

TRANSONIC DATA MEMORANDUM 76002

FIRST-ORDER METHOD FOR ESTIMATING THE SUBCRITICAL PRESSURE DISTRIBUTION ON A TWO-DIMENSIONAL AEROFOIL IN COMPRESSIBLE VISCOUS FLOW**SUMMARY**

The Memorandum outlines the principal features of a method (developed by Firmin at the RAE, Farnborough) for estimating the pressure distribution, force and moment coefficients and boundary layer and wake characteristics for a two-dimensional aerofoil in subcritical viscous flow.

A brief history of earlier attempts to account for the effects of viscosity is given, leading to a description of the current method which uses the first-order RAE Standard Method to calculate the inviscid flow over a profile comprising the aerofoil plus a displacement surface calculated by Thwaites' method for the laminar portion of the boundary layer and by the Head-Green method for the turbulent portion.

Some examples of pressure distributions obtained by a computer program of the method are compared with fully-corrected experimental data.

Reference is made to a detailed evaluation of the method that compared calculated results with experimental data selected from a range of wind tunnels.

1. INTRODUCTION

The development of methods for calculating the pressure distribution on two-dimensional aerofoils in compressible viscous flow was given an impetus in the mid-1960's by the facility provided by digital computers to iterate rapidly between successive approximations to the boundary-layer flow, the aerofoil pressure distribution (calculated in inviscid flow for the aerofoil-plus-displacement surface) and their mutual interaction. Just such an iterative scheme has been formulated by Firmin¹ – successfully consolidating and improving upon earlier ideas, to construct and program a method in which the inviscid theory is restricted to first order but which incorporates terms that account for the effects of wake curvature.

The accuracy of this method has now been assessed in some detail in Reference 2 by comparison with as-measured experimental data, the effects of tunnel constraint being taken into account in an extension of the method.

The basic formula is given in Section 3 and a brief history of earlier viscous flow methods that have led to the current method is given in Section 4. Sections 5 and 6 present an outline of the inviscid and viscous flow methods employed and their combination is described in Section 7; some brief details of computer programs of the method are given in Section 8. Finally, Section 9 presents comparisons with fully-corrected experimental data for one particular aerofoil and comments on some of the limitations of the method.

Issued December 1975
With Amendment A, September 1990

2. NOTATION

B	Wilby compressibility factor
C_L	lift coefficient
$C_{m^{1/4}}$	pitching-moment coefficient about $1/4$ -chord, positive nose-up
C_p	pressure coefficient
C_p^*	pressure coefficient for local Mach number of unity
H	viscous layer shape parameter ($= \delta^* / \theta$)
H_T	equivalent incompressible shape parameter
M_∞	freestream Mach number
q	non-dimensional total velocity
R_c	Reynolds number based on chord length
R_θ	Reynolds number based on momentum thickness
$S^{(n)}(x)$	functions in standard first-order equation for velocity on aerofoil
t	non-dimensional aerofoil thickness
u	non-dimensional perturbation velocity in x -direction
w	non-dimensional perturbation velocity in z -direction
x	non-dimensional coordinate in chordwise direction
x_T	chordwise location of transition from laminar to turbulent flow
z	non-dimensional coordinate measured from and normal to chord-line or its linear extension
α	incidence
α_E	effective incidence (see Section 7)
α_0	incidence for $C_L = 0$
γ	ratio of specific heats of air (taken to be 1.4)
$\Delta S^{(4)}(x)$	correction to $S^{(4)}(x)$ due to upwash correction
$\Delta w(x)$	upwash correction due to wake vorticity correction

$\Delta\gamma(x)$	vorticity correction due to wake curvature
δ^*	non-dimensional displacement thickness of boundary layer or wake
θ	non-dimensional momentum thickness of boundary layer or wake
ρ	non-dimensional aerofoil leading-edge radius

Suffixes

i	denotes incompressible flow value
l	denotes lower surface value
max	denotes maximum value
s	pertaining to camber
t	pertaining to thickness
u	denotes upper surface value
α	pertaining to incidence

3. THE BASIC FORMULA

Firmin's method calculates the inviscid flow over a profile comprising the aerofoil plus a displacement surface obtained from a boundary-layer calculation, using the equation

$$q = \frac{1 + \frac{1}{B}[S^{(1)}(x) \pm S^{(4)}(x) \pm \Delta S^{(4)}(x)] \pm \frac{\alpha_E}{B} \sqrt{\frac{1-x}{x}} \left[1 + \frac{S^{(3)}(x)}{B}\right] - \frac{1}{2} \frac{\alpha_E^2}{B^2}}{\left[1 + \frac{1}{B^2} \left(\frac{dz_t}{dx} \pm \frac{dz_s}{dx}\right)^2\right]^{1/2}} \quad (3.1)$$

in which the symbol $\pm$ refers to the upper and lower surfaces, the positive sign being taken for the upper surface and the negative sign for the lower surface. The definitions of the various functions are given in subsequent sections.

Equation (3.1) is derived from Equation (5.13) in Reference 3 by omitting most of the second-order terms. Consequently, although some second-order terms are retained, Equation (3.1) is usually referred to as being of first order (see Section 5). The term $\Delta S^{(4)}(x)$ has no counterpart in Reference 3 since it is introduced to account for the effects of vorticity in the wake which is treated by Firmin as a curved rotational flow (see Section 7).

4. HISTORICAL BACKGROUND

The basic proposition that the viscous flow around an aerofoil could be represented by the inviscid flow around a modified shape consisting of the aerofoil plus the boundary-layer displacement surface was first propounded by Preston⁴. He demonstrated the procedure⁵ by using measured boundary-layer data to

estimate the loss of lift caused by viscous effects, obtaining favourable comparisons with experiment. A similar demonstration was attempted by Brebner and Bagley⁶ on a more-modern aerofoil, again using carefully measured boundary-layer data.

Spence and Beasley⁷ used a similar method to calculate lift coefficients for six symmetrical aerofoils at incidence, but using calculated momentum and displacement thicknesses. They demonstrated satisfactorily the ability of their method to account for viscous effects on lift, including the effects of transition position and Reynolds number, provided that an adequate method was available to define the boundary layer.

All of these early workers in the field concentrated on predicting the loss of lift due to viscous effects. They recognised the possibility of calculating the detailed pressure distributions but were handicapped by the lack of fast computing devices and a sufficiently accurate viscous theory for the boundary layer and the lack of any suitable method to treat the wake.

The advent of digital computers led Wilkinson⁸, using the Head⁹ method for the turbulent boundary layer, and Powell¹⁰ and Lock *et al*¹¹, using the Nash and Macdonald¹² method, to construct iterative schemes to calculate the detailed pressure distribution over aerofoils in two-dimensional viscous flow. Wilkinson and Powell used modified forms of the approximate inviscid method of Weber¹³ while Lock *et al* used an improved form of the RAE Standard Method (which is the first-order method outlined in Reference 3). An improvement to Powell's method¹⁰ was achieved by Horton¹⁴, who replaced the Nash and Macdonald boundary-layer method with his own entrainment history method^{15, 16}.

Wilkinson, Powell, Lock *et al* and Horton followed earlier workers in assuming no pressure difference across the boundary layer at the trailing edge of the aerofoil or across the wake. This assumption, based on the theorem of Taylor¹⁷, had been proved incorrect by Preston¹⁸ but was not dispensed with until the curved rotational nature of the flow in the wake was taken into account by Firmin¹.

Firmin set up an iterative procedure, similar in concept to that of Lock *et al*¹¹, for the calculation of the pressure distribution, force and moment coefficients, the boundary layer and wake details and the pressure recovery downstream along the wake for a two-dimensional aerofoil. He used a numerical integration technique to enumerate a modified form (see Section 5) of the RAE Standard Method³ for the inviscid flow and Green's¹⁹ extension of Head's⁹ entrainment method for the turbulent viscous layers. The Head-Green entrainment method has been found to be superior to that of Nash and Macdonald¹², particularly in the calculation of displacement thickness, and it provides a solution for the wake characteristics.

In the current method Firmin uses a first-order inviscid flow solution, including some second-order terms, in the calculation of subcritical viscous flows. An analogous extension to supercritical flow without or with shocks has been prepared²⁰ using the RAE Transonic Small Perturbation^{21, 22} inviscid solution and a more advanced boundary-layer method²³.

5. THE INVISCID FLOW METHOD

The total velocity on the surface of an aerofoil, at incidence α radians to the freestream, is given by Equation (5.13) of Reference 3 as

$$q = \frac{1 + \frac{1}{B}[u_{it} \pm u_{is} \pm u_{i\alpha}] + \frac{1}{B^2}[(u_{itt} + u_{iss} + u_{is\alpha} - \frac{1}{2}\alpha^2) \pm (u_{ist} + u_{i\alpha t})]}{\left[1 + \frac{1}{B^2}\left(\frac{dz_t}{dx} \pm \frac{dz_s}{dx}\right)^2\right]^{1/2}}, \quad (5.1)$$

in which u_i is the incompressible perturbation velocity and subscripts s , t and α taken singly denote first-order terms due to camber, thickness and incidence respectively and in combination denote second-order terms; thus u_{ist} is the incompressible perturbation velocity due to the interaction between camber and thickness.

In Equation (5.1) the effects of compressibility are accounted for by the Wilby factor,

$$B = [1 - M_\infty^2(1 - M_\infty C_{pi})]^{1/2} \quad (5.2)$$

where $C_{pi} = (1 - q_i^2).$ (5.3)

For aerofoils with only moderate thickness and camber the second-order terms u_{itt} , u_{iss} , $u_{is\alpha}$ and u_{ist} may be omitted so that Equation (5.1) reduces to

$$q = \frac{1 + \frac{1}{B}[u_{it} \pm u_{is} \pm u_{i\alpha}] + \frac{1}{B^2}[-\frac{1}{2}\alpha^2 \pm u_{i\alpha t}]}{\left[1 + \frac{1}{B^2}\left(\frac{dz_t}{dx} \pm \frac{dz_s}{dx}\right)^2\right]^{1/2}} \quad (5.4)$$

which, although it retains some second-order terms, is usually called a first-order equation.

Equation (5.4) may be expressed in a form analogous to the first-order RAE Standard Method, the development of which from the original work of Küchemann and Weber²⁴ is described in Section 4.1 of Reference 3, by writing the individual components in terms of the Weber functions, $S^{(n)}(x)$, as shown in Reference 3, Equations (5.2) to (5.4) and (5.12). It then becomes

$$q = \frac{1 + \frac{1}{B}[S^{(1)}(x) \pm S^{(4)}(x)] \pm \frac{\alpha_E}{B} \sqrt{\frac{1-x}{x}} \left[1 + \frac{S^{(3)}(x)}{B}\right] - \frac{1}{2} \frac{\alpha_E^2}{B^2}}{\left[1 + \frac{1}{B^2}\left(\frac{dz_t}{dx} \pm \frac{dz_s}{dx}\right)^2\right]^{1/2}} \quad (5.5)$$

where B is given by Equations (5.2) and (5.3) in which q_i is obtained from Equation (5.5) by putting $B = 1.0$.

In Equations (5.1), (5.4) and (5.5) the alternative signs refer to the upper and lower surfaces respectively.

The functions $S^{(1)}(x)$, $S^{(3)}(x)$ and $S^{(4)}(x)$ are defined by integral relationships in Reference 3 (Equations (5.18), (5.20) and (5.21)). Two techniques for their enumeration are outlined in Reference 3; one is a direct numerical integration technique evolved by Jones²⁵ while the other is an approximate summation technique based on the Weber summation formulae (also defined in Reference 3). The slopes dz_l/dx and dz_s/dx may be evaluated by any convenient means, including the use of Weber summation formulae defined in Reference 3 by $S^{(2)}(x)$ and $S^{(5)}(x)$. The direct numerical integration technique of Jones has been incorporated into a computer program (program 121.203 in Reference 26) that provides both first- and second-order results.

A detailed evaluation of the inviscid program, in both first- and second-order forms, was made in Reference 27 by comparison with exact theories for a large number of cases. To provide examples of the results obtained by the method two cases are considered in Figures 1 and commented on in Section 9.1.

6. THE VISCOUS LAYER METHOD

Firmin calculates the laminar-flow boundary layer by Thwaites' approximation^{28, 29} with a transformation for compressible flow developed independently by Illingworth³⁰ and Stewartson³¹ which is described in Reference 32. No attempt is made to predict the transition position, a suitable value having to be specified as input to the program. The laminar calculation should be started at the stagnation point, but as this is not available exactly from the inviscid flow method (as used in the program) the point closest to the stagnation value is taken as the origin of the boundary layer[†].

For the turbulent flow regions there is a wide choice of methods which yield quite accurate results for the viscous layers as long as the flow on the aerofoil remains attached up to the trailing edge. In the present context the entrainment methods are particularly suitable because they combine accuracy with the speed of solution necessary for an iterative scheme, perhaps the most reliable of these being the entrainment history method of Green *et al*²³. However, in association with a subcritical first-order approximate solution, a simpler method is adequate. Thompson³³, in a critical review of methods existing in the 1960's, recommended Head's⁹ entrainment method and this, extended by Green¹⁹ to include the wake and account for compressibility, is used by Firmin in his program. Another very suitable method in frequent use is that of Horton^{15, 16}, who extended Head's method to include a lag entrainment equation.

For cases involving thin boundary layers at transition (*e.g.* with well-forward transition locations, and here it should be noted that the method has not been evaluated for transition locations further aft than 15% chord) the development of the turbulent boundary layer by the entrainment method is insensitive to the initial value of the shape parameter, H , and so the laminar value is used. Thus the resulting boundary layer represents a rapid change from laminar to turbulent flow at the transition point whereas in practice the transition would be gradual and spread over a small finite distance. It has been assumed that the growth of the momentum thickness is continuous through the transition point, although a facility to specify a jump has been included in the program. This jump can be used to raise the value of R_θ at transition to 320, considered to be the minimum for which turbulent flow is possible[‡].

Figure 2, described in Section 9.2, shows comparisons of displacement and momentum thicknesses on the aerofoil and in the wake obtained from Firmin's method (*i.e.* using a calculated pressure distribution and the Head-Green boundary-layer method) and experiment. In this example the agreement is very good, but it is pertinent to record that comparatively little experimental data exist for comparisons of this type to be made.

[†] The ARA version (program 230.215 in Reference 26) uses an interpolation procedure to determine the location of the stagnation point.

[‡] The value of R_θ is not yielded directly by the program and must be obtained manually.