SC Figure 42
FF–CC Figures 7 and 10	FS–CC Figures 16 and 19	FC–CC Figure 25	SS–CC Figure 31	SC–CC Figure 37	CC–CC Figure 43

FIGURE 1 FIGURE LOCATIONS FOR FREQUENCY COEFFICIENTS

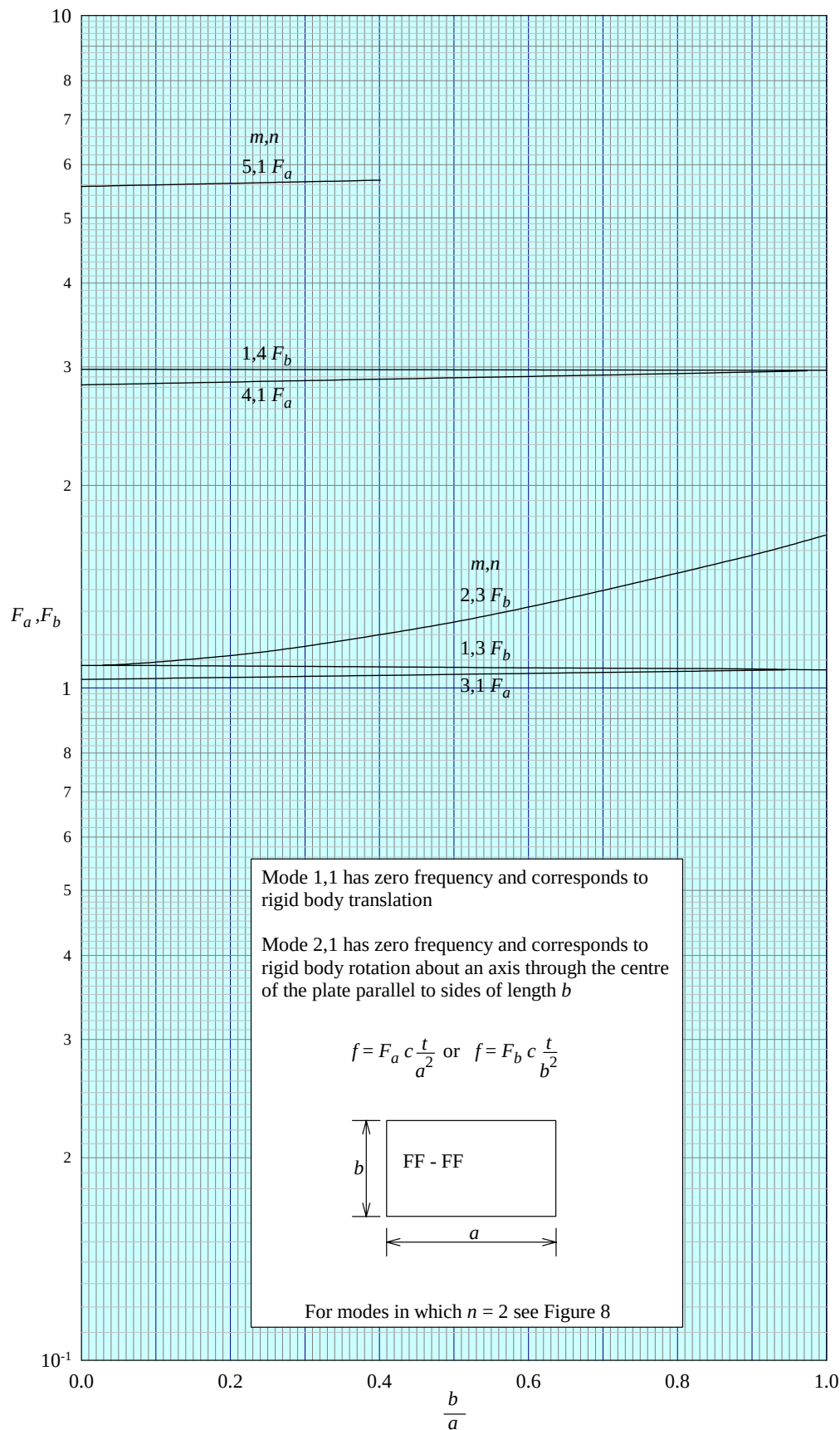


FIGURE 2 FF-FF

