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Geometrical properties of cranked and straight-tapered wing planforms



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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU Aero W.00.01.01, was monitored and guided by the Aerodynamics Committee. This Committee first met in 1942 and now has the following membership:

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GEOMETRICAL PROPERTIES OF CRANKED AND STRAIGHT-TAPERED WING PLANFORMS

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GEOMETRICAL PROPERTIES OF CRANKED AND STRAIGHT-TAPERED WING PLANFORMS

1. NOTATION AND UNITS (see Sketches 2.1, 3.1 and 3.2)

		SI	British
A	aspect ratio, b^2/S		
b	wing span	m	ft
c	local chord parallel to wing centre-line	m	ft
$\bar{c}$	geometric (or standard) mean chord	m	ft
$\bar{\bar{c}}$	aerodynamic mean chord	m	ft
c_i	chord at i^{th} crank, <i>i.e.</i> outboard chord of i^{th} wing panel	m	ft
c_0	centre-line chord	m	ft
c_t	tip chord (also denoted as c_{m+1})	m	ft
m	number of cranks in half-wing, excluding crank at wing centre-line (see Section 3)		
S	wing area, $b\bar{c}$	m ²	ft ²
S_i	area of i^{th} wing panel, $\eta_i(b/2)\bar{c}_i$	m ²	ft ²
s_i	spanwise distance, measured from axis of symmetry and normal to wing centre-line, of chord c_i	m	ft
x_0	chordwise distance, parallel to wing centre-line, of leading edge aft of wing apex	m	ft
x_{0i}	chordwise distance, parallel to wing centre-line, of leading edge of chord c_i aft of wing apex	m	ft
$\bar{\bar{x}}_{1/4}$	chordwise distance, along wing centre-line, of quarter-chord point of aerodynamic mean chord aft of wing apex	m	ft
y	spanwise coordinate, measured from axis of symmetry and normal to wing centre-line	m	ft
η_i	spanwise extent of i^{th} wing panel as fraction of semispan, $\frac{s_i - s_{i-1}}{b/2}$		

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λ	taper ratio, c_t/c_0		
λ_i	taper ratio of i^{th} wing panel, c_i/c_{i-1}		
$\Lambda_{n,i}$	sweepback of n^{th} chord line, between spanwise stations s_{i-1} and s_i for $i = 1, 2, \dots, m$	deg	deg

Subscripts

i	denotes properties associated with wing panel between chords c_{i-1} and c_i , i.e. i^{th} panel, or its outboard end.
j	dummy panel identifier used in calculation of $\bar{x}_{1/4}$ (Equation (3.2))
n	denotes properties associated with n^{th} chord line (n is fraction of local chord measured from leading edge) $n = 0$ denotes leading edge $n = 1/4$ denotes quarter-chord line $n = 1$ denotes trailing edge
p	denotes properties associated with p^{th} chord line (Equation (5.4))

2. INTRODUCTION

Aerodynamic calculations on wings often require the use of various reference lengths, areas and reference points, see Derivation 1. Here, a 'wing' is taken to be the usual pair of symmetrically disposed identical half-wings, such as in Sketches 2.1, 4.1 or 5.1. The main text of this Item presents information concerning the reference lengths and reference points commonly used for cranked and uncranked wings with straight edges and chordwise or pointed tips, see Derivation 2. Information concerning the representation of cranked wings by means of an equivalent straight-tapered wing planform is given in Addendum A which has been worded so that it can be used independently of the main text of this Item.

