



ESDU 75031

Issued November 1975
With Amendments A to D
September 2016
Supersedes ESDU 73028

Drag of two-dimensional steps and ridges immersed in a turbulent boundary layer for Mach numbers up to 3

Associated software: ESDUpac A7531



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THE PREPARATION OF THIS DATA ITEM

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DRAG OF TWO-DIMENSIONAL STEPS AND RIDGES IMMERSED IN A TURBULENT BOUNDARY LAYER FOR MACH NUMBERS UP TO 3

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. GEOMETRIES CONSIDERED AND SCOPE OF ITEM	2
3. PRESENTATION OF DATA	2
3.1 Plain Steps and Ridges	3
3.2 Chamfered and Radiused Steps and Radiused Ridges	3
3.2.1 Chamfered forward-facing steps	4
3.2.2 Chamfered rearward-facing steps	4
3.2.3 Radiused forward-facing steps	5
3.2.4 Radiused rearward-facing steps	5
3.2.5 Radiused rectangular ridges	5
4. NOTES ON APPLICABILITY	5
5. ACCURACY	7
6. DERIVATION AND REFERENCES	8
7. EXAMPLE	9
FIGURES 1a to 9	11-21
APPENDIX A DEVELOPMENT OF THE FUNCTIONS P AND Q	22
A1. ADDITIONAL NOTATION	22
A2. DEVELOPMENT OF P AND Q	22
A2.1 Numerical Evaluation of A and B for Use in Equation (A2.7)	24
APPENDIX B COMPUTER PROGRAM FOR ESTIMATION OF DRAG OF TWO-DIMENSIONAL STEPS AND RIDGES IMMERSED IN A TURBULENT BOUNDARY LAYER FOR MACH NUMBERS UP TO 3	37
B1. INTRODUCTION	37
B2. PROGRAM	37
B2.1 Input	37
B2.2 Output	39
B2.3 Running the Program	39