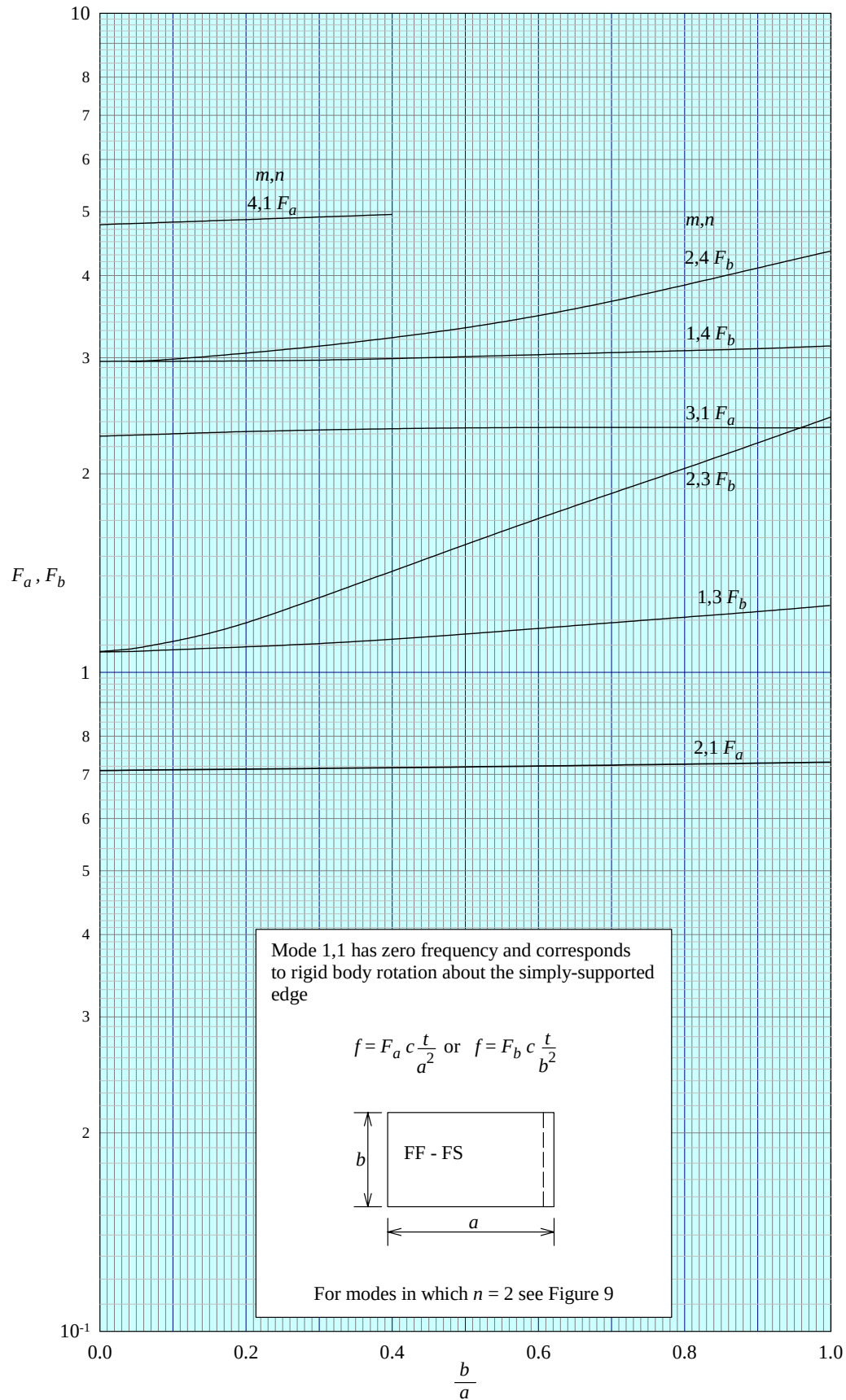


FIGURE 3 FF-FS

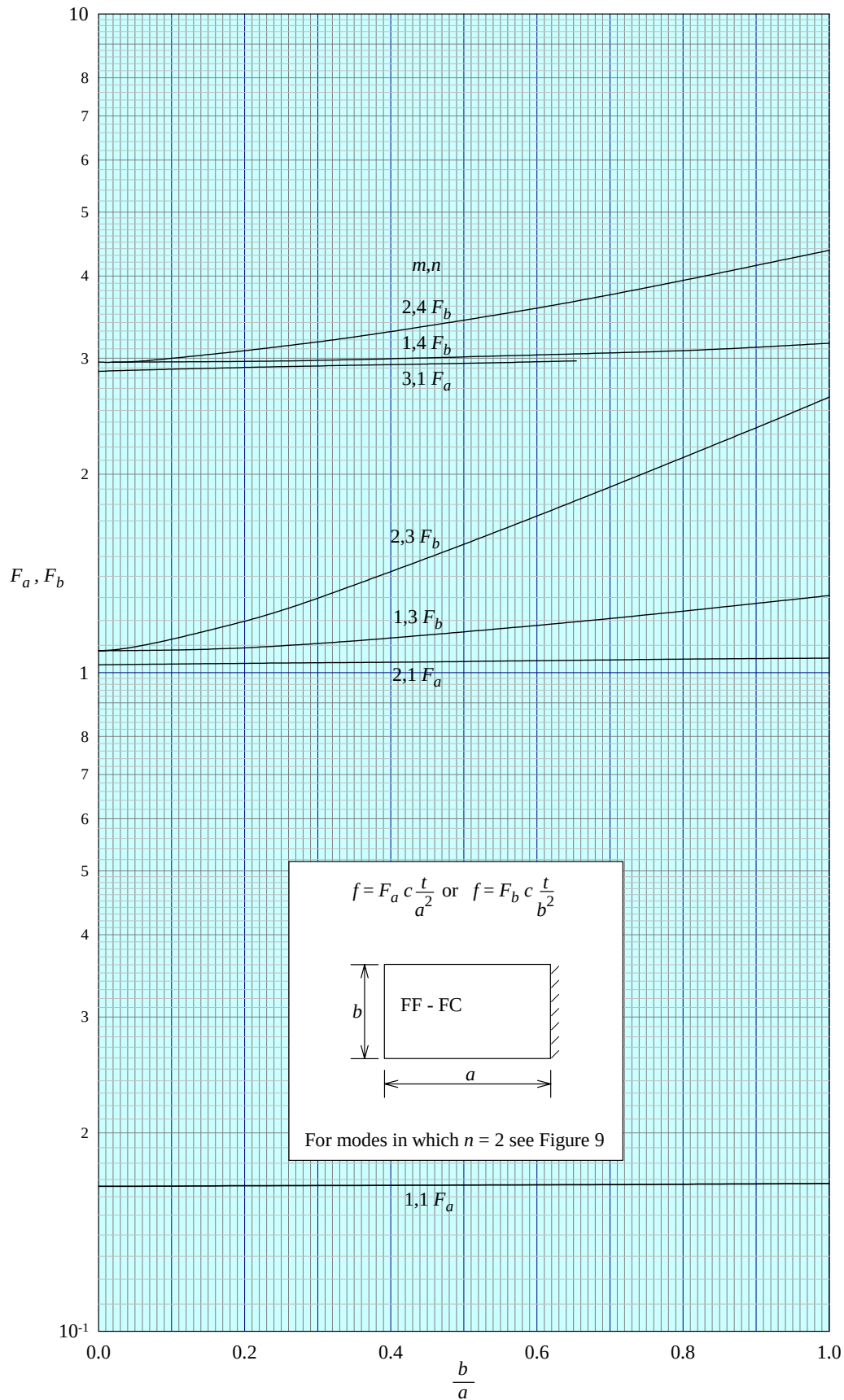