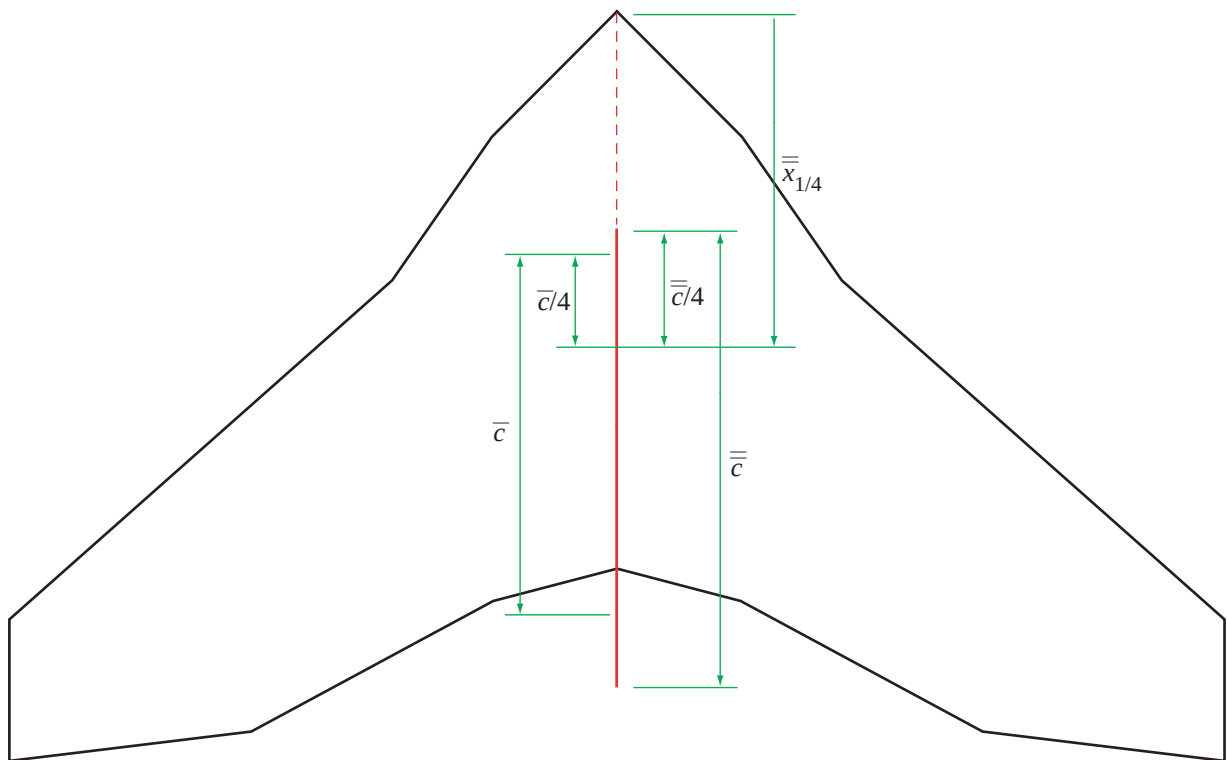
Thin aerofoil theory carries the implication that the aerodynamic centre of each chordwise section of a wing is located at the local quarter-chord point. Further, if the sectional lift coefficient due to incidence is assumed to be uniform spanwise, the aerodynamic centre will be calculated to lie at the quarter-chord point of the aerodynamic mean chord, positioned on the wing centre-line. Although this result is invalid for the spanwise loadings that occur in practice, it is through these simple concepts that the aerodynamic centre has become classically linked with the quarter-chord point of the aerodynamic mean chord. Consequently, pitching moments are frequently related to an axis through the quarter-chord point of the aerodynamic mean chord and are often non-dimensionalised with respect to this chord. It is emphasised that in practical situations the aerodynamic centre of a wing does not usually occur at the quarter-chord point of the aerodynamic mean chord*, and the latter is used as a reference point purely as a matter of convenience.

The geometric (or standard) mean chord of a wing is also used as a reference length to non-dimensionalise pitching moment coefficients. In practice, for convenience, this chord is positioned on the wing centre-line so that its quarter-chord point is coincident with the quarter-chord point of the aerodynamic mean chord. The positioning of the reference aerodynamic and geometric mean chords on the wing planform centre-line is illustrated in Sketch 2.1. It is important to note, however, that the aerodynamic mean chord is not associated with a spanwise location. The aerodynamic mean chord is defined from consideration of pitching moments and thus has a longitudinal position only.

