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Generalisation of smooth continuous stress-strain curves for metallic materials

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GENERALISATION OF SMOOTH CONTINUOUS STRESS-STRAIN CURVES FOR METALLIC MATERIALS

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GENERALISATION OF SMOOTH CONTINUOUS STRESS-STRAIN CURVES FOR METALLIC MATERIALS

1. INTRODUCTION

This Item describes a generalised method for representing the stress-strain curves of materials that exhibit a smooth curve. The method uses a reference stress and a characteristic index, in addition to the modulus of elasticity, and this Item indicates how these two quantities may be determined. The Item is valid for both tension and compression and provides a convenient method of accounting for the characteristics of materials beyond the linear region of the stress-strain curve.

2. NOTATION

c_n	0.n cent proof stress in compression	N/m^2	lbf/in^2
E	modulus of elasticity (which may be different in tension and compression)	N/m^2	lbf/in^2
E_s	secant modulus	N/m^2	lbf/in^2
E_t	tangent modulus	N/m^2	lbf/in^2
f	stress	N/m^2	lbf/in^2
f_n	stress at which $E_t = ½E$	N/m^2	lbf/in^2
f_R	reference stress	N/m^2	lbf/in^2
f_R'	second reference stress chosen suc1 h that $f_R > f_R'$	N/m^2	lbf/in^2
f_t	tensile strength	N/m^2	lbf/in^2
j , k	see Equations (6.1) and (5.1) respectively		
m	material characteristic (see Equation (3.1))		
t_n	0.n per cent proof stress in tension	N/m^2	lbf/in^2
β , γ	see Equations (5.1) and (6.1), respectively		
ε	total strain		
ε_R	non-proportional strain corresponding to stress f_R (see Section 4)		
ε_R'	non-proportional strain corresponding to stress f_R' (see Section 4)		
ν	Poisson's ratio		

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ν_e fully elastic value of Poisson's ratio

ν_p fully plastic value of Poisson's ratio

Both SI and British units are quoted but any coherent system of units may be used.

3. GENERALISED EQUATIONS FOR STRESS-STRAIN CURVES

Derivation 2 shows that a smooth continuous stress-strain curve can be represented by the equation

$$\frac{\varepsilon E}{f_n} = \frac{f}{f_n} + \frac{1}{m}\left[\frac{f}{f_n}\right]^m \tag{3.1}$$

in which f_n is taken to be the stress at which the tangent modulus is one half of the modulus of elasticity and the index m has been found to characterise the shape of the stress-strain curve and is therefore referred to as the material characteristic. Equation (3.1) only applies if f/f_n is positive and this needs to be taken into account in cases where the chosen sign convention leads to either f or ε being of opposite sign to f_n. Figure 1 is a graph of Equation (3.1) plotting f/f_n against $\varepsilon E/f_n$ for a number of values of m. For methods of calculating f corresponding to a given value of strain see Section 5.

Equation (3.1) can be rearranged and the value of $E_t (= \mathrm{d}f/\mathrm{d}\varepsilon)$ obtained as

$$E_t = E\left[1 + \left[\frac{f}{f_n}\right]^{m-1}\right]^{-1}. \tag{3.2}$$

This can then be rearranged to give

$$\frac{E}{f_n}\frac{f}{E_t} = \frac{f}{f_n} + \left[\frac{f}{f_n}\right]^m. \tag{3.3}$$

Equations (3.3) and (3.2) are presented graphically in Figures 2 and 3 respectively. For methods of calculating f corresponding to given values of f/E_t see Section 6.

The secant modulus, E_s, is the ratio of stress to total strain and may be determined directly from Equation (3.1).

$$E_s = E\left[1 + \frac{1}{m}\left[\frac{f}{f_n}\right]^{m-1}\right]^{-1}. \tag{3.4}$$

From Equations (3.2) and (3.4) the simple relationship between E_s and E_t follows.

$$\left[\frac{E}{E_s} - 1\right] = \frac{1}{m}\left[\frac{E}{E_t} - 1\right]. \tag{3.5}$$

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Figure 4 gives the variation in Poisson's ratio with increasing stress for a range of values of m and three values of ν_e, the elastic value of Poisson's ratio, and is based on the equation

$$\nu = \nu_p - (E_s/E)(\nu_p - \nu_e) = \frac{\nu_e + \nu_p \frac{1}{m}\left[\frac{f}{f_n}\right]^{m-1}}{1 + \frac{1}{m}\left[\frac{f}{f_n}\right]^{m-1}} . \tag{3.6}$$

As the material yields, Poisson's ratio will increase from its elastic value up to its fully plastic value. The plastic value of Poisson's ratio, ν_p, is assumed to be 0.5 in all cases. Values of Poisson's ratio developing from elastic values of 0.25, 0.3 and 0.33 are presented in Figure 4.

4. DETERMINATION OF *m* AND f_n

The value of m, the material characteristic, can be found by standard fitting procedures if the full stress-strain curve is known. However, it may be estimated, provided that at least two points on the non-proportional region of the curve are known, from the equation

$$m = \frac{\log(\varepsilon_R/\varepsilon_R')}{\log(f_R/f_R')} , \tag{4.1}$$

where f_R and f_R' are two known stress values on the stress-strain curve of the actual material and ε_R and ε_R' are the strains in excess of the elastic strain corresponding to these stresses. For example, if f_R' and f_R are t_1 and t_2, the 0.1 per cent and 0.2 per cent tensile proof stresses of the material, then ε_R' and ε_R are 0.001 and 0.002 respectively. The strains will be referred to as non-proportional strains as they correspond to the strains produced in excess of recoverable elastic strain. A nomogram of Equation (4.1) is given in Figure 5.

For a known value of m the reference stress, f_n, may be evaluated from the equation

$$f_n = f_R\left[\frac{m\varepsilon_R E}{f_R}\right]^{-1/(m-1)} \tag{4.2}$$

or from the nomogram of this expression in Figure 6.

The accuracy of results obtained from the generalised curves depends upon how close the reference stresses used to determine the curves are to the stresses and strains for which data are required. The highest accuracy is obtained when the stress value for which data are required lies between f_R' and f_R. Variation in modulus causes only small changes in f_n. For example, if $m = 10$ variations in modulus of ± 20 per cent cause changes in f_n of less than $\pm 2½$ per cent. The generalised stress-strain equation used in this Item may be applied to either tension or compression provided that the material has a smooth stress-strain curve, which can be represented by Equation (3.1), and that the three required values, E, f_R, and f_R', are known for the appropriate load direction.

Proof stresses for use in design are usually available as specified minima or as design allowables determined to a clearly defined statistical basis. The associated values of moduli are usually average values. Thus,

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calculation of f_n and m using a pair of proof stresses (to the same basis) coupled with an average value of modulus is not strictly a coherent calculation. Nevertheless, such a calculation produces values that give a stress-strain curve that passes through both proof stresses, approaches the average modulus in the linear region and is thus consistent with normal stressing procedures. Section 8 provides example calculations employing data given in Reference 4 for one particular specification material.

Compressive generalised stress-strain curves are of particular use in buckling calculations. Table 9.1 lists values of m and f_n obtained using the available compressive data currently in Reference 4. These values are all calculated using the 0.1 and 0.2 per cent proof stresses, to the basis detailed in the table, together with the average value of the compressive modulus. The accuracy of the derived stress-strain curves may therefore be expected to be highest in the region between these two proof stresses. Table 9.1 illustrates that there are significant differences among the values of m and f_n for a material of given composition and heat treatment. It is therefore advisable to obtain the latest data relevant to the particular material specification if accurate values are required. For example, Reference 4 should be consulted for the latest information on British and European specification aircraft materials.

5. APPROXIMATE CALCULATION OF STRESS FROM STRAIN

Equation (3.1) cannot be solved algebraically when it is required to determine the stress corresponding to a given strain. In such cases Figure 1 may be used to estimate the stress. Alternatively, the following equation may be used to obtain an approximate value of f.

$$\frac{f}{f_n} = \frac{\varepsilon E}{f_n}\left[1 + \beta\left(\left|\frac{\varepsilon E}{f_n}\right|\right)^k + \frac{1}{m}\left(\left|\frac{\varepsilon E}{f_n}\right|\right)^{m-1}\right]^{-1/m}, \tag{5.1}$$

where $\beta = (1 + 1/m)^{(m-1-k)} - (1 + 1/m)^{-k}$

and $k = 0.790\ 44m - 0.869\ 77$.

Equations (5.1) and (3.1) satisfy the following statements.

a) $E = f/\varepsilon$ at low values of f or ε.

b) The equations have identical asymptotes at $\varepsilon(E/f_n) \to \infty$

c) The equations have identical values at $f/f_n = 1.0$.

d) The slopes given by both equations are identical at $f/f_n = 1.0$.

Provided that $m \geq 3$, values of f/f_n estimated using Equation (5.1) are within +1.8 per cent and –3.2 per cent of their true value. In the range $0 \leq \varepsilon E/f_n \leq 5$ and with $m \geq 3$ values of f/f_n are within ±1.8 per cent of their true value.

Approximate values of f/f_n obtained from Equation (5.1) may be checked by substitution into Equation (3.1). If the value requires refining an iterative method, such as the Newton-Raphson method, can be used and a single correction will achieve sufficient accuracy for most practical purposes. Example 8.5 provides an example calculation.

ESDUpac A7616 provides a Fortran subroutine for the calculation of f corresponding to a given strain to high accuracy. The subroutine has been tested over the region $2 \leq m \leq 10\ 000$ and $-100 \leq \varepsilon E/f_n \leq +100$. Appendix A details the subroutine input and output.

Approximate values of f corresponding to a given value of ε should not be used in Equation (3.2) to calculate the corresponding value of E_t. This is because small errors in f produce significant errors in E_t. The method set out in Section 6 is recommended for estimates of E_t.

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6. APPROXIMATE CALCULATION OF STRESS CORRESPONDING TO A VALUE OF f/E_t

Equation (3.3) cannot be solved algebraically when it is required to determine the stress corresponding to a given value of f/E_t. In such cases Figure 2 may be used to estimate the stress. Alternatively, the following equation may be used to obtain an approximate value of f.

$$\frac{f}{f_n} = \frac{f}{E_t}\frac{E}{f_n}\left[1 + \gamma\left(\left|\frac{f}{E_t}\frac{E}{f_n}\right|\right)^j + \left(\left|\frac{f}{E_t}\frac{E}{f_n}\right|\right)^{m-1}\right]^{-1/m},$$

where $\gamma = 2^{(m-1-j)} - 2^{-j}$ (6.1)

and $j = 0.983\ 42m - 2.357\ 48$.

Equations (6.1) and (3.3) satisfy the following statements.

a) $E_t = E$ at low values of f.

b) The equations have identical asymptotes at $(f/E_t)(E/f_n) \to \infty$

c) The equations have identical values at $f/f_n = 1.0$.

d) The slopes given by both equations are identical at $f/f_n = 1.0$.

Provided that $m \geq 5$, values of f/f_n estimated using Equation (6.1) are within ± 3 per cent of the true value.

Approximate values of f/f_n obtained from Equation (6.1) may be checked by substitution into Equation (3.3). If the value requires refining an iterative method, such as the Newton-Raphson method, can be used and a single correction will achieve sufficient accuracy for most practical purposes. Example 8.6 provides an example calculation.

ESDUpac A7616 provides a Fortran subroutine for the calculation of f corresponding to a given f/E_t to high accuracy. The subroutine has been tested over the region $2 \leq m \leq 10\ 000$ and $-100 \leq (f/E_t)(E/f_n) \leq +100$. Appendix A details the subroutine input and output.

7. DERIVATION AND REFERENCES

7.1 Derivation

This section lists selected sources that have assisted in the preparation of this Item

.

1. RAMBERG, W. OSGOOD, W.R. — Description of stress-strain curves by three parameters. NACA tech. Note 902, April 1943.

2. BARRATT, A.J. MICHAEL, M.E. — Generalised stress-strain data for aluminium alloys and certain other materials. *Jl r. aero. Soc.*, Vol. 59, No. 530, pp. 152-158, February 1955.

3. GERARD, G. BECKER, H. — Handbook of structural stability. Part 1 – Buckling of flat plates. NACA tech. Note 3781, January 1959.

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References

This section gives recommended sources of information supplementary to that in this Item.

4.	ESDU	Metallic materials data handbook. DEF STAN 00-932. ESDU, London, 1990.
5.	USA DoD	Military standardisation handbook. Metallic materials and elements for aerospace vehicle structures. MIL-HDBK-5. USA Department of Defence, Washington.
6.	VAN KATWIJK, K. GRAYLEY, M.E.	Generalised stress-strain curves. Estimation of stress corresponding to strain or to an f/E_t ratio. ESDU Memorandum No. 78, August 1991.

8. EXAMPLES

8.1 It is required to find the equation for the compressive stress-strain curve for loading transverse to the grain direction for aluminium alloy clad sheet to British Standard BS 3L 73.

Reference 4 provides the following values of compressive proof stress*, modulus and Poisson's ratio for BS 3L 73 loaded transverse to the grain direction.

$$c_1 = 356 \text{ MN/m}^2,$$
$$c_2 = 370 \text{ MN/m}^2,$$
$$\nu_e = 0.33$$

and $$E = 68 \times 10^3 \text{ MN/m}^2 \text{ (in compression).}$$

The value of m may be calculated using Equation (4.1) or the nomogram of Figure 5. From the notation:

$$f_R = 370 \text{ MN/m}^2, \qquad f_R' = 356 \text{ MN/m}^2,$$
$$\varepsilon_R = 0.002, \qquad \varepsilon_R' = 0.001,$$

so that

$$\varepsilon_R/\varepsilon_R' = 2 \text{ and } f_R/f_R' = 1.039.$$

Using Figure 5, a straight line from 2 on the left-hand scale to 1.039 on the right-hand scale intersects the m scale at 18. The same result could be obtained using Equation (4.1). Then, using the determined value of m,

$$\frac{m\varepsilon_R E}{f_R} = \frac{18 \times 0.002 \times 68 \times 10^3}{370} = 6.62.$$

Using Figure 6, a straight line joining 6.62 on the left-hand scale to 18 on the diagonal scale intersects the f_n/f_R scale at 0.895. Hence

$$f_n = 0.895 \times 370 = 331 \text{ MN/m}^2.$$

* Compressive proof stresses to an A basis, see footnote to Table 9.1, will be used since they provide the most reliable values.

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