FIGURE 4 FF-FC

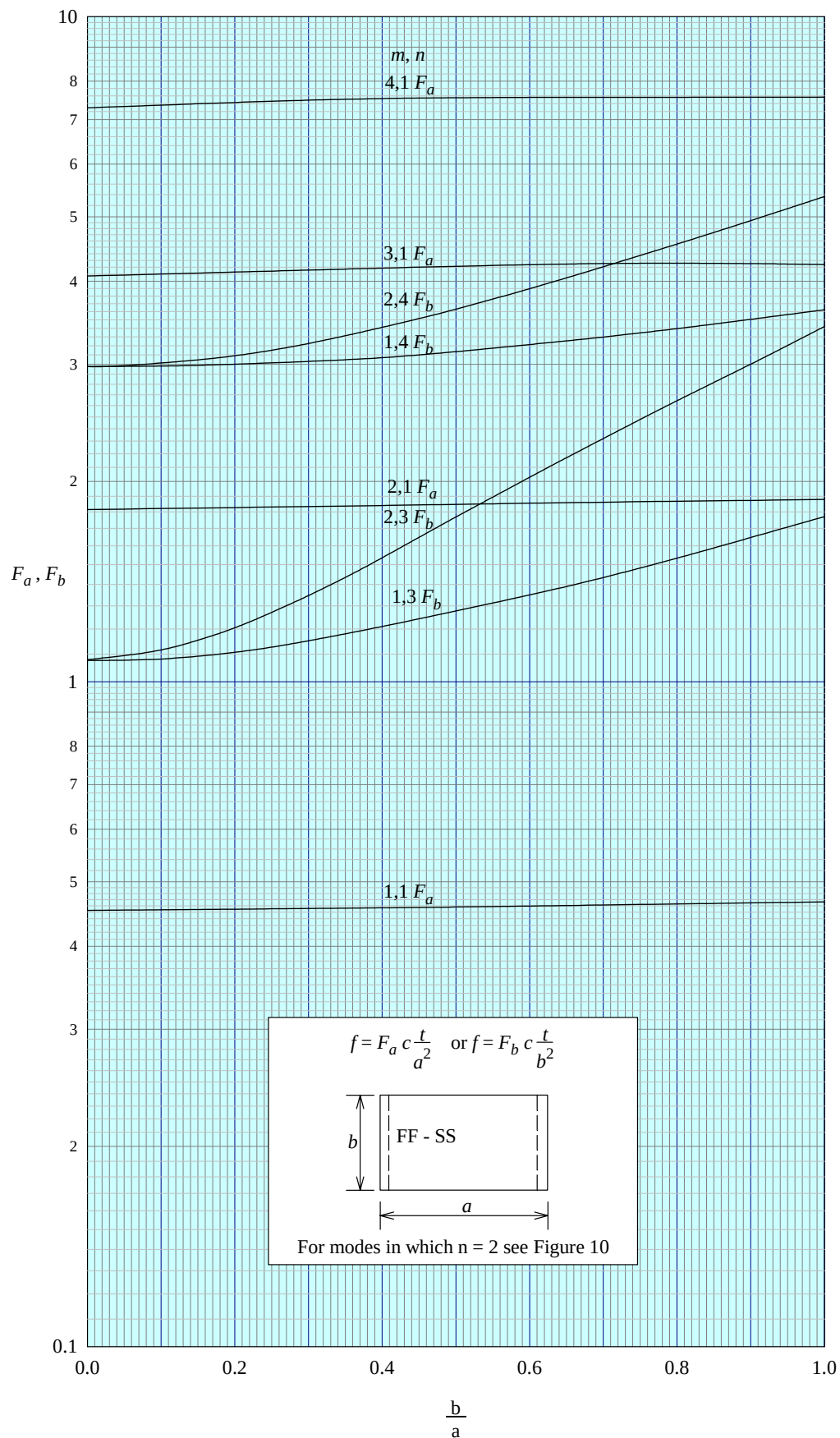


FIGURE 5 FF-SS