* The aerodynamic centre of a straight-tapered wing in inviscid flow can be found from ESDU 70011 and ESDU 70012.

The aerodynamic and geometric mean chords and the chordwise position of their quarter-chord points with respect to the wing apex are expressed in generalised form for multi-crank wings in Section 3, while Sections 4 and 5 deal respectively with the special cases of single-crank wings and straight-tapered wings. The geometrical properties of straight-tapered wings are given in both analytical and graphical form. For a straight-tapered wing, Addendum B gives a simple geometrical construction for obtaining the size and longitudinal location of the aerodynamic mean chord.

Derivation 2 supplies analysis which provides the derivation and substantiation of equations given here for the geometrical properties of straight-tapered and multi-panel wings.

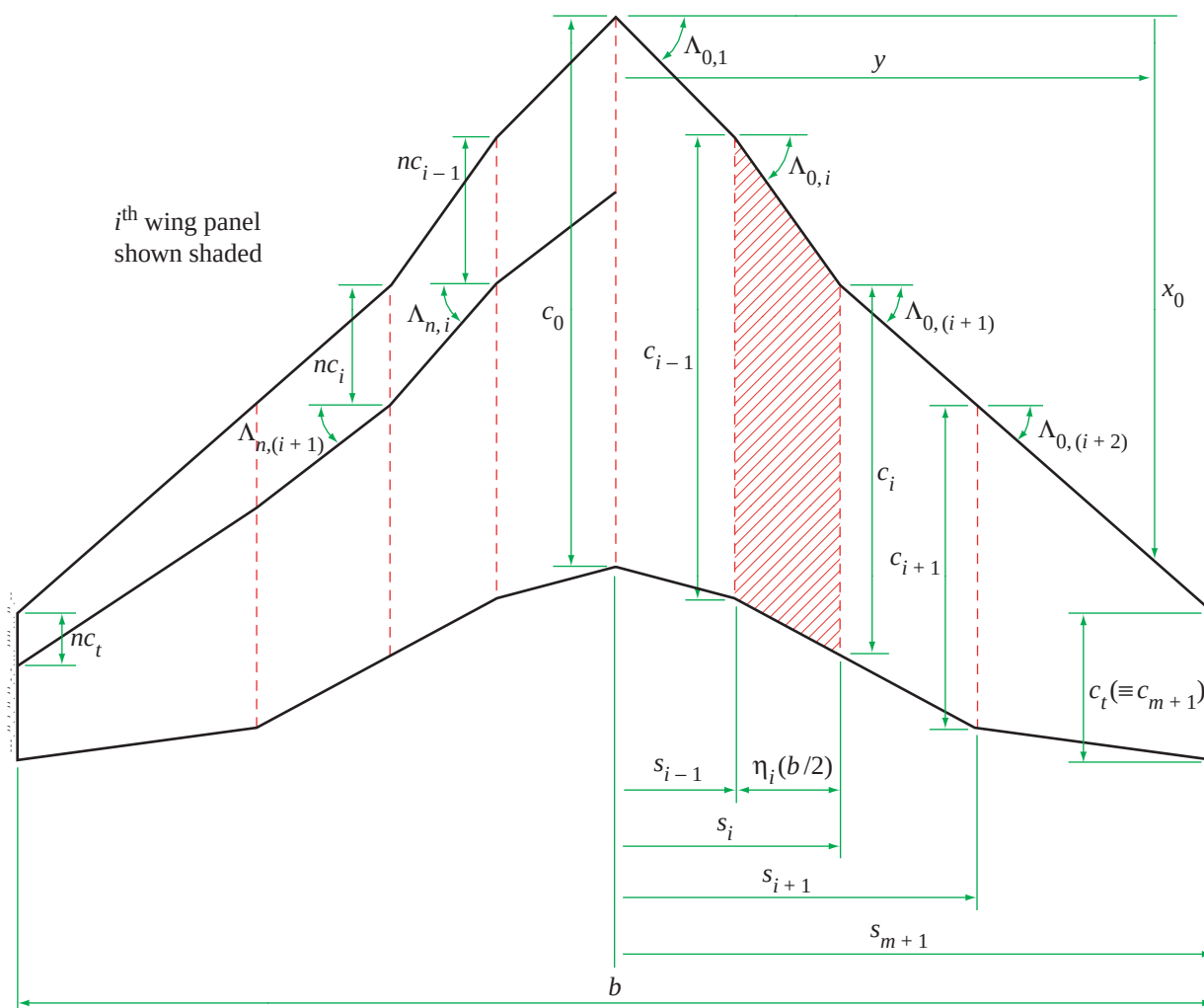


Sketch 2.1 Position of reference chords