DRAG OF TWO-DIMENSIONAL STEPS AND RIDGES IMMERSED IN A TURBULENT BOUNDARY LAYER FOR MACH NUMBERS UP TO 3

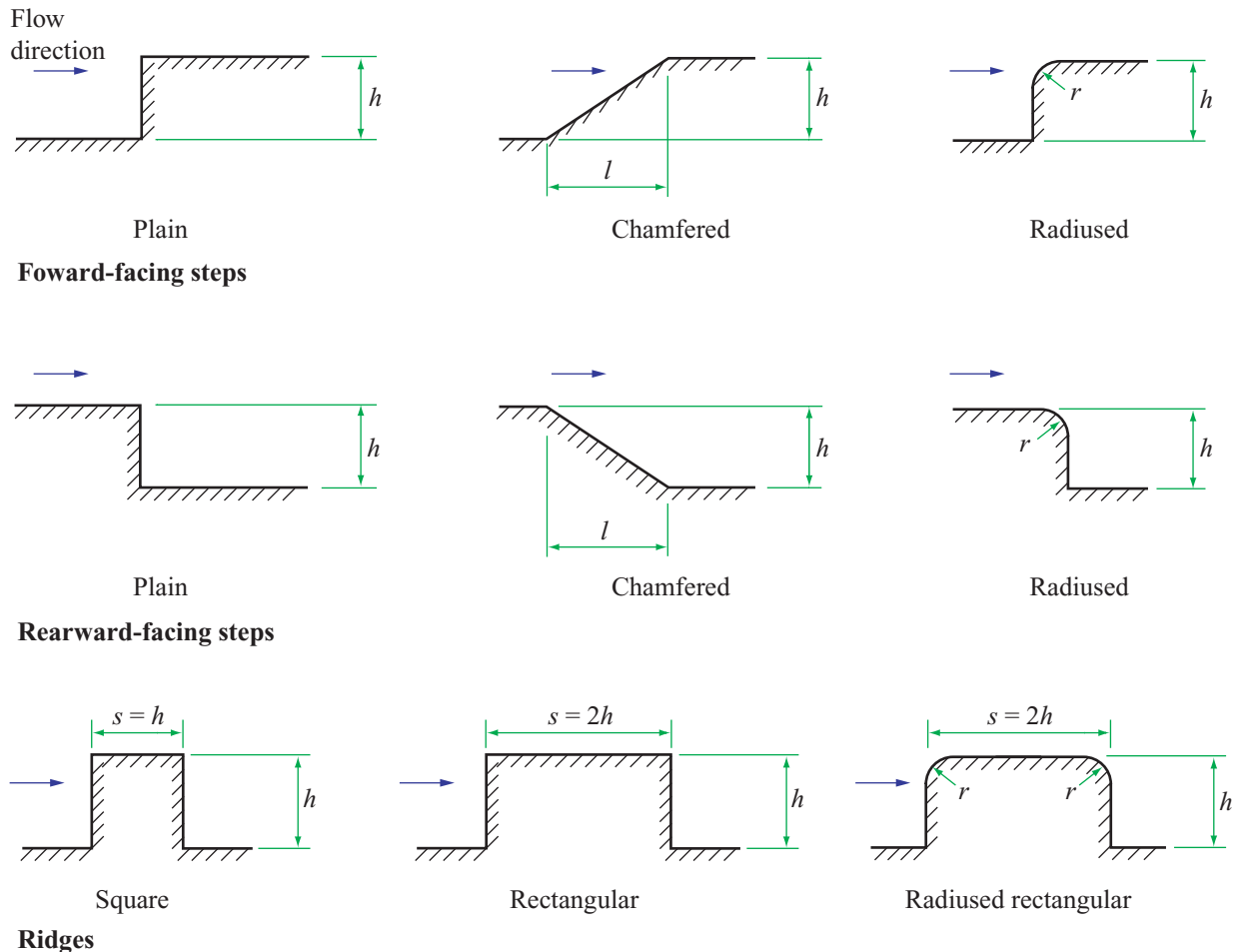
1. NOTATION AND UNITS

		<i>SI</i>	<i>British</i>
C_D	incremental drag coefficient due to two-dimensional excrescence, based on its height (h) and kinetic pressure at edge of boundary layer in absence of excrescence		
C_f	local skin friction coefficient in absence of excrescence, based on kinetic pressure at edge of boundary layer		
h	height of step or ridge, see Sketch 2.1	m	ft
k_c	factor relating drag of chamfered step to that of plain step (Equation (3.2))		
k_r	factor relating drag of radiused step or ridge of that of plain step or ridge (Equation (3.3))		
l	streamwise length of chamfered step, see Sketch 2.1	m	ft
M_1	Mach number at edge of boundary layer in absence of excrescence		
P	function in Equation (3.1), see Appendix A		
Q	function in Equation (3.1), see Appendix A		
R_h	Reynolds number based on velocity and kinematic viscosity at edge of boundary layer in absence of excrescence, and h		
r	edge radius of step or ridge, see Sketch 2.1	m	ft
s	streamwise length of ridge, see Sketch 2.1	m	ft
u_τ	friction velocity $(\tau_w/\rho_w)^{1/2}$	m/s	ft/s
δ	$[= \delta(0.999)]$, local boundary-layer thickness, in absence of excrescence, for which velocity in boundary layer is 0.999 times that at edge of boundary layer	m	ft
ν_w	kinematic viscosity at wall in absence of excrescence	m ² /s	ft ² /s
ρ_w	density at wall in absence of excrescence	kg/m ³	slug/ft ³
τ_w	wall shear stress in absence of excrescence	N/m ²	lbf/ft ²

Issued November 1975
 With Amendments A to D, September 2016 – 39 pages
 This page Amendment C

2. GEOMETRIES CONSIDERED AND SCOPE OF ITEM

The two-dimensional excrescences considered in this Item are shown in Sketch 2.1. Insufficient data are available for the analysis of chamfered ridges.



Sketch 2.1

The charts given in this Item were derived from experimental data for excrescences transverse to the stream in a turbulent boundary layer on a flat wall (corresponding to a flat plate at zero incidence), covering the Mach number range up to 3; they apply to airflow with zero heat transfer and zero pressure gradient and are considered valid for excrescences with heights up to about one tenth³ of the boundary-layer thickness. Guidance on other aspects of applicability, including the presence of streamwise pressure gradients, is given in Section 4. Notes on the accuracy are given in Section 5.

A computer program of the method is provided as ESDUpac A7531 and is described in Appendix B.

3. PRESENTATION OF DATA

The equations and curves presented were derived essentially from two series of tests, one at RAE Bedford^{3,5} and the other at ARA⁶ on the same models covering a range of excrescence heights (1.27, 2.54 and 3.81 mm for the steps and 3.81 mm for the ridges). The RAE tests were for Mach numbers of 0.2 and 0.8 and 1.4 to 2.8 while the ARA tests covered the range from 0.6 to 1.4. Derivation⁴ presents formulae for the calculation of the parameters involved in the RAE correlation and these formulae are used as the basis of the present Item. Other supporting data are contained in Derivations 1 and 2.