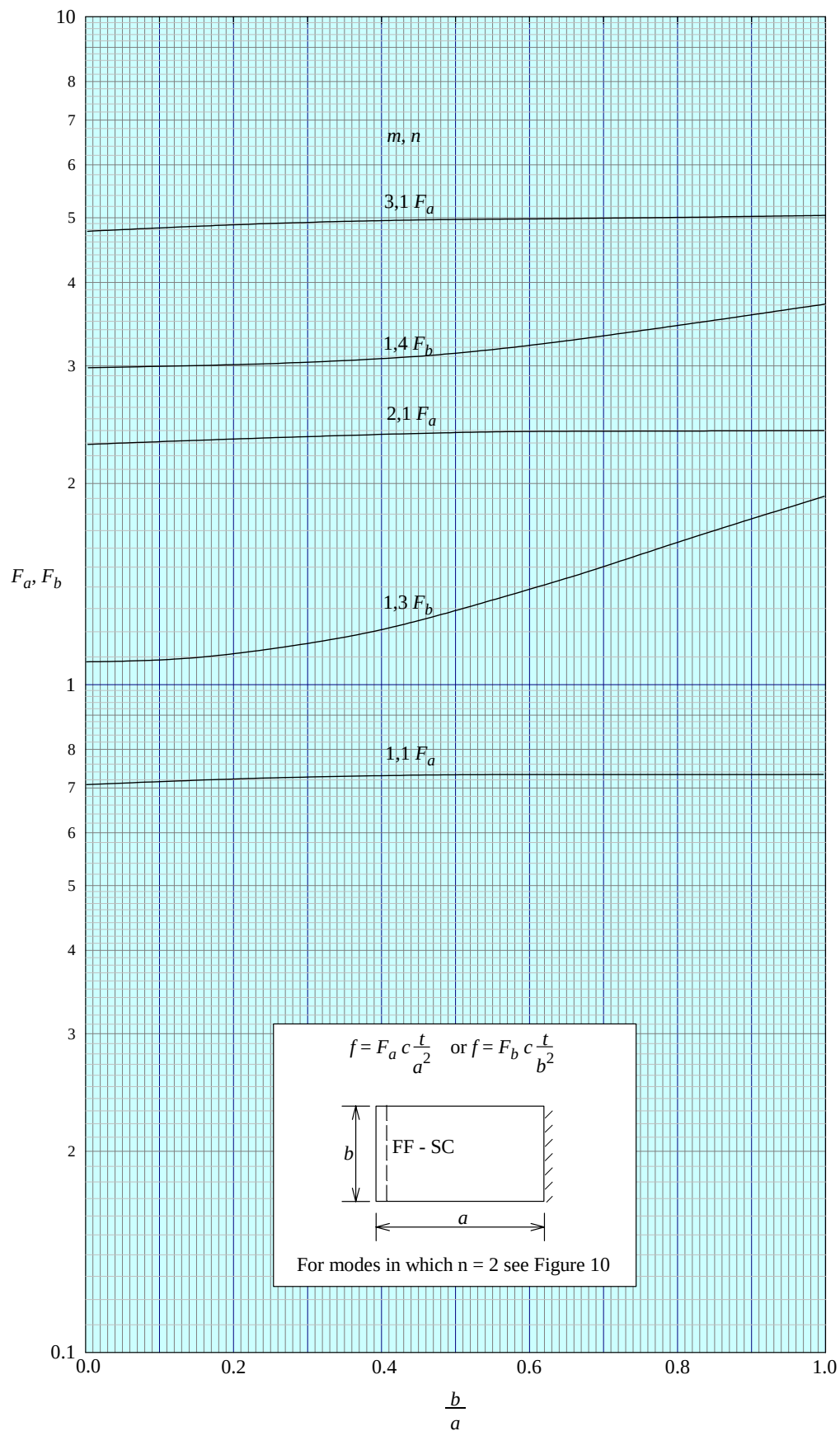


FIGURE 6 FF-SC

