

FIRST APPROXIMATION TO TAKE-OFF FIELD LENGTH OF MULTI-ENGINED TRANSPORT AEROPLANES

1. NOTATION AND UNITS*

		SI	British
C_L	lift coefficient, $L / \frac{\rho}{2} \left(\frac{rV}{3600} \right)^2 S$		
D	drag	N	lbf
F_N	total installed net thrust	N	lbf
f_N	installed net thrust of one engine	N	lbf
g	acceleration due to gravity	m/s ²	ft/s ²
k	ratio V_1/V_2		
L	lift	N	lbf
m	aeroplane mass	kg	slug
n	number of engines installed		
r	conversion factor (1000 m/km, 1852 m/n.mile, 6076.12 ft/n.mile)	m/km, m/n.mile	ft/n.mile
S	wing area	m ²	ft ²
s	horizontal distance	m	ft
V	true airspeed	km/h, kn	kn
V_{MCG}	minimum directional control speed on the ground (true airspeed)	km/h, kn	kn
V_R	rotation speed (true airspeed)	km/h, kn	kn
V_{screen}	speed at screen with all engines operating (true airspeed)	km/h, kn	kn
V_1	decision speed (true airspeed)	km/h, kn	kn
V_2	take-off safety speed (true airspeed)	km/h, kn	kn
W	aeroplane weight, mg	N	lbf

* It is customary British practice to work in terms of aeroplane weight; however, when using SI units aeroplane mass is used. The equations and expressions in this Item are presented in terms of aeroplane weight only, but they are equally applicable if aeroplane mass is used provided that the substitution $W = mg$ is made.

γ_c	climb angle	degree	degree
ρ	density of air	kg/m ³	slug/ft ³
σ	relative density, ρ/ρ_0		

Subscripts

<i>seg 2</i>	denotes value in second segment take-off climb (see Section 3)
<i>stall</i>	denotes value at stall
<i>stop</i>	denotes average value during deceleration phase of abandoned take-off
0	denotes value at sea level in International Standard Atmosphere (Item Nos 68046 and 72018 give numerical values)
0.7	denotes value at $0.7V_2$
1	denotes value at V_1
2	denotes value at V_2

2. INTRODUCTION

The Item presents data in Figures 1 to 4 for obtaining first approximations to the take-off field length of multi-engined transport aeroplanes for take-off over a 35 ft (10.7 m) screen from hard runways of zero gradient in still-air conditions*. The “basic parameter” used to correlate field-length data in Figures 1 and 2 is

$$\frac{W^2}{C_{L_2} n f_{N_{0.7}} S \sigma}$$

This parameter is derived in Appendix A, Section A1 and a physical interpretation is obtained by noting that it is proportional to the distance taken to accelerate the aeroplane from rest to the take-off safety speed[†], V_2 , assuming a constant acceleration, zero retarding force and zero change of potential energy (see also Derivations 1, 2 and 5). A modified form of the basic correlation parameter is derived in Appendix A, Section A2 and used in Figure 3 to provide a means of correlating data for different levels of deceleration during an abandoned take-off. The effect of the aeroplane thrust-minus-drag in the airborne phase of the take-off is accounted for by using the second segment climb gradient, $\tan \gamma_{c_{seg 2}}$, as a convenient, secondary correlating parameter in Figures 1 to 3 (see Section 3).

Values of thrust, f_N , for jet and fan engines may be estimated using References 10 and 11 and propeller thrusts using Reference 12. See Section 6 for detailed information concerning the correlations and the original data for field length, take-off safety speed, thrust and climb gradient.

* See Item No. 82033 for take-off distance to 50 ft (15.2 m) for light and general aviation aeroplanes.

† The take-off safety speed, V_2 , is the minimum permitted speed at the screen during a continued take-off with one engine failed.

3. TAKE-OFF FIELD LENGTH AND SECOND SEGMENT CLIMB GRADIENT FOR CIVIL TRANSPORT AEROPLANES

For the purposes of this Item, the take-off field length may be defined as the greater of the following distances (see References 8 and 9):

- (i) 1.15 times the unfactored, all-engines-operating distance to 35 ft screen height,
- (ii) the balanced field length with a single engine failure.

