

ELASTIC STRESS CONCENTRATION FACTORS. DOUBLE RADIUS FILLETS IN SHOULDERED SHAFTS IN TORSION

1. NOTATION

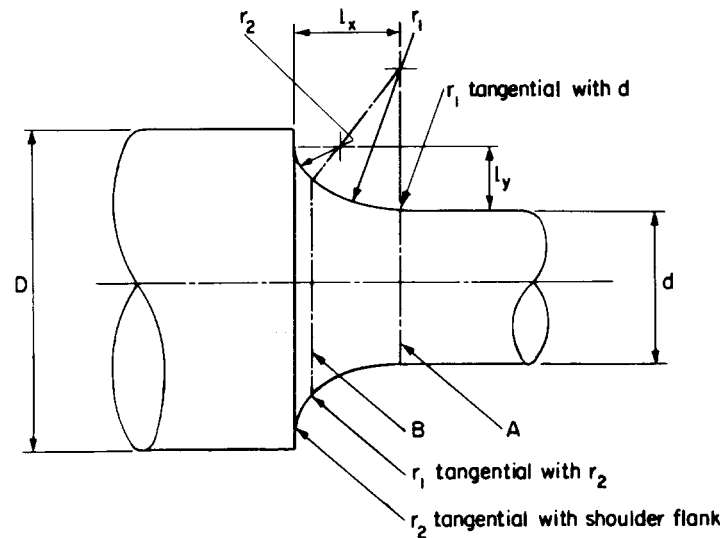
D	maximum diameter of shaft	m	in
d	minimum diameter of shaft	m	in
K'_A, K'_B	stress concentration factors at circumferential lines A and B, respectively (see Sketch 2.1), in double radius fillets defined by $q_A = K'_A q_{ref}$ and $q_B = K'_B q_{ref}$		
K'	stress concentration factor for single radius fillets defined by $q_{max} = K' q_{ref}$		
l_x	length of fillet in direction of shaft axis (see Sketch 2.1)	m	in
l_y	radial height of fillet (see Sketch 2.1)	m	in
q_A, q_B	shear stresses at circumferential lines A and B, respectively (see Sketch 2.1), in double radius fillets	N/m ²	lbf/in ²
q_{max}	maximum shear stress in shaft with single radius fillet	N/m ²	lbf/in ²
q_{ref}	$\frac{16T}{\pi d^3}$	N/m ²	lbf/in ²
r	radius of single radius fillet (equal to lesser of l_x or l_y)	m	in
r_1	fillet radius at end of double radius fillet that runs out on shaft	m	in
r_2	fillet radius at end of double radius fillet that runs out on shoulder flank	m	in
T	torque applied to shaft	N m	lbf in

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

The stress concentration factor at a shouldered shaft subjected to torsion can be reduced to any desired value by the introduction of a sufficiently large single radius fillet. In practice this is not always possible, usually due to restrictions on the axial length and the radial height of the fillet. In such circumstances the use of a carefully chosen double radius fillet, of the type shown in Sketch 2.1, can slightly reduce the stress concentration below that which can be obtained within the constraint on l_x and/or l_y with a single radius fillet of radius r . Depending upon r_1/r_2 and whether l_x or l_y governs the fillet, reductions to about $0.92K'$ and $0.8K'$ respectively are possible. Values of K' are given in Section 2 of Item No. 69021 "Elastic stress concentration factors, geometric discontinuities in rods and the tubes of isotropic materials".

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Sketch 2.1

In double radius fillets there are two separate peak concentrations of stress; they occur at the circumferential lines marked *A* and *B* on Sketch 2.1. Within the constraints on l_x and/or l_y , the lowest peak shear stress is obtained with the largest fillet for which $q_A = q_B$, that is $K'_A = K'_B$. Therefore, Figures 1 and 2 give r_1/d plotted against l_x/d and l_y/d for $r_1/r_2 = 3$ and 6, respectively, for the condition $K'_A = K'_B$ and Figure 3 gives the associated reduction in $K'_A/K' (= K'_B/K')$ plotted against r_1/r_2 .

These data relate to geometrically perfect fillets. The peak shear stress q_B occurs at, or close to, the point of common tangency between the two radii. It is necessary to have a good blend between the two radii at their point of common tangency to ensure that q_B does not exceed q_A . Small variations in r_1 and r_2 to accommodate manufacturing requirements should be considered to ensure that the full benefits of the double radius fillet are realised. This is particularly so with regard to r_2 since a 10 per cent reduction in r_2 only can lead to a 10 per cent increase in q_B .

3. DERIVATION

1. BATTENBO, H. Numerical stress concentrations for stepped shafts in torsion with circular and stepped fillets. *J. Strain Anal.*, Vol. 2, No. 2, pp. 90-101, February 1974.
BAINES, B.H.

4. EXAMPLE

It is required to provide a fillet having a stress concentration factor of less than 1.2 at a shoulder in a transmission shaft. The shaft diameters either side of the shoulder are 86 mm and 130 mm. A spacing washer carrying the maximum allowable 45° chamfer of 8 mm to accommodate the fillet is provided.

Assuming a single radius fillet with $r = l_x (= l_y$ since a 45° chamfer is provided) then

$$\frac{D}{d} = \frac{130}{86} = 1.51 \text{ and } \frac{r}{d} = \frac{8}{86} = 0.093.$$

Then, from Figure 2.10 of Item No. 69021, $K' = 1.29$. This value is too high so resorting to a double

radius fillet with a stress concentration factor of less than 1.2 requires that

$$\frac{K'_A}{K'} = \frac{K'_B}{K'} = \frac{1.2}{1.29} = 0.93 .$$

Since l_x and l_y are restricted to 8 mm maximum and since with a double radius fillet l_x will exceed l_y then $l_x = 8$ mm is the governing restriction. Then, from the upper curve of Figure 3 it can be seen that $r_1/r_2 = 3$ will satisfy this requirement.

Thus, from Figure 1 with l_x at the maximum allowable value of 8 mm giving $l_x/d = 0.093$ and following the path indicated on the figure by the chain dotted line,

$$\frac{r_1}{d} = 0.135 \quad \text{at which value} \quad \frac{l_y}{d} = 0.059 .$$

$$\begin{aligned} \text{Thus,} \quad r_1 &= 0.135 \times 86 &= 11.6 \text{ mm} , \\ l_y &= 0.059 \times 86 &= 5.1 \text{ mm} \\ \text{and} \quad r_2 &= 11.6/3 &= 3.87 \text{ mm} . \end{aligned}$$