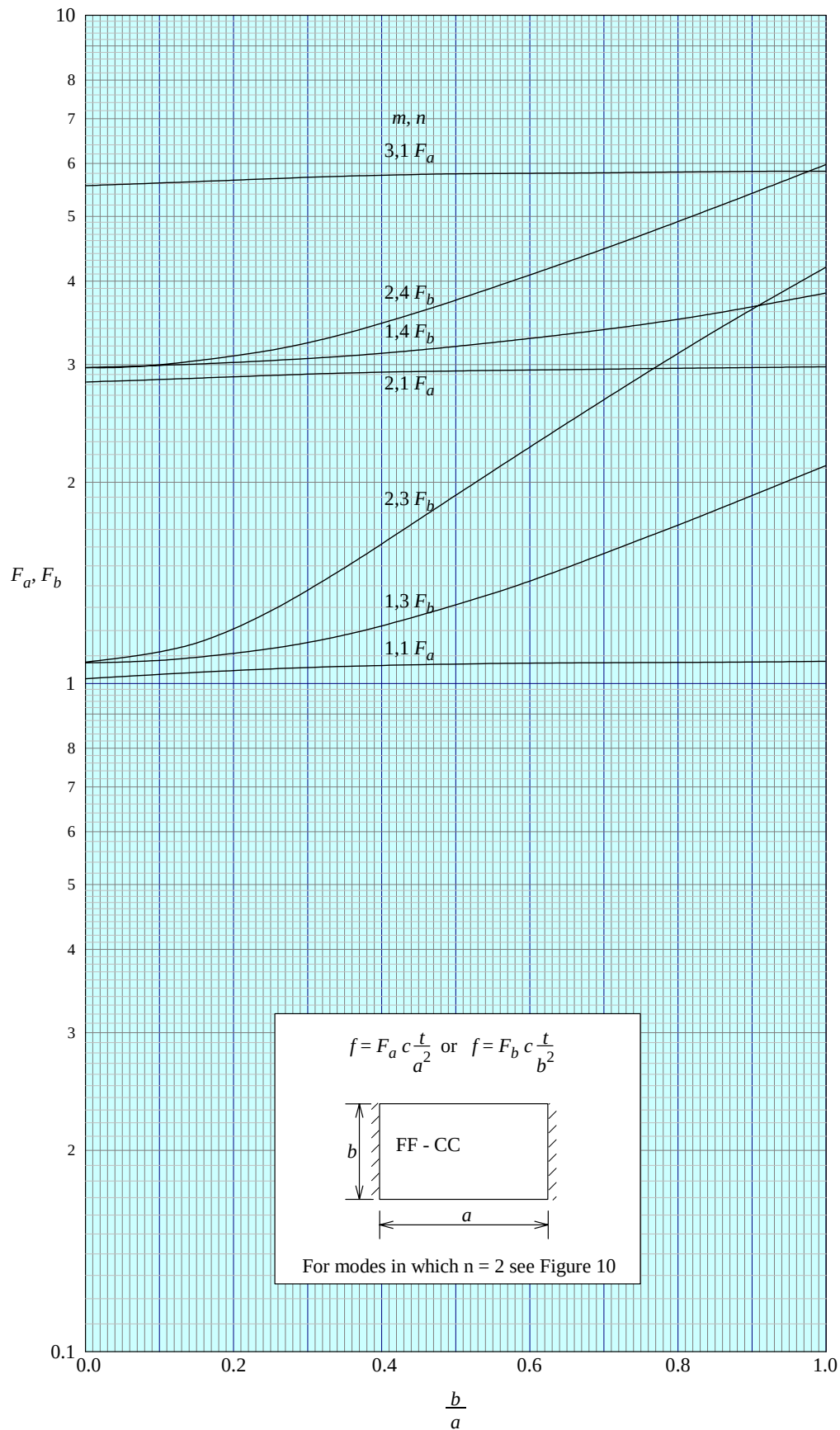


FIGURE 7 FF-CC

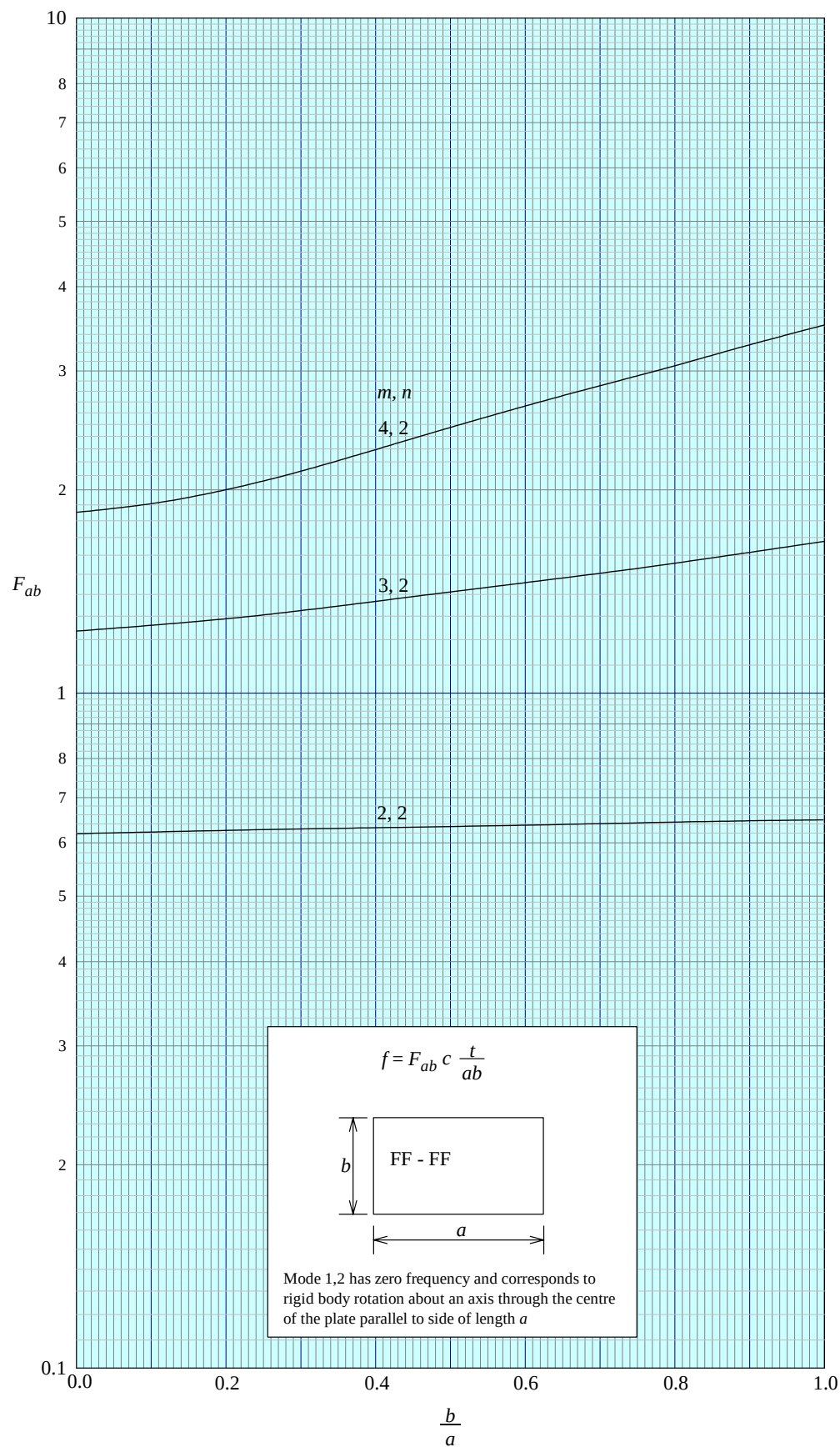