The data from which values of (i) and (ii) may be obtained are presented in Sections 4 and 5 respectively.

For the period immediately after take-off, References 8 and 9 divide the flight path into several segments in each of which the aeroplane steady climb gradient must equal or exceed a specified minimum value. For the second segment climb the minimum gradients are given in the following table.

n	2	3	4
$\tan \gamma_{c_{seg\ 2}}$	0.024	0.027	0.03

The associated conditions at which the aeroplane must meet the required minimum second segment climb gradient are as follows:

- (i) aeroplane configuration as for take-off but with undercarriage retracted,
- (ii) critical engine inoperative,
- (iii) speed V_2 .

Climb gradient can be calculated from Equations (3.7) and (3.9) of Item No. 70023 which take full account of the magnitudes and directions of the forces acting on the aeroplane. However, for the purposes of the present Item satisfactory results should be obtained by using the following approximate relationship

$$\tan \gamma_{c_{seg\ 2}} \cong \left(\frac{F_N - D}{L} \right)_2 \cong \frac{(n - 1)f_{N_2}}{W} - \left(\frac{D}{L} \right)_2. \quad (3.1)$$

In Equation (3.1), $(D/L)_2$ corresponds to the aeroplane configuration defined above and, ideally, should include the effects of asymmetric flight. See Sections 6.2 and 6.4 regarding the use made of second segment climb data in this Item. See References 10, 11 and 12 for thrust estimation methods.

4. UNFACTORED ALL-ENGINES-OPERATING DISTANCE TO 35 ft (10.7 m)

Figure 1 gives values of the *unfactored*, all-engines-operating distance from the start of the ground run to a screen height of 35 ft (10.7 m) above the plane of the runway. Note that although Figure 1 deals with the all-engines-operating distance, the correlation makes use of $\tan \gamma_{c_{seg\ 2}}$ and V_2 (or C_{L_2}) which relate to an engine-failed take-off. During a normal, all-engines-operating take-off the rate of pitch used is usually insufficient to prevent the speed at the screen, V_{screen} , exceeding V_2 . The speed increment $(V_{screen} - V_2)$ is greatest for two-engined aeroplanes and may be expected to decrease as the number of engines is increased (see Section 6.2).

5. BALANCED FIELD LENGTH

Values of the balanced field length are given in Figures 2 and 3 for transport-type aeroplanes. This is the most frequently used measure of take-off performance when the effects of an engine failure during take-off are considered. The balanced field length occurs at the condition where, following an engine failure, the distance taken to abandon the take-off and to stop is equal to the distance taken to continue the take-off and to attain the safety speed, V_2 , at the screen height. The current, practical method of defining (and, during take-off, of identifying) this condition is via the decision speed, V_1 , which is related to the engine failure speed.

Whether the balanced field length is viewed in terms of the continued or the abandoned take-off, the significant variables are the number of engines, n , and the speed V_1 . The larger the value of n , the smaller is the effect of an engine failure. The value of V_1 is determined by the ability of the aeroplane to accelerate after an engine failure relative to its ability to stop, when using the available means of retardation, see Figure 4.

Owing to regulatory limitations on the range of values that V_1 may take, it is not always possible to obtain a balanced field length. If a relatively high level of deceleration is possible, the value of V_1 required for a balanced field length may be greater than the rotation speed, V_R , and, since the regulations do not permit $V_1 > V_R$, the critical case becomes the continued take-off* with $V_1 = V_R$ (at speeds greater than V_R the aeroplane is considered committed to take-off). Alternatively, if the available level of deceleration is relatively low, or the thrust-to-weight ratio high, the value of V_1 for a balanced field length may be less than the minimum directional control speed (with the critical engine inoperative) on the ground, V_{MCG} . In such instances the critical case is the abandoned take-off* with $V_1 = V_{MCG}$.

The correlations of balanced field length given in Figures 2 and 3 are considered separately in Sections 5.1 and 5.2. Both correlations apply to the estimation of balanced field length for a 35 ft (10.7 m) screen height and both can be used to obtain estimates in accordance with the requirements of References 8 and 9.

Note, however, that Reference 9 requires consideration of two balanced field cases associated with

- (i) a dry runway and a 35 ft (10.7 m) screen height,
- (ii) a wet runway and a screen height not less than 15 ft (4.6 m).

In general, the balanced field length obtained in case (ii) is close to that in case (i) and, therefore, when using this Item to estimate field lengths in accordance with the requirements of Reference 9 it is recommended that the distance obtained for case (i) be assumed to apply to case (ii) also.

5.1 Balanced Field Length Using the Basic Parameter

Figure 2 presents balanced field length data for typical certification requirements plotted against the basic parameter for lines of constant second segment climb gradient. In this Figure the differences that exist between various sets of Certification Requirements are not accounted for. However, the effect of such differences on the balanced field length is usually less than the scatter of the data used to construct Figure 2 (see Section 6.1). For cases where it is required to consider the effects on the balanced field length of different levels of deceleration, see Section 5.2.

* The usual effect of these limits is to increase the field length by a relatively small amount from the "balanced" value. Section 5.2 considers the case of continued take-off from V_R but the Item does not consider separately the abandoned take-off from V_{MCG} . However, the latter can result in field lengths much greater than those of Figures 2 and 3 if V_{MCG} is unusually high relative to V_2 and if the level of deceleration available is relatively low.

5.2 Balanced Field Length Using a Modified Correlating Parameter

Figure 3 presents curves of balanced field length for constant values of $\tan \gamma_{c_{seg2}}$ plotted against the modified correlating parameter

$$\frac{W^2}{C_{L_2} n f_{N_{0.7}} S \sigma} \left(\frac{n - k^2}{n - 1} \right)$$

where k is the ratio V_1/V_2 (see Appendix A, Section A2). The factor $(n - k^2)/(n - 1)$ is included to take account of the effects of different levels of deceleration during an abandoned take-off and, in addition, permits the inclusion of data for the case of the continued take-off with $V_1 = V_R$. From Section 5 the value of k may be expected to vary with the ratio

$$\frac{\text{mean acceleration after engine failure}}{\text{mean deceleration using the available means of retardation}}.$$

In Figure 4 the following approximation to this ratio,

$$\frac{(n-1)f_{N_{0.7}}}{W} \frac{g}{\ddot{s}_{stop}},$$

is used to correlate values of k for a range of aeroplane types and levels of deceleration. Figure 4 presents values of k for both the balanced field and the continued take-off (with $V_1 = V_R$) cases when associated with a 35 ft (10.7 m) screen height; two curves are given, corresponding to the aeroplane maximum and minimum take-off weights. Note that the average deceleration, $\ddot{s}_{stop}$, applies to the part of the abandoned take-off during which the wheel brakes are applied and beginning about four seconds after the engine failure occurs. The use of Figure 4 in conjunction with Figure 3 allows values of field length to be obtained for a screen height of 35 ft for which the available level of deceleration during an abandoned take-off may itself be specified. However, if the user wishes to estimate the balanced field length on a wet runway when coupled with a reduced screen height (*i.e.* as required in Reference 9), he should follow the advice of Section 5.

In the absence of more precise information, $\ddot{s}_{stop}$ may be estimated from the following table which gives some typical values of both the overall aeroplane deceleration and of the deceleration due to some of the usual contributory factors.

<i>Case</i>	$-\frac{\ddot{s}_{stop}}{g}$	<i>Notes</i>
Typical deceleration on dry runway	0.3 to 0.4	Derivation 4 suggests average value of 0.37 based on study of 15 transport aeroplanes
Typical deceleration on wet runway	0.2 to 0.3	
Typical "worst case" deceleration	0.1	<i>e.g.</i> icy runway or no anti-skid } plus aerodynamic drag only
Typical "brakes-alone" value for project work (dry runway)	0.3	see Item No. 71026
Aerodynamic drag with take-off flap	0.02 to 0.03	higher values can be obtained by use of lift dumpers, air brakes (see Item No. 70015), parachutes, propellers idling in ground fine pitch (see Item No. Perf ED1/1)
Reverse thrust	0.02 to 0.04 per engine	for braking thrust of propellers, see Item No. Perf ED1/2

6. NOTES ON THE CORRELATIONS

The Figures in this Item take into account the principal factors affecting take-off performance and it is found that about 95 per cent of the field-length data used lie within ± 8 per cent of the values obtained from the curves in Figures 1 to 3. Details of the data used are given in the following sections.

6.1 Field-Length Data

The field-length data used here were extracted from the flight manuals of multi-engined transport aeroplanes. The data presented in a flight manual are usually generated by computation following the certification trials of the aeroplane and, as such, represent the performance of the aeroplane as agreed by the Manufacturer and Certification Authority. In consequence, these data are likely to have been influenced by such factors as

- (i) the extent and repeatability of the flight-test results and the degree to which the computed take-off performance can be correlated with the measured values (it is unlikely that, for any given aeroplane, the take-off field-length performance can be consistently calculated to within ± 2 or 3 per cent of the measured values although it is to be expected that the calculated performance will always be within ± 5 per cent of measured values).
- (ii) differences in national airworthiness requirements,
- (iii) special features in the handling and performance characteristics of the aeroplane.

The following table gives a guide to the number and types of aeroplanes for which data were analysed in the preparation of this Item.

Engine type	Number of types of aeroplane	Approximate range of values of aeroplane	
		weight, lbf	mass, kg
Turbo-jet	3	10 000 to 400 000	4 000 to 180 000
Low bypass ratio turbo-fan	14	40 000 to 350 000	18 000 to 160 000
Medium to high bypass ratio turbo-fan	7	10 000 to 800 000	4 000 to 360 000
Turbo-prop and piston	12	6 000 to 180 000	3 000 to 80 000

6.2 Take-Off Safety Speed Data

The take-off safety speed or the corresponding lift coefficient is used for the correlation of all field-length data in this Item because of the ease with which values of V_2 can be obtained from aeroplane flight manuals. However, as noted in Section 4, in the all-engines-operating case V_{screen} is usually greater than V_2 . The effect of this anomaly on the correlation of all-engines-operating distance is small and, to some extent, overcome by the use of second segment climb gradient as a secondary correlating parameter, since both the speed increment ($V_{screen} - V_2$) and $\tan \gamma_{c_{seg2}}$ are functions of the single-engine thrust-to-weight ratio, f_N/W .

The criteria that determine the minimum value of V_2 for any particular aeroplane are stated in the appropriate Airworthiness Requirements; for initial project calculations it is usual to assume that

$$V_2 = 1.2V_{stall} \quad (6.1)$$

$$\text{or} \quad C_{L_2} = \frac{C_{L_{stall}}}{1.44} \quad (6.2)$$

However, the definitive value of V_2 may be governed by any of a number of factors, for example the minimum directional control speed in the air, see References 8 and 9.

6.3 Engine Thrust Data

In most cases the engine thrust data used in calculating values of the correlating parameters were the appropriate *installed* values for the engine type and model concerned. Where installed-thrust data were not available, uninstalled values were used with an estimated reduction for installation effects and oftakes. Similarly, thrust data for some propeller-driven aeroplanes were calculated using the appropriate engine power and propeller efficiency curves with either given or estimated factors for slipstream and body effects.

In calculating values of the “basic” and “modified” parameters the total thrust, $F_{N_{0.7}}$, was taken as the sum of individual engine thrusts. Differences in the magnitude of f_N were taken into account in such cases as the centre and outboard engines on three-engined aeroplanes but differences in the thrust lines of individual engines were not considered. In addition, no account was taken of the angle, ϵ , between the total thrust vector and the flight path in the airborne phase of the take-off (but see also Section 6.4). The data used in constructing this Item relate to aeroplanes for which ϵ is unlikely to be greater than about 15 degrees.

When using the Item the user can in general assume that the total engine thrust is nf_N or $(n-1)f_N$ as appropriate – this is the notation adopted in Figures 1 to 4. Values of f_N can be estimated using References 10 and 11 and values of propeller thrust using Reference 12.