This page Amendment D

In both the RAE and ARA tests the excrescences were mounted on the circular plate (of diameter 368 mm) of a floating-element drag balance set into the wind-tunnel wall. The excrescences were effectively of infinite span, end effects being avoided by means of extensions fixed to the wind-tunnel wall. The measured drag represents the load on the excrescence itself plus any changes in skin friction on the balance plate due to the presence of the excrescence less the skin friction on that part of the plate covered by the excrescence, but does not include the effect of drag changes induced downstream of the plate.

3.1 Plain Steps and Ridges

The incremental drag coefficient due to a plain step or ridge in two-dimensional flow may be represented by

$$C_D = (P + Q)C_f \quad (3.1)$$

The functions P (dependent on M_1 and R_h) and Q (dependent on M_1 and C_f) are developed in Appendix A and presented respectively in Figure 1a and 1b (forward-facing steps), Figure 2a and 2b (rearward-facing steps), Figure 3a and 3b (square ridges) and Figure 4a and 4b (rectangular ridges with $s/h = 2$).^{*} In application to flat plate flows with zero pressure gradient C_f may be obtained from ESDU 68020. For other applications see Section 4. The ranges of the experimental data on which the charts are based are shown in Table 4.1 and the appropriate ranges of R_h are indicated on the charts. Outside these ranges the charts should be used with caution.

At supersonic speeds data for plain steps and plain square ridges were available for at least three Mach numbers, and a trend with Mach number could be established. For rectangular ridges, however, the tests were performed at $M_1 = 1.4$ and 1.7 only; in the construction of Figure 4a and 4b a dependence on M_1 similar to that exhibited by square ridges was assumed and the supersonic range was terminated at $M_1 = 2$.

For forward-facing steps (Figure 1a), square ridges (Figure 3a) and rectangular ridges (Figure 4a), the charts for P have areas on them, indicated by hatched lines for which, as $\log_{10}(R_h)$ approaches 3 the charts may become less meaningful; indeed, a negative drag may be indicated in some instances. For these cases the implication is that Equation (A2.1) has been extrapolated beyond its limit of applicability and C_D should be taken to be zero.

Appendix A contains numerical data developed from Figures 1a to 4b that allow the calculation of C_D/C_f for steps and ridges in two-dimensional flow without the need to read from graphs.

3.2 Chamfered and Radiused Steps and Radiused Ridges

The drag coefficient of a chamfered step is presented in terms of that of a plain step of the same height. Thus,

$$C_D(\text{chamfered step}) = k_c C_D(\text{plain step}), \quad (3.2)$$

where the values of k_c are given in Figures 5 and 6 for forward-facing steps and rearward-facing steps respectively.

The drag coefficient of a radiused step or a radiused rectangular ridge is similarly presented. Thus,

$$C_D(\text{radiused step or ridge}) = k_r C_D(\text{plain step or ridge}), \quad (3.3)$$

^{*} For rectangular ridges of different s/h values, interpolation may be made between the data of Figures 3a and 4a, and Figures 3b and 4b, together with the assumption that for a rectangular ridge having $s/h > 5$ the drag is the sum of the drags of a forward-facing step and a rearward-facing step. This interpolation process should cater for a maximum C_D in the range $1 < s/h < 5$ for most combinations of M_1 , R_h and C_f .

where Figures 7, 8 and 9 show the values of k_r for forward- and rearward-facing steps and rectangular ridges respectively.

Figures 5 and 6 were compiled from experimental data^{3, 5, 6} relating to chamfers given by values of l/h of 0, 1, 5, 10 and 30. The $l/h = 10$ case was not tested for rearward-facing chamfers. Figures 7 and 9 were derived from data^{3, 5, 6} relating to radiusing given by values of r/h of 0, 1/3, 2/3 and 1, while Figure 8 was derived from data⁶ relating to radiusing given by values of r/h of 0, 1/3 and 2/3 for transonic speeds only. These values of chamfer angle and radius parameter are shown as appropriate on each of Figures 5 to 9. The experimental data were obtained from excrescences with heights, h , equal to the largest used for plain steps and ridges (3.81 mm). Fewer experimental data were available to construct Figures 6 and 8 which should be regarded as less soundly based than Figures 5, 7 and 9. The ranges of the experimental data on which Figures 5 to 9 are based are shown in Table 4.1. Outside these ranges the Figures should be used with caution.

Where only subsonic and supersonic data are available (Figures 5 and 7) the values of k_c and k_r are indicated by two hatched areas, each encompassing the effects of M_1 and R_h . Where transonic data are also available (Figures 6, 8 and 9) an attempt has been made to indicate mean lines for individual Mach numbers and to quote accuracy bands for these according to the spread of R_h covered in the test data. The presentations for the five configurations are described in Sections 3.2.1 to 3.2.5.

3.2.1 Chamfered forward-facing steps

Figure 5 gives the values of k_c for chamfered forward-facing steps. It shows two hatched areas for subsonic and supersonic speeds, which indicate the bounds of experimental data from Derivation 5 and supporting data from Derivation 1. No data are available for chamfered forward-facing steps at transonic speeds. There is a tendency at supersonic speeds for the higher Mach number data ($M_1 = 2.0$) to lie towards the upper boundary, and the lower Mach number data ($M_1 = 1.4$) towards the lower boundary. No consistent Reynolds number effects were in evidence. At subsonic speeds it is difficult to separate the effects of M_1 and R_h .

3.2.2 Chamfered rearward-facing steps

The data for k_c for chamfered rearward-facing steps, obtained from Derivations 5 and 6 for a roughness Reynolds number ($u_\tau h/\nu_w$) of 1000, were found to correlate in terms of M_1 as shown in Figure 6. Attention is drawn to three important features of Figure 6:

- (i) For small chamfer angles the factor k_c rises with Mach number up to $M_1 = 1.2$ and then decreases again.
- (ii) There exists a range of chamfer lengths (l) where the chamfer factor k_c is greater than 1.0, indicating a drag greater than that for a plain step. This occurs where the chamfer angle $\tan^{-1}(h/l)$ is between 8° and 50° ; the exact range and maximum value is Mach number dependent with the maximum value and range occurring at $M_1 = 1.2$. This effect is substantiated by low-speed data of Derivation 1.
- (iii) In practice, chamfering a rearward-facing step is only beneficial if the chamfer angle is small; however, as noted in (ii), the beneficial range for chamfer angles will be dependent on the onset Mach number (M_1).

Experimental data^{1, 5} are available for roughness Reynolds numbers down to a value of about 200. When expressed in the form of Equation (3.2) these data yield values of k_c which in the majority of cases lie within ± 0.2 of the appropriate curve in Figure 6. However because only very few data are available and because of the difficulty involved in measuring excrescence drag accurately at low roughness Reynolds numbers, no firm conclusions can be reached concerning the variation of k_c with roughness Reynolds number.

3.2.3 Radiused forward-facing steps

As there were no transonic data available, Figure 7 shows the values of k_r for forward-facing steps as two hatched areas, encompassing the effects of M_1 and R_h , for subsonic and supersonic speeds. At subsonic speeds the Mach number effect was negligible, the spread of data within the hatched area being almost entirely due to Reynolds number effects. The lower Reynolds number data ($R_h \approx 6 \times 10^3$) tended to lie towards the upper boundary while the higher Reynolds number data ($R_h \approx 5 \times 10^4$) tended to lie towards the lower boundary.

At supersonic speeds no Reynolds number effects were in evidence, and data for the higher Mach number ($M_1 = 2.0$) tended to lie towards the upper boundary and the data for the lower Mach number ($M_1 = 1.4$) towards the lower boundary.

3.2.4 Radiused rearward-facing steps

For this case, only the transonic data from Derivation 6 were available, covering the range $0.6 \leq M_1 \leq 1.4$ for roughness Reynolds numbers of about 1000 ($R_h \approx 5 \times 10^4$). It was impossible to determine any Mach number dependency and Figure 8 shows the value of k_r as a mean line through the data. The scatter over the Mach number range considered is up to ± 0.04 in k_r , the larger scatter occurring at the higher values of r/h .

3.2.5 Radiused rectangular ridges

Figure 9 shows the values of k_r for radiused rectangular ridges, deduced from the data of Derivations 5 and 6 for a roughness Reynolds number ($u_\tau h/\nu_w$) of 1000, given as mean lines for a range of Mach numbers.

Data⁵ are available for $u_\tau h/\nu_w = 200$ at subsonic and supersonic speeds only. For $r/h > 0.3$ there is evidence that reducing $u_\tau h/\nu_w$ to 200 reduces k_r by up to 0.08 for the supersonic cases ($M_1 \geq 1.4$) and increases k_r by up to 0.2 for the subsonic cases ($M_1 \leq 0.8$).

4. NOTES ON APPLICABILITY

The ranges of experimental data on which the charts are based are summarised in Table 4.1 and the appropriate ranges of R_h are indicated on Figures 1a to 4b. Outside these ranges the Figures should be used with caution.

Information for this Item was derived from two-dimensional data but for the specific case of ridges, tests on finite length square ridges¹ indicate that the Item may be valid for planform aspect ratios down to 8.

This Item is restricted to excrescences in the wall region of the boundary layer. The heights of the excrescences tested^{3, 5, 6} ranged up to about 0.05 times the boundary-layer thickness, but it is considered that the data may be applied to heights up to about one tenth of the boundary-layer thickness.

TABLE 4.1 Details of Experimental Data for Two-dimensional Steps and Ridges

(a) Plain

<i>Excrescence</i>	<i>Derivation</i>	M_1	h/δ	s/h	$C_f \times 10^3$	$\log_{10}(R_h)$
Forward-facing steps	1	0.07	0.04 to 0.3	—	2.6 and 3.0	3.6 to 4.4
	3, 5	0.2	0.01 to 0.03	—	1.7 to 2.2	3.3 to 4.7
	3, 5	0.8	0.01 to 0.03	—	1.6 to 2.2	3.3 to 4.7
	3, 5	1.4	0.01 to 0.04	—	1.5 to 2.0	3.3 to 4.7
	3, 5	1.7	0.04	—	1.5 to 1.8	3.9 to 4.5
	3, 5	2.2	0.02 to 0.05	—	1.2 to 1.6	3.3 to 4.6
	3, 5	2.8	0.02 to 0.05	—	1.0 to 1.4	3.1 to 4.7
	6	0.6 to 1.4	0.03 to 0.04	—	1.4 to 1.6	4.6 to 4.7
Rearward-facing steps	1	0.07	0.4 to 0.3	—	2.6 and 3.0	3.6 to 4.4
	3, 5	0.2	0.01 to 0.03	—	1.7 to 2.2	3.3 to 4.7
	3, 5	0.8	0.01 to 0.03	—	1.7 to 2.2	3.3 to 4.5
	3, 5	1.4	0.01 to 0.04	—	1.6 to 2.0	3.3 to 4.5
	3, 5	1.7	0.04	—	1.4 to 1.9	3.8 to 4.7
	3, 5	2.0	0.04	—	1.3 to 1.6	3.9 to 4.6
	3, 5	2.2	0.02 to 0.05	—	1.3 to 1.6	3.3 to 4.4
	3, 5	2.8	0.02 to 0.05	—	1.0 to 1.5	3.1 to 4.5
Ridges	1	0.07	0.03 to 0.16	0.1 to 19.6	2.6	3.6 to 4.2
	3, 5	0.2	0.01 to 0.03	1	1.7 to 2.2	3.3 to 4.7
	3, 5	0.8	0.01 to 0.03	1	1.8 to 2.2	3.3 to 4.4
	3, 5	1.4	0.01 to 0.04	1	1.7 to 2.4	3.3 to 4.3
	3, 5	2.2	0.02 to 0.05	1	1.3 to 1.6	3.3 to 4.5
	3, 5	2.8	0.02 to 0.05	1	1.0 to 1.5	3.1 to 4.5
	3, 5	0.2	0.03	2	1.7 to 2.2	3.8 to 4.7
	3, 5	0.8	0.03	2	1.8 to 2.2	3.8 to 4.4
	3, 5	1.4	0.04	2	1.6 to 2.2	3.6 to 4.3
	3, 5	1.7	0.04	2	1.4 to 1.9	3.8 to 4.6
	6	0.6 to 1.4	0.02 to 0.04	1	1.4 to 1.6	4.5 to 4.7
	6	0.6 to 1.4	0.03 to 0.04	2	1.4 to 1.6	4.6 to 4.7

(b) Chamfered

<i>Excrescence</i>	<i>Derivation</i>	M_1	h/δ	$\tan^{-1}(h/l)$ (deg)	$C_f \times 10^3$	$\log_{10}(R_h)$
Forward-facing steps	1	0.07	0.15	24 to 90	2.6	4.2
Rearward-facing steps	5	0.2, 0.8, 1.4 and 2.0	0.03 to 0.04	2, 6, 12, 45	1.4 to 2.2	3.8 to 4.7
	1	0.07	0.15	24 to 90	2.6	4.2
	5	0.2, 0.8, 1.7 and 2.0	0.03 to 0.04	2, 12, 45	1.4 to 2.2	3.8 to 4.7
	6	0.8, 1.0, 1.2 and 1.4	0.03 to 0.04	2, 12, 45	1.4 to 1.5	4.2 to 4.7

(c) Radiused

<i>Excrescence</i>	<i>Derivation</i>	M_1	h/δ	s/h	r/h	$C_f \times 10^3$	$\log_{10}(R_h)$
Forward-facing steps	5	0.2, 0.8, 1.4 and 2.0	0.03 to 0.04	—	1/3, 2/3 and 1.0	1.4 to 2.2	3.8 to 4.7
Rearward-facing steps	6	0.8, 1.0, 1.2 and 1.4	0.01 to 0.04	—	1/3 and 2/3	1.4 to 1.5	4.2 to 4.7
Ridges	1	0.07	0.02 to 0.37	2	1	2.6 and 3.0	3.2 to 4.5
	5	0.2, 0.8, 1.4, 1.7 and 2.0	0.03 to 0.04	2	1/3, 2/3 and 1.0	1.4 to 2.2	3.8 to 4.7
	6	0.8, 1.0, 1.2 and 1.4	0.03 to 0.04	2	1/3, 2/3 and 1.0	1.4 to 1.5	4.6 to 4.7

Although the Item was derived from excrescences in a turbulent boundary layer on a wind-tunnel wall with zero streamwise pressure gradient, the data would be expected to apply to excrescences on a parallel circular cylinder in axisymmetric flow provided that lateral curvature effects were small. In practical terms this condition is satisfied if the displacement thickness is less than five per cent (say) of the cylinder radius.

In flows with pressure gradients the incremental drag coefficient is changed (*i.e.* amplified or attenuated) compared with its flat plate value. Some allowance could be made for upstream effects on the drag due to the excrescence by using in Figures 1a to 4b and Equation (3.1) a local value of C_f that is consistent with the pressure gradient. This value might be calculated by means of theoretical methods such as References 8 to 11. Having done this it would still be necessary to account for the changes in drag that arise downstream as a result of the changes in boundary-layer characteristics. No immediate recommendation can be made on this, except at sub-critical Mach numbers where Reference 7 provides useful guidance. Note that in Reference 7 a correction factor was derived in which the *total* incremental drag coefficient due to an excrescence was based on the kinetic pressure in the free-stream, whereas the *local* incremental drag coefficient was based on the local kinetic pressure at the edge of the boundary layer, as in this Item.

5. ACCURACY

The ranges of the RAE and ARA tests^{3, 5, 6} are shown in terms of R_h on Figures 1a to 4b. For estimating the accuracy of this Item within these ranges the following table gives, approximately, the maximum differences between the experimental values of C_D and values predicted from this Item for plain steps and ridges. In this comparison the test results from the larger excrescences were primarily considered, since they yield the larger drags and should have provided the more reliable balance measurements. A similar examination covering all excrescence heights tested showed in some cases differences in C_D of as much as $\pm 30C_f$.

TABLE 5.1 Accuracy of Method for Plain Steps and Ridges

Excrescence Type	Deviation band on C_D		
	$M_1 \leq 0.8$	$0.8 \leq M_1 \leq 1.4$	$1.4 \leq M_1 \leq 3.0$
Forward-facing plain steps	$\pm 5C_f$	$\pm 10C_f$	$\pm 15C_f$
Rearward-facing plain steps	$\pm 5C_f$	$\pm 10C_f$	$\pm 10C_f$
Square or rectangular ridges	$\pm 10C_f$	$\pm 10C_f$	$\pm 15C_f$

For radiused and chamfered steps and ridges the accuracy is likely to be of the same order as shown in the table for those geometries for which measurements were made. For intermediate geometries however, and particularly for chamfered rearward-facing steps (Figure 6), the errors are possibly much greater.

6. DERIVATION AND REFERENCES

Derivation

The Derivation lists selected sources that have assisted in the preparation of this Item.

1. WIEGHARDT, K. Erhöhung des turbulenten Reibungswiderstandes durch Oberflächenstörungen.
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The References are sources of information supplementary to that in this Item.

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RAE tech. Rep. 72231, 1972.

7. EXAMPLE

A change in the skin thickness at a circumferential joint on an aircraft fuselage, of local radius 1.75 m, creates a forward-facing step of height 3.0 mm at a point in the turbulent boundary layer defined by $x = 10$ m, where x is measured along the surface from the virtual origin of the turbulent boundary layer. Given that the Mach number, velocity, kinematic viscosity and kinetic pressure at the edge of the boundary layer, in the absence of the step, are 0.7, 215 m/s, 2.9×10^{-5} m²/s and 1.204×10^4 N/m² respectively, check that this Item is applicable in that the ratio of step height to local boundary-layer thickness is less than about 0.1 and the local boundary-layer displacement thickness is less than five per cent of the local fuselage radius, then find the drag increment associated with this step. Find also the minimum drag reduction due to chamfering the step by 45°.

For the purposes of this example it will be assumed that the flow is axisymmetric. In addition, pressure gradient effects will be ignored, but it should be pointed out that they may not be negligible in practice.

To determine that the Item is applicable it is first necessary to estimate the thickness and displacement thickness of the boundary layer. These are determined from ESDU 68020 by first evaluating the Reynolds number based on x ,

$$R_x = 215 \times 10 / (2.9 \times 10^{-5}) = 7.41 \times 10^7.$$

Hence, from ESDU 68020, with $R_x = 7.41 \times 10^7$ (and in the Notation of that Item),

$$R_{\delta(0.999)} = 9.8 \times 10^5.$$

Thus $\delta(0.999) = 9.8 \times 10^5 \times 2.9 \times 10^{-5} / 215 = 0.132$ m and, therefore, $h/\delta(0.999) = 3.0 \times 10^{-3} / 0.132 = 2.3 \times 10^{-2}$, which is less than 0.1.

With $M_1 = 0.7$ and $R_x = 7.41 \times 10^7$ ESDU 68020 also yields, in the Notation of that Item,

$$R_{\theta_r} = 7.8 \times 10^4$$

and $H_r = 1.475.$

Hence from Equation (4.1) of ESDU 68020

$$R_{\delta^*} = 7.8 \times 10^4 \times 1.475 = 1.151 \times 10^5.$$

Thus, $\delta^* = 1.151 \times 10^5 \times 2.9 \times 10^{-5} / 215 = 0.0155$ m

and, therefore, $\delta^*/(\text{local fuselage radius}) = 0.0155/1.75 = 0.0089$, which is less than five per cent (0.05).

Therefore, since $h/\delta(0.999)$ is less than 0.1 and $\delta^*/(\text{local fuselage radius})$ is less than 0.05, this Item is applicable.

Also from ESDU 68020, with $M_1 = 0.7$ and $R_x = 7.41 \times 10^7$,

$$C_f = 0.0018.$$

The Reynolds number based on h ($3 \text{ mm} = 3 \times 10^{-3} \text{ m}$) is

$$R_h = 215 \times 3.0 \times 10^{-3} / (2.9 \times 10^{-5}) = 2.22 \times 10^4,$$

and therefore,

$$\log_{10}(R_h) = 4.35.$$

From Figures 1a and 1b with $M_1 = 0.7$, $\log_{10}(R_h) = 4.35$ and $C_f = 0.0018$,

$$P = 76$$

and $Q = 17$.

Hence, by Equation (3.1),

$$\begin{aligned} C_D &= (76 + 17) \times 0.0018 \\ &= 93 \times 0.0018 \\ &= 0.167. \end{aligned}$$

(See footnote.*)

The frontal area of the step $= \pi(1.753^2 - 1.75^2) = 0.03302 \text{ m}^2$.

Therefore as the kinetic pressure at the edge of the boundary layer is $1.204 \times 10^4 \text{ N/m}^2$ the drag of the step is $0.167 \times 0.03302 \times 1.204 \times 10^4 = 66.4 \text{ N}$.

For the effect of chamfering the step, reference to Section 3.2.1 indicates no consistent Mach number or Reynolds number trends within the subsonic hatched area of Figure 5. The minimum drag reduction is therefore predicted by the upper boundary of this hatched area.

With $\tan^{-1}(h/l) = 45^\circ$,

$$k_c \leq 0.57.$$

Hence, chamfering reduces the drag coefficient by at least

$$0.167 \times (1 - 0.57) = 0.072,$$

which corresponds to a drag reduction of 28.6 N.

* Numerical data in Appendix A allow the calculation of C_D/C_f without the need to use Figures 1a to 4b. In this case, see page 24, the numerical data yield $C_D/C_f = 93.8$; the difference from the value obtained graphically is well within both the reading accuracy of Figures 1a and 1b and the deviation band of ± 5 for $M_1 \leq 0.8$ quoted for this excrescence type in Section 5.

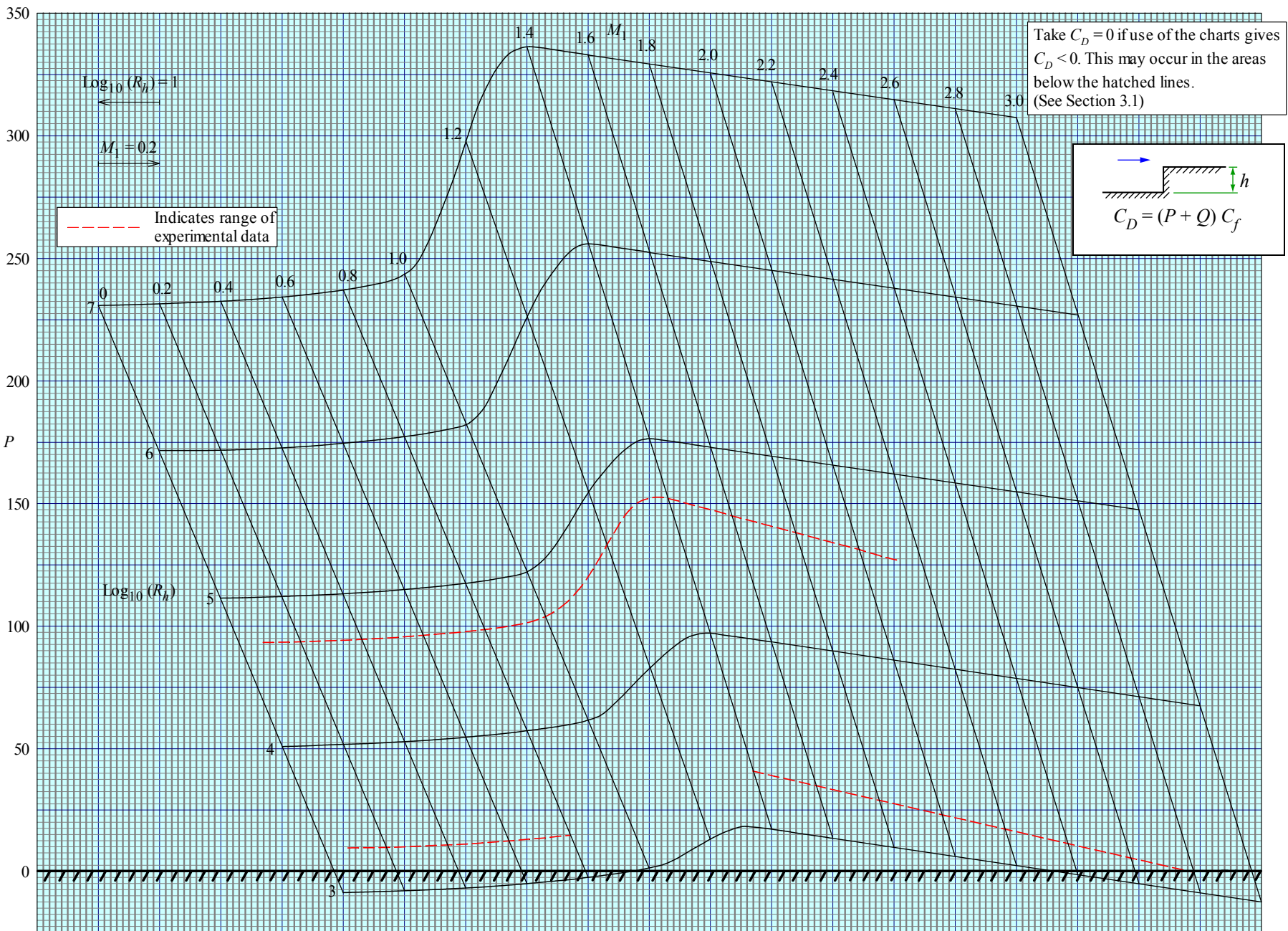


FIGURE 1a FUNCTION P FOR FORWARD-FACING STEPS