FIGURE 8 FF-FF

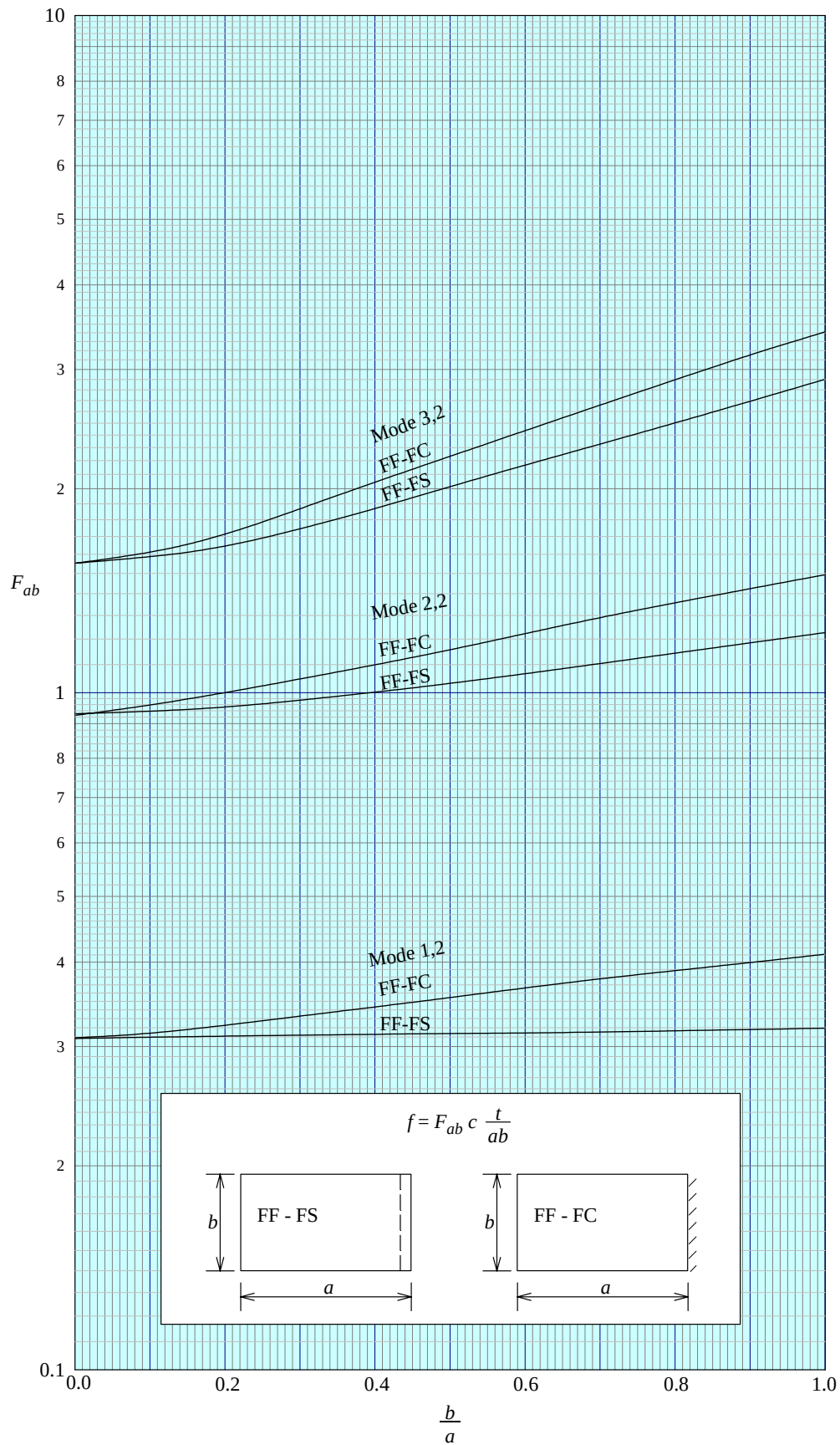


FIGURE 9 FF-FS, FF-FC