3. GENERALISED MULTI-CRANK WING

3.1 General

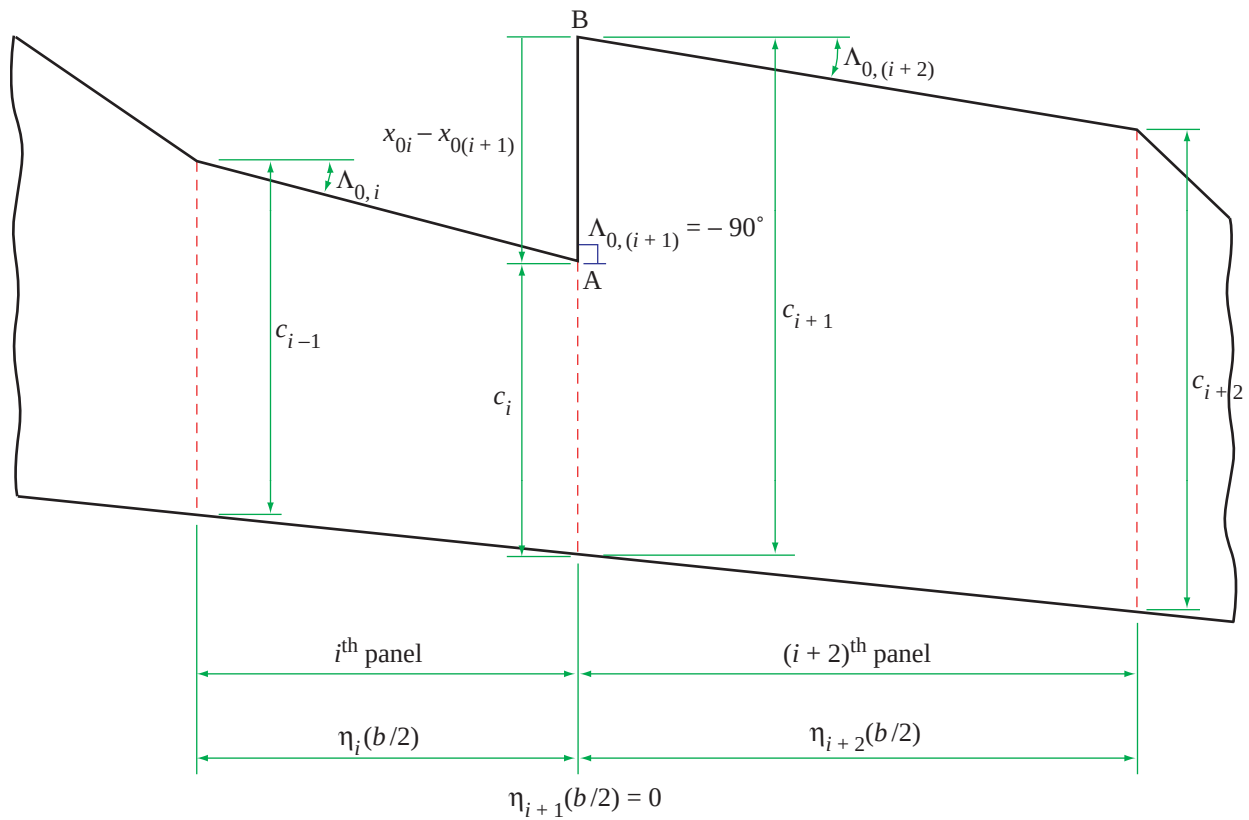
In general there can be any number of cranks in the wing leading edge and a different number, occurring at different spanwise stations, in the trailing edge. For the purposes of calculation, wherever there is a crank in one edge a crank is also assumed to exist in the other edge at the same spanwise station, so that the wing planform becomes divided into a series of trapezoidal panels. Of course, it may happen that some of the cranks in one edge or the other are dummies at which there is no change in sweep. Sketch 3.1 shows the planform of a multi-crank wing, and illustrates the notation employed when panel sweep angles are all less than 90° . If there are cranks at m distinct spanwise stations on a half-wing, excluding the wing centre-line, the half-wing is divided into $m + 1$ trapezoidal panels. When $m = 0$ the half-wing has straight leading and trailing edges.



Sketch 3.1 Multi-crank wing

On some wing planforms one edge (usually the leading edge) may have a sawtooth, so that it has two cranks at the same spanwise station, the sweep between them being $\pm 90^\circ$. In order to be able to use the same equations for such planforms, a wing panel of zero span is assumed to exist at a sawtooth. Sketch 3.2 illustrates the notation of chords and sweeps that applies when there is a sawtooth at the outboard end of the i^{th} panel and a sweep of -90° between the i^{th} and $(i+1)^{\text{th}}$ cranks (points A and B). In the sketch the $(i+1)^{\text{th}}$ panel has zero span.

When there are panel sweeps of $\pm 90^\circ$ in the leading or trailing edge of a wing, m is equal to the number of spanwise stations on the half-wing at which there are cranks (excluding the wing centre-line), with stations where there are sawteeth being counted twice. With this system of notation, the equations developed in this Item may be applied directly to planforms with panel sweeps of $\pm 90^\circ$, with the exception that in Equation (3.2) the term $\eta_i \tan \Lambda_{0,i}$ is set equal to its limiting value of $(x_{0i} - x_{0(i-1)})/(b/2)$ if $\Lambda_{0,i} = \pm 90^\circ$. Similarly, $\eta_j \tan \Lambda_{0,j}$ is set equal to $(x_{0j} - x_{0(j-1)})/(b/2)$ if $\Lambda_{0,j} = \pm 90^\circ$.



Sketch 3.2 Wing with Sawtooth

3.2 Aerodynamic Mean Chord

The aerodynamic mean chord of a wing is defined by the equation

$$\bar{c} = \frac{\int_0^{b/2} c^2 dy}{\int_0^{b/2} c dy}.$$

For a multi-crank wing this can be written

$$\bar{c} = \frac{\sum_{i=1}^{m+1} \bar{c}_i S_i}{\sum_{i=1}^{m+1} S_i}, \text{ where } \bar{c}_i = \frac{1}{S_i} \int_{s_{i-1}}^{s_i} c^2 dy.$$

It follows that

$$\bar{c} = \frac{2}{3} \left\{ \frac{\sum_{i=1}^{m+1} c_{i-1}^2 (1 + \lambda_i + \lambda_i^2) \eta_i}{\sum_{i=1}^{m+1} c_{i-1} (1 + \lambda_i) \eta_i} \right\}. \quad (3.1)$$

3.3 Chordwise Position of Aerodynamic Mean Chord

The location of the aerodynamic mean chord (positioned on the wing centre-line) is defined by establishing the chordwise position of its quarter-chord point. Relative to the wing apex this position is given by

$$\bar{x}_{1/4} = \frac{\int_0^{b/2} \left(x_0 + \frac{c}{4} \right) c \, dy}{\int_0^{b/2} c \, dy}.$$

For a multi-crank wing $\bar{x}_{1/4}$ can be written

$$\bar{x}_{1/4} = \frac{\sum_{i=1}^{m+1} \bar{x}_{1/4i} S_i}{\sum_{i=1}^{m+1} S_i},$$

where $\bar{x}_{1/4i} = x_{0(i-1)} + (x_{0i} - x_{0(i-1)}) \frac{(1 + 2\lambda_i)}{3(1 + \lambda_i)} + \frac{\bar{c}_i}{4}$

and $S_i = \frac{\eta_i b (1 + \lambda_i) c_{i-1}}{4}.$

If there are no sawteeth in the wing (i.e. $s_i \neq s_{i-1}$), $x_{0i} - x_{0(i-1)}$ can be written

$$x_{0i} - x_{0(i-1)} = (s_i - s_{i-1}) \frac{(x_{0i} - x_{0(i-1)})}{(s_i - s_{i-1})} = \eta_i \left(\frac{b}{2} \right) \tan \Lambda_{0,i},$$

and $x_{0(i-1)} = \sum_{\substack{j=1 \\ (i>1)}}^{i-1} (x_{0j} - x_{0(j-1)}) = \sum_{\substack{j=1 \\ (i>1)}}^{i-1} \eta_j \left(\frac{b}{2} \right) \tan \Lambda_{0,j}.$

It follows that

$$\bar{x}_{1/4} = \frac{b}{6} \left\{ \frac{\sum_{i=1}^{m+1} \eta_i c_{i-1}}{\sum_{i=1}^{m+1} c_{i-1} (1 + \lambda_i) \eta_i} \left[3(1 + \lambda_i) \sum_{\substack{j=1 \\ (i>1)}}^{i-1} \eta_j \tan \Lambda_{0,j} + (1 + 2\lambda_i) \eta_i \tan \Lambda_{0,i} \right] + \frac{\bar{c}}{4} \right\}$$

or, as $A = b^2/S$

$$\bar{x}_{1/4} = \frac{A}{12} \left\{ \sum_{i=1}^{m+1} \eta_i c_{i-1} \left[3(1 + \lambda_i) \sum_{\substack{j=1 \\ (i>1)}}^{i-1} \eta_j \tan \Lambda_{0,j} + (1 + 2\lambda_i) \eta_i \tan \Lambda_{0,i} \right] \right\} + \frac{\bar{c}}{4} . \quad (3.2)$$

If there are sawteeth in the wing, the term $\eta_i \tan \Lambda_{0,i}$ in Equation (3.2) is replaced by its limiting value of $(x_{0i} - x_{0(i-1)})/(b/2)$ whenever $\Lambda_{0,i} = \pm 90^\circ$. Similarly, $\eta_j \tan \Lambda_{0,j}$ is replaced by $(x_{0j} - x_{0(j-1)})/(b/2)$ if $\Lambda_{0,j} = \pm 90^\circ$.

In Equation (3.2) $\bar{x}_{1/4}$ is expressed in terms of panel leading-edge sweeps. Should the sweeps of the n^{th} chord lines be known instead, then the relationship

$$\tan \Lambda_{0,i} = \tan \Lambda_{n,i} + \frac{2nc_{i-1}(1 - \lambda_i)}{\eta_i b}$$

may be used for individual panels to calculate the leading-edge sweeps from the n^{th} chord sweeps.

3.4 Geometric (or Standard) Mean Chord

The geometric (or standard) mean chord of a wing is defined by the equation

$$\bar{c} = \frac{\int_0^{b/2} c \, dy}{\int_0^{b/2} dy} .$$

For a multi-crank wing this can be written

$$\bar{c} = \frac{\sum_{i=1}^{m+1} \int_{s_{i-1}}^{s_i} c \, dy}{\sum_{i=1}^{m+1} \int_{s_{i-1}}^{s_i} dy